1. Let $h$ be a quadrature mirror filter associated to a scaling function, $\phi(t)$. Prove that if $H(\omega)$ is $p$-regular then $\Phi^l(2\pi k) = 0$ for any integer $k \neq 0$ and $l < p$.

Note: $\Phi^l(2\pi k)$ refers to the $l$th derivative of the scaling function in the frequency domain.

2. Suppose that $h, \tilde{h}$ define a pair of perfect reconstruction filters in a biorthogonal system.
   a) Show that $H^*(\omega) \tilde{H}(\omega) + H^*(\omega + \pi) \tilde{H}(\omega + \pi) = 2$.
   b) Prove that $h_{new}[n] = \frac{1}{2}(h[n] + h[n-1]), \tilde{h}_{new}[n] = \frac{1}{2}(\tilde{h}[n] + \tilde{h}[n-1])$ defines a new pair of perfect reconstruction filters.
   c) The Deslauriers-Dubuc filters are $H(\omega) = 1$ and $\tilde{H}(\omega) = \frac{1}{16}(-e^{-3j\omega} + 9e^{-j\omega} + 16 + 9e^{j\omega} - e^{3j\omega})$. Compute $h_{new}$ and $\tilde{h}_{new}$ as well as the corresponding biorthogonal wavelets $\psi_{new}, \tilde{\psi}_{new}$.

3. Image Mosaic: Let $f_0[n_1, n_2]$ and $f_1[n_1, n_2]$ be two images of $N^2$ pixels. We want to merge the center of $f_0[n_1, n_2]$ for $N/4 \leq n_1, n_2 < 3N/4$ in the center of $f_1$. Compute the wavelet coefficients of $f_0$ and $f_1$. At each scale $j$, and orientation $1 \leq k \leq 3$, replace the wavelet coefficients corresponding to the center of $f_1$ by the wavelet coefficients of $f_0$. Do this for at least 3 levels using Daubechies wavelet with 3 vanishing moments. Reconstruct an image from this manipulated wavelet representation. Summarize your observations. You can use wavemenu for this experimentation. You need to download the two images from the course website.

4. Prove that the spectrogram is time and frequency shift invariant.

5. Compute the time-bandwidth product for the spectrogram of $x(t) = e^{j\omega_0 t}$, computed with the window function $h(t)$. Your answer will be in terms of $h(t)$ and $\omega_0$. 