MICHIGAN STATE UNIVERSITY

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 863: Analysis of Stochastic Systems

Fall Semester, 2005

ECE 863 WEB SITE – `http://www.angel.msu.edu`

INSTRUCTOR'S CONTACT INFORMATION

Instructor: J.R. Deller, Jr. , Professor of Electrical and Computer Engineering

Office: 3209 Engineering Building
Phone: (517) 353-8840
Internet: deller@msu.edu www.egr.msu.edu/∼deller

Office Hours: MW 1:00–2:30 p.m. (in EB 3209)
Appt. by email: Send 2 suggested times that you could meet on MON. or FRI.– *24 hrs. notice, please.* Email response will confirm.

COURSE PREREQUISITES / COREQUISITES

- Math background typical of undergraduate major in engineering, math, physics, statistics
- One or more courses in signal and linear system theory typical of those taught in junior-year ECE curricula (e.g., at MSU: minimally ECE 366 (or 360), others that would be very useful include ECE 313, ECE 457, ECE 466, STT 441)

STUDY MATERIALS

Lecture notes: Posted on course web site.

Textbook: A. Leon-Garcia, *Probability and Random Processes for Electrical Engineers* (2nd edition), Addison-Wesley, 1994.

Supplementary reading: Excerpts from J.R. Deller, Jr., *Discrete-Time Signal Processing with Speech Processing Motivations*, in prep. for IEEE Press, 2005.

DSP software package: MATLAB. Occasional use in homework. Available on college network.

Supplementary references: Posted on course web site as available.

COURSE DESCRIPTION AND POLICIES ....

Topic Coverage and Emphasis

- What is covered in the course?
  - ECE 863 covers fundamental analytical techniques that are used to model and process random variables and random processes in many areas of engineering, science, and other quantitative disciplines. The emphasis of this course is on the use of stochastic techniques in modern signal processing and systems modeling. Lecture examples and homework will support this focus.
  - The topic coverage appears in the **Topic Outline, Related Lecture Slides and Related Reading** table on pages vi–xi of this syllabus.
  - The schedule of topic coverage, as well as a list of important (exam, holiday) dates, appear in the **Course Schedule and Important Dates** table on page xii of this syllabus.
  - Material discussed in class and related problems will be emphasized on exams.

Homework (HW)

- When and how are HW assignments made?

  HW assignments will be posted on the class web site approximately every week. New postings will always be announced in class.

- How is HW graded?
  - HW will *not* be collected and graded. Why?
    - The pervasive availability of solutions and the increased tendency for some students to copy one another's solutions (as opposed to working together, which is *encouraged* for those who find it helpful) has made it difficult to fairly grade HW.
    - HW scores rarely change students' grades, because those who work hard on HW (hence, get good HW scores), are generally those who do well on exams. (The converse is also true.)
  - You may keep a "HW Notebook" and hand it in near the end of the semester if you choose (date will be announced). This is *optional* – choosing not to hand in such a notebook will *not* reduce your course grade. Effort exhibited in this book, as well as interactions with students concerning HW problems, will be considered in assigning course grades for those who are on "borderlines" between grades.
  - Four "midterms" plus the Final Exam will provide ample data with which to assess performance.
  - Examinations will include problems that strongly resemble the HW problems and examples worked in the lecture notes and in class, *including* MATLAB *exercises.*

- Is it important to do the HW assignments?
  - You will not succeed in this course unless you diligently work the assigned problems. These problems force you to struggle with the concepts, and to learn the important analytical tricks necessary to solve exam problems. It is important to *try the problems and exercises as soon as possible after the relevant class.* Doing so will strongly reinforce the new concepts.
  - Examinations will include problems that strongly resemble the HW problems and examples worked in the lecture notes and in class.

- Are solutions to HW problems provided?

  HW will be discussed in and outside of class to the extent that students have questions, and to the extent that certain problems might be particularly illustrative of some points. Solutions will be posted on the class web site approximately one week after problems are assigned.

Examinations

- How many exams will there be, and when are they scheduled?

  There will be four "Midterm" Exams, and a one-hour Final Exam. Dates for the five exams are shown on the **Examination and Holiday Schedule** included on page xii of this syllabus. Midterms 1, 2, 3, and 4 cover roughly the first, second, third, and fourth quarters of the course. The Final Exam covers the same material as Midterms 1, 2, and 3. (Note that Midterm 4 and the Final Exam will both be given during the university-scheduled final exam period.) The Final is *optional.* It is entirely your decision whether you want to take it. Although the material might have to be adjusted, the *exam dates will almost certainly not change*, so you can plan accordingly.

- Will the lowest midterm score be "dropped" (excluded from the final grade computation)?
  - Sort of. The Final Exam score will replace the lowest of the scores on Midterms 1, 2, and 3. However, this will happen even if the Final Exam score is *lower* than the lowest midterm score (i.e., if you take the Final Exam, you must study for it and take it seriously).
  - The only exception is if the lowest midterm score is 0% because of an unexcused absence. In that case, the Final Exam score will replace the second-lowest midterm score (of Midterms 1,2,3) and the 0% will remain in the computation.

- Are sample exams available?

  Sample exams from previous semesters will be posted on the class web site. It is recommended that these be used as an indicator of important topics, and to get a sense for the style of the exams. Using old exams as an exclusive study strategy is generally not effective.

- What if I miss an exam?

  A student may take a "makeup" exam only if a legitimate case of illness or personal emergency arises which is documented by a physician or other appropriate official. A student who finds it necessary to miss a midterm should contact the professor *before* the exam, if at all possible, to explain the circumstances. The Final Exam will serve as the "Makeup Exam" for students with an *excused* absence from Midterm 1, 2, or 3. (Merely "skipping" a midterm does not guarantee that the Final will be considered a "Makeup.") Students who miss more than one midterm are usually required to drop the course because such absences are indicative of a chronic health or other personal problem which makes it impossible to stay current with the material.

- How do I decide whether to take the "optional" Final Exam?
  - To maximize learning, it is useful to study for the Final Exam. The option to omit the Final is recommended only for persons whose scores are very high (3.5, 4.0) on the three midterms. However, you can make some simple calculations based on your existing QPA scores to predict your final course grade with and without the Final, then decide.
  - If you missed Midterm 1, 2, or 3, and you have been given permission to take the Makeup Exam, you must take the Final Exam because this will serve as the Makeup. Failure to do so will result in a 0% on the missed midterm, and a 0% on the Final Exam. In this case, your course grade will be based on three Midterm scores and the Final Exam score.
  - If you missed Midterm 1, 2, or 3, and you have been *denied* permission to take the Makeup Exam, you should take the Final Exam because you will have already have 0% score on the missed Midterm. Failure to do so will result in a 0% on the missed midterm, and a 0% on the Final Exam.

Grading

- Are "INCOMPLETES" possible? How about "DEFERRED" grades?
  - INCOMPLETE (I) grades will be given only in unusual cases of illness or other personal emergency which causes the student to miss a significant amount of the course. The instructor must concur that the missed work can be completed in a reasonable amount of time. This grade will *not* be given for any other reason.
  - There is no appropriate reason for the DEFERRED grade (DF) in this course.
  - A student who misses more than one examination will generally be required to repeat the course.
  - A student who misses the Final Exam (this includes Midterm 4 which is given in the same period) without satisfactory explanation will receive a failing grade in the course. This is university policy.

- How is the final grade determined? How will I know where I stand in the course after Midterm 1, 2, etc.?

  Grades will be determined as follows:

4 Midterm Exams*	100 points each
Final Exam	100 points
TOTAL possible points*	400

  *Final Exam score replaces lowest of MT 1,2,3 scores if the Final is taken

  On each exam you will receive a "report card" (or "quality point average" (QPA)) grade: $4.0, 3.5, 3.0, \ldots, 0.0$ [sometimes a finer quantization used], as well as a usual point (percentage) score. At the end of the semester, the professor will determine a grade for each person based on a curve of the total 400 points. A second grade will also be based on your individual "QPA" scores. Let

$$
\begin{aligned}
Q_{\mathrm{mt1}} &= \text{“QPA” grade on Midterm 1} \\
Q_{\mathrm{mt2}} &= \text{“QPA” grade on Midterm 2} \\
Q_{\mathrm{mt3}} &= \text{“QPA” grade on Midterm 3} \\
Q_{\mathrm{mt4}} &= \text{“QPA” grade on Midterm 4} \\
Q_{\mathrm{fin}} &= \text{“QPA” grade on the Final Exam} \\
G &= \text{grade determined in the conventional way using percentage scores}
\end{aligned}
$$

  Assuming that you opt to take the Final Exam, your final course grade will be

$$\max\left\{G, \left[\frac{100\,Q_{\mathrm{mt1}} + 100\,Q_{\mathrm{mt2}} + 100\,Q_{\mathrm{mt3}} + 100\,Q_{\mathrm{mt4}} + 100\,Q_{\mathrm{fin}} - 100\min\{Q_{\mathrm{mt1}}, Q_{\mathrm{mt2}}, Q_{\mathrm{mt3}}\}}{400}\right]\right\}.$$

  where, $[\,\cdot\,]$ indicates rounding to the nearest half integer. If you legitimately choose not to take the Final Exam, then your grade will be

$$\max\left\{G, \left[\frac{100\,Q_{\mathrm{mt1}} + 100\,Q_{\mathrm{mt2}} + 100\,Q_{\mathrm{mt3}} + 100\,Q_{\mathrm{mt4}}}{400}\right]\right\}.$$

  See Fig. 1 for an example grade computation.

Figure 1: EXAMPLE COURSE GRADE COMPUTATION

Note: This example was made up for an undergraduate class which uses the same grading system. We generally do not expect such low grades in graduate courses. Just accept this example as illustrative of the algorithm, not as reflective of typical scores and grades in ECE 863.

André-Marie "Andy" Amperè receives the following "percentage" scores on the exams: MT1 40 (1.0), MT2 43 (1.5), MT3 78 (3.25), MT4 71 (3.0) Final 80 (3.5). Andy's original MT1 score of 40 (1.0) is replaced by the Final Exam score 80 (3.5). The test scores correspond to the "QPA" scores shown in parentheses in each case.

Method 1 ("Subjective"). At the end of the semester, Andy's "percentage" scores are averaged to give a course percentage of

$$\frac{80+43+78+71}{400} \times 100\% = 68.0\% \to 2.5$$

The class average for this aggregate measure is 73%. The instructors decide that Andy's result is a 2.5 for the course after considering the overall class performance.

Method 2 ("Objective"). Then Andy's "QPA" scores are averaged

$$\frac{(3.5 \times 100) + (1.5 \times 100) + (3.25 \times 100) + (3.0 \times 100)}{400} = 2.81 \text{ round to } \to 3.0$$

The Verdict. Andy receives a 3.0 in ECE 863.

ECE 863 — Fall Semester, 2005

TOPIC OUTLINE, RELATED LECTURE SLIDES, and RELATED READING

Updated August 31, 2005

0: INTRODUCTION TO THE COURSE (Lec. Slides: 1-5)

I: INTRO TO PROBABILITY THEORY AND RANDOM VARIABLES

A. Random Phenomena (Lec. Slide: 6; Read: Deller-Sec. 8.1, LG-Ch. 1)

B. Probability Spaces

1. Sample spaces (Lec. Slide: 7; Read: Deller-Sec. 8.2.1, LG-Sec. 2.1)
2. Event spaces (Lec. Slides: 8-10; Read: Deller-Sec. 8.2.1, LG-Sec. 2.1)
3. Probability measures
    - a. Concept (Lec. Slide: 11; Read: Deller-Sec. 8.2.1, LG-Secs. 2.2-2.3)
    - b. Mass analogy (Lec. Slides: 12-13; Read: Deller-Sec. 8.2.1)
    - c. Properties of prob. measures (example proofs) (Lec. Slide: 14; Read: Deller-Sec. 8.2.1)
    - d. Joint prob., cond'l prob., independence (Lec. Slides: 15-15.05; Read: Deller-Sec. 8.2.1, LG-Secs. 2.4-2.5)
    - e. Classic example: Binary channel (Lec. Slides: 15.1-15.5; Read: Deller-Ex. 8.7)
4. APPENDIX: Set Theory Review (7 unnumbered slides)

C. Combined Probability Spaces

1. Concepts (Lec. Slide: 17; Read: Deller-Sec. 8.2.1, LG-Sec. 2.6)
2. Classic example: Errors in bitstream – Introduce binomial (Lec. Slides: 17-18; Read: Deller-Sec. 8.2.1, Ex. 8.9)

D. Review Points to Date (Lec. Slide: 20)

E. Random Variables (r.v.'s)

1. Concept (Lec. Slides: 21-25; Read: Deller-Secs. 8.3.1-8.3.2, LG-Sec. 3.1)
2. Discrete, continuous, mixed types (Lec. Slides: 26-27; Read: Deller-Sec. 8.3.1, LG-Sec. 3.2)
3. Creation of a new prob. space by a r.v. (Lec. Slides: 28-29; Read: Deller-Sec. 8.3.2)
4. TABLE OF COMMON DISTRIBUTIONS FOR R.V.'S (Lec. Slide: 30; Read: Deller-Tbls. 8.6,8.7, LG-Tbls. 3.1,3.2)
5. Cumulative distribution function (Lec. Slide: 30; Read: Deller-Sec. 8.3.3, LG-Sec. 3.2)
    - a. Definition and analysis (Lec. Slide: 31; Read: Deller-Sec. 8.3.3, LG-Sec. 3.2)
    - b. Properties (Lec. Slides: 32-38; Read: Deller-Sec. 8.3.3, LG-Sec. 3.2)
6. Probability mass function (pmf) and density function (pdf)
    - a. pdf for a continuous r.v. (Lec. Slides: 39-44; Read: Deller-Sec. 8.3.3, LG-Sec. 3.3)
    - b. pmf for a discrete r.v. (Lec. Slide: 45; Read: Deller-Sec. 8.3.3, LG-Sec. 3.3)
    - c. pdf for a mixed or discrete r.v. (Lec. Slide: 46; Read: Deller-Sec. 8.3.3, LG-Sec. 3.3)
7. Moments of a r.v.
    - a. General (Lec. Slides: 47-48; Read: Deller-Sec. 8.3.4, LG-Secs. 3.6,4.7)
    - b. First- and second-order moments (Lec. Slides: 49-56; Read: Deller-Sec. 8.3.4, LG-Sec. 3.6)
    - c. Characterizing a pdf by moments (Lec. Slide: 57)
8. Characteristic function (ch.fcn.)
    - a. Definitions (Lec. Slide: 57.1; Read: LG-Sec. 3.9)
    - b. Properties (Lec. Slide: 57.2; Read: LG-Sec. 3.9)
    - c. Moment generating (MG) property & MG function (Lec. Slides: 57.2-57.3; Read: LG-Sec. 3.9)
    - d. Important theorem (Lec. Slide: 57.3; Read: LG-Sec. 3.9, p. 147)

e. MG property for the jt. ch. fcn. (Lec. Slide: 57.4; Read: LG-Sec. 3.9)
f. Probability generating function (Lec. Slide: 57.4; Read: LG-Sec. 3.9)
g. Examples (Lec. Slides: 57.4-57.6; Read: LG-Sec. 3.9)

F. Functions of a Random Variable

1. Introduction (Lec. Slides: 58-58.1; Read: LG-Sec. 3.5)
2. Transformations of distributions (Lec. Slides: 59-68; Read: LG-Sec. 3.5)
3. Expected value of a function of a r.v. (Lec. Slides: 69-72; Read: Deller-Ex. 8.3.4, LG-Sec. 3.6)
   a. Continuous r.v. (Lec. Slide: 69; Read: LG-Sec. 3.4)
   b. Discrete r.v. (Lec. Slides: 76-78; Read: Deller-Ex. 8.9, LG-Sec. 3.4)

G. Important Distribution Models for Discrete Random Variables

1. Bernoulli (Lec. Slides: 76-78; Read: Deller-Ex. 8.9, LG-Sec. 3.4)
2. Binomial (Lec. Slides: 79-84; Read: Deller-Ex. 8.9, LG-Sec. 3.4)
3. Geometric
   a. Definition and discussion (Lec. Slides: 85-86; Read: LG-Sec. 3.4)
   b. Memoryless property (Lec. Slides: 87-90; Read: LG-Sec. 3.4)
4. Poisson
   a. Definition and discussion (Lec. Slides: 91-98; Read: Deller-Ex. 8.13, LG-Sec. 3.4)
   b. Moments (Lec. Slides: 99-101; Read: Deller-Ex. 8.17, LG-Sec. 3.4)
   c. Extension to multiple time units (Lec. Slide: 102; Read: LG-Sec. 3.4)
   d. As a limiting case of the binomial (Lec. Slides: 103-106; Read: LG-Sec. 3.4)
   e. Interarrival time r.v. (Lec. Slide: 107; Read: LG-Sec. 3.4)

H. Important Distribution Models for Continuous Random Variables

1. Exponential
   a. Definition and discussion (Lec. Slides: 108-113; Read: LG-Sec. 3.4)
   b. Moments (Lec. Slide: 114; Read: LG-Sec. 3.4)
   c. Memoryless property (Lec. Slide: 115; Read: LG-Sec. 3.4)
   d. As a limiting case of the geometric r.v. (Lec. Slides: 117-118; Read: LG-Sec. 3.4)
   e. SUMMARY TABLE "Bernoulli Islands" (Lec. Slide: 119; Read: LG-Sec. 3.4)
2. Gaussian
   a. Definition, discussion, pdf (Lec. Slide: 120; Read: Deller-Ex. 8.15, LG-Sec. 3.4)
   b. cdf and "$Q(\ )$" function (Lec. Slide: 121; Read: Deller-Ex. 8.15, LG-Sec. 3.4)
   c. Example illustrated distributions (Lec. Slides: 122-124; Read: Deller-Ex. 8.15, LG-Sec. 3.4)
   d. Classic example: Using tabulations (Lec. Slides: 125-127.2; Read: Deller-Ex. 8.15, LG-Sec. 3.4)
3. Laplacian
   a. Definition and discussion (Lec. Slides: 128-129; Read: Deller-Ex. 8.16, LG-Sec. 3.4)
   b. Example: Speech amplitudes (Lec. Slides: 130-131; Read: Deller-Ex. 8.16, LG-Sec. 3.4)
4. Gamma
   a. Definition and discussion (Lec. Slides: 132-136; Read: LG-Sec. 3.4)
   b. Special cases (Lec. Slide: 137; Read: LG-Sec. 3.4)
5. Cauchy (Lec. Slide: 137; Read: LG-Sec. 3.4)
6. Rayleigh (Lec. Slides: 137-138; Read: Deller-Ex. 8.18, LG-Sec. 3.4)

J. Multiple Random Variables (Focus on two r.v.'s)

1. Introduction and definitions
2. Joint cdf and pdf
   a. Definition - joint cdf (Lec. Slide: 143; Read: LG-Sec. 4.2)
   b. Properties - joint cdf (Lec. Slides: 143-148; Read: LG-Sec. 4.2)
   c. Definition - joint pdf (Lec. Slide: 149; Read: LG-Sec. 4.2)
   d. Properties - joint pdf (Lec. Slide: 149; Read: LG-Sec. 4.2)
3. Conditional cdf and pdf
   a. Introduction and definitions (Lec. Slides: 149.1-151; Read: LG-Sec. 4.4)
   b. Properties (Lec. Slide: 151; Read: LG-Sec. 4.4)

c. Interval vs. point conditioning (Lec. Slides: 151-153; Read: LG-Sec. 4.4)

4. Statistical independence of random variables
   a. Definition and consequences (Lec. Slide: 154; Read: LG-Sec. 4.3)
   b. Linear (in)dependence (a preview) (Lec. Slide: 155)
5. Summary examples on multiple (two) random variables
   a. Revisiting the binary detection in Gaussian noise problem
      (Lec. Slides: 156-158; Read: Deller-Ex. 8.15)
   b. A good review example by Prof. Radha (Lec. Slides: 159-167)
6. Conditional expectation (Lec. Slide: 168; Read: LG-Sec. 4.4)

K. Functions of Two (or More) Random Variables (Selected Topics)

1. Sums of independent random variables (transformation of distribution)
   a. pdf of the sum variable (Lec. Slides: 169-171; Read: LG-Preview Sec. 5.1)
   b. A preliminary look at the Central Limit Theorem (Lec. Slide: 171) Preview Sec. 5.3
   c. Example: Difference variable between two Gaussian r.v.'s (Lec. Slide: 172)
2. Expectations of scalar functions of multiple random variables
   a. Definitions and discussion (Lec. Slides: 173-174; Read: LG-Sec. 4.7)
   b. Joint moments (Special expectations of scalar fcns. of two r.v.'s)
      (i) Definitions (Lec. Slide: 175; Read: LG-Sec. 4.7)
      (ii) Correlation and covariance (Special moments, two r.v.'s)
      (Lec. Slides: 175-176; Read: LG-Sec. 4.7)
      (iii) Interpretive example: MMSE estimation
      (Lec. Slides: 177-183; Read: LG-Preview Sec. 4.9)

L. Jointly-Gaussian (jt-G) Random Variables (Focus on two r.v.'s)

1. Definition and analysis
   a. Definition of jt-G random variables (Lec. Slides: 183-186; Read: LG-Sec. 4.8)
   b. Equal-density contours (Lec. Slides: 187-188; Read: LG-Sec. 4.8)
   c. Effects of variances and correlation coefficient (Lec. Slides: 189-197; Read: LG-Sec. 4.8)
   d. Circular symmetry (Lec. Slides: 198-208)
2. Properties of jt-G r.v.'s (Lec. Slides: 209-223) Sec. 4.8
3. Conditional pdf, Y conditioned by jt-G X (Lec. Slides: 224-225; Read: LG-Sec. 4.8)
4. Summary of main points about jt-G r.v.'s (Lec. Slide: 226)

M. Minimum Mean-Square-Error (MMSE) Estimation and Prediction

1. Unconstrained MMSE estimation (Lec. Slides: 227-232; Read: LG-Sec. 4.9)
2. Linear MMSE estimation
   a. Linear vs. affine (Lec. Slides: 233-241.3)
   b. Orthogonality principle and calculations of MSE (Lec. Slides: 242-246)

N. Further Topics in Multiple Random Variable Theory
(n random variables – Focus on random vector formulation)

1. Random vectors: Def'n (Lec. Slides: 247-248; Read: LG-pp. 217-20)
2. cdf and pdf (Lec. Slides: 248-250; Read: LG-pp. 217-20)
3. Random matrices
   (including correlation and covariance matrices) (Lec. Slides: 250-253)
4. Scalar functions of random vectors
   (esp., joint moments among r.vct. elements) (Lec. Slide: 254)
5. Characteristic functions (Lec. Slide: 255; Read: LG-pp. 235-36)
6. Revisiting the jt-G case (Lec. Slides: 256-258.1; Read: LG-pp. 240-46)
7. Statistical relations between r.vcts.
   (incl'g indep., cross-cov., cross-cor., etc.) (Lec. Slides: 259-261)
8. Generalizing the MMSE est'n / pred'n problem
   to multiple observations (Lec. Slides: 259-261; Read: LG-pp. 249-51 (sort of))
9. APPENDIX TO SECTION N. Notes on complex random variables (Lec. Slides: 267-269)

II: SUMS AND SEQUENCES OF RANDOM VARIABLES

A. Sums of r.v's

1. Sums of two r.v's – Review (Lec. Slides: 270-271; Read: LG-Example 4.31)
    a. pdf and ch. fcn.
    b. Moments
2. Sums of n r.v's (Lec. Slides: 272-278; Read: LG-Sec. 5.1 to p. 274)
    a. pdf and ch. fcn.
    b. Moments
3. Sums of a random number of r.v's (Lec. Slides: 279-282; Read: LG-pp. 274-5)

B. Laws of large numbers

1. Sample mean (Lec. Slides: 283-288; Read: LG-pp. 275-77)
    a. Definition
    b. Statistical characterization
    c. Convergence
        (i) Convergence in MS
        (ii) Convergence in probability
2. Weak law of large numbers (Lec. Slide: 288; Read: LG-pp. 277-8)
3. Strong law of large numbers (Lec. Slide: 289; Read: LG-pp. 278-9)
4. APPENDIX TO SECTION B:
    Summary of stochastic convergence types to date (Lec. Slides: 290-291; Read: LG-Sec. 5.5)

C. A more detailed study of the convergence of random sequences

1. Modes of convergence (Lec. Slides: 292-299; Read: LG-Sec. 5.5)
    a. Deterministic sequences (Review of Calculus 101)
    b. Random sequences
2. Relationships among convergence modes (Lec. Slide: 300, 291; Read: LG-Sec. 5.5)
3. Cauchy criteria for convergence (Lec. Slides: 301-303; Read: LG-Sec. 5.5, p. 300)
4. Further results on MS convergence (Lec. Slides: 303.1-303.6)

D. A more rigorous look at the laws of large numbers (Lec. Slides: 304-306)

E. Central limit theorem (Lec. Slide: 307; Read: LG-Sec. 5.3)

III: DISCRETE-TIME RANDOM PROCESSES

A. Definitions (Lec. Slides: 308-312)

B. Mean, covariance, independence

1. Intra-process concepts (Lec. Slides: 313-315)
2. Cross-process concepts (Lec. Slides: 315-317)

C. Stationarity

1. Intra-process concepts (Lec. Slides: 318-325)
2. Cross-process concepts (Lec. Slide: 326)
3. Properties of auto- and cross-correlation for WSS r.p's (Lec. Slides: 326-328)
4. Asymptotic stationarity (Lec. Slide: 328)

D. Example classes and instances of DTrp's

1. A generic class of DTrp's – IID r.p's
   a. General introduction to the class (Lec. Slides: 329.1-331.1; Read: LG-Sec. 6.3, p. 339)
   b. Example: Bernoulli process
      (Lec. Slide: 331.1 (below); Read: LG-Sec. 6.3, Examples 6.11,6.12)
2. A generic class of DTrp's – DTrp's created by summing IID r.v's
   a. General introduction to the class (Lec. Slides: 331.2-331.4; Read: LG-Sec. 6.3, pp. 341-2)
   b. Example: Binomial counting process
      (Lec. Slides: 332.1-331.3; Read: LG-Sec. 6.3, Example 6.13)
   c. General: Covariance analysis (Lec. Slides: 332.4-333.3; Read: LG-Sec. 6.3, pp. 344-5)
   d. General: Stat'y & indep. increments property
      (Lec. Slides: 333.4-335.4; Read: LG-Sec. 6.3, pp. 343-4)
   e. Example: Random walk process (Lec. Slides: 336.1-338.2; Read: LG-Sec. 6.3, Example 6.14)

IV: CONTINUOUS-TIME RANDOM PROCESSES

A. Definitions (Lec. Slides: 339-341; Read: LG-Secs. 6.1-6.2)

B. Mean, covariance, independence

1. Intra-process concepts (Lec. Slides: 341-343; Read: LG-Secs. 6.1-6.2)
2. Cross-process concepts (Lec. Slides: 344-345; Read: LG-Secs. 6.1-6.2)

C. Stationarity

1. Intra-process concepts (Lec. Slides: 345-346; Read: LG-Sec. 6.5)
2. Cross-process concepts (Lec. Slides: 346-347; Read: LG-Sec. 6.5)
3. Properties of auto- and cross-correlation for WSS r.p's
   (Lec. Slides: 347-348; Read: LG-Sec. 6.5)
4. Asymptotic stationarity (Lec. Slides: 348-349; Read: LG-Sec. 6.5)

D. Example classes and instances of CTrp's

1. Sinusoids with random parameters and other introductory examples
   (Lec. Slides: 349-358; Read: LG-Sec. 6.4)
2. Independent increment processes (Lec. Slides: 359-362; Read: LG-Sec. 6.4)
   a. General theory (Lec. Slides: 331.2-331.4; Read: LG-Sec. 6.4)
   b. Example: Poisson counting process (Lec. Slides: 363-369; Read: LG-Sec. 6.4)
   c. Example: Wiener process (Lec. Slides: 370-371; Read: LG-pp. 354-6)
3. Gaussian r.p. (Lec. Slides: 371-371.1; Read: LG-p. 362)

E. Power spectral density for a WSS r.p.

1. Single r.p.
   a. Theory (Lec. Slides: 372-375; Read: LG-Sec. 7.1 thru Example 7.5)
   b. Examples (Lec. Slides: 376-377; Read: LG-Examples 7.2–7.5)
2. Two r.p's (Lec. Slides: 378-379)

F. Stochastic calculus

1. What do calculus operations mean? (Lec. Slides: 380-381; Read: LG-Sec. 6.6)
2. Mean-square continuity (Lec. Slides: 381-387; Read: LG-Sec. 6.6)
3. Mean-square differentiation (Lec. Slides: 388-397; Read: LG-Sec. 6.6)

ECE 863 – Fall Semester, 2005: COURSE SCHEDULE and IMPORTANT DATES

TOPIC SCHEDULE

Week of Aug. 29	Topics 0, I-A thru I-C
Weeks of Sep. 5,12	Topics I-D thru I-H
Weeks of Sep. 19,26	Topics I-J thru I-N
Weeks of Oct. 3,10	Topic II (all)
Week of Oct. 17	Topic III (all)
Weeks of Oct. 24,31	Topics IV-A thru IV-E
Week of Nov. 7	Topics IV-F & G
Weeks of Nov. 14,21,28	Topic V (all)
Week of Dec. 5	TBD

EXAMINATION & HOLIDAY SCHEDULE

Monday, Sep. 5	*No class - Labor Day Holiday*
Monday, Sep. 26 (regular class period)	**Midterm 1**
Monday, Oct. 24 (regular class period)	**Midterm 2**
Tuesday, Nov. 22 (evening - time to be determined)	**Midterm 3**
Wednesday, Nov. 23	*No class - You took an exam last night!*
Friday, Nov. 25	*No class - Thanksgiving Holiday*
Thursday, Dec. 15 (10 a.m. – 12 noon)	**Midterm 4 & Final Exam**