ECE 360
HOMEWORK #3
Due September 25, 2002

- Read 4.5 and 6.1-6.5 from Ambardar.
- Office Hours: M,T 10:00-11:30 am, F 12:30-2:00 pm (for this week only)

1. a) Given an LTI system with the output given by
\[ y(t) = \int_{-\infty}^{t} e^{-2(t-\tau)} x(\tau - 1) d\tau \]
i) Find the impulse response of this system.
ii) Is this system causal?
iii) Is this system stable?

Note: To find the impulse response of a system given the input-output relationship, you need to plug in the impulse function as the input and then compute the corresponding output.

b) Given an LTI system with the output given by
\[ y(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)} x(\tau - 1) d\tau \]
i) Find the impulse response of this system.
ii) Is this system causal?
iii) Is this system stable?

2. 6.3 (a, c, e) from Ambardar

3. 6.6 (a) from Ambardar (Hint: Use properties of convolution, you can find the result without computing any convolution integrals.)

4. 6.40 (a,c,d) from Ambardar

5. Finding the impulse response of a first order system: Consider the first-order system \( y'(t) + ay(t) = x(t) \). There are two ways of finding the impulse response of this system:
   a) Find the response of the relaxed system to \( x(t) = u(t) \). This will be the step response of the system, and then take the derivative of this response to obtain the impulse response, \( h(t) \), of the system. (Recall that the impulse response is the derivative of the step response since delta function is the derivative of the unit step function.)
b) Solve the differential equation assuming zero initial conditions and \( x(t) = \delta(t) \). This is equivalent to solving the following differential equation:

\[
y'(t) + ay(t) = 0, \quad y(0) = 1
\]

It is equivalent to finding zero-input response with \( y(0)=1 \) as the initial condition. The change in the initial conditions is due to the application of impulse function at time zero. (Read pages 82-84 from the book for more examples.)

Using one of the approaches described above, solve 6.24 (a,b) for circuit 1.
Note: Circuit 1 is a RC lowpass filter.

6. Consider an LTI system with the input and output related by

\[
y(t) = \int_{0}^{t} e^{-\tau} x(t - \tau) d\tau
\]

a) Find the impulse response of the system.

b) Is this system causal?

c) Determine the response of the system, \( y(t) \), when \( x(t) = u(t + 1) \).

d) Consider the interconnections of the LTI systems given below, where \( h(t) \) is the function found in part (a). Find the impulse response of the whole system.

e) Solve for the response of the system of part (d) to the input of part c).

Hint: The output can be written by inspection.