Instructions

- This exam is due on Tuesday November 25, 2003 by 5:00 pm. Please hand in your exam to me, do not leave it under a door or in a mailbox.

- During the exam you may use the lecture notes, the internet, any textbook in the library, and any software package. However, please make sure that you give proper citation for material quoted.

- Do not discuss the exam with anyone until at least 24 hours past the end of the examination period. Any such discussions, even between students who have completed their exams, will be treated as violations of the honor code.

- There are four questions on this exam. Please answer all questions as thoroughly as you can. To accelerate grading, please write only on one side of your papers, and start each problem at the top of a new piece of paper. For full credit, cross out any incorrect intermediate steps. If you need to make any additional assumptions in solving a problem, state them clearly. Legible writing will help when it comes to partial credit.

- Office Hours: Monday 4:00-5:00 p.m., Tuesday 10:20-11:40 a.m. (There will be no lecture on Tuesday)

- When appropriate, I will post the frequently asked questions and answers on the website.
1. [30] Musical sounds and voiced speech segments can be modeled with sums of sinusoidal partials, \( f(t) = \sum_{k=1}^{K} A_k e^{j\omega_k t} u(t - t_k) \) where \( A_k \)'s may be complex and \( u(t) \) is the unit step function. You may assume that \( 0 \leq t_1 \leq t_2 \leq t_3 \ldots \leq t_k \) to simplify your computations.

a) [16] Design a kernel in the ambiguity domain such that when this kernel is applied to the signal the resulting time-frequency distribution does not have any cross-terms. (Delta functions are useful only in theory and not practical for implementations.) Hint: Think of filter design in the ambiguity domain. Be sure to specify all the properties of the filter (e.g. lowpass/highpass/bandpass, cut-off frequencies etc.).

Hint: The Fourier transform of \( u(t) \) is \( \pi \delta(\omega) + \frac{j}{\omega} \).

b) [8] Derive an expression for the time-frequency distribution, \( C(t, \omega) \), corresponding to the kernel found in part a), in terms of the Wigner distribution. Interpret your results with respect to the operations that are performed on the Wigner distribution.

c) [4] Explain in terms of the kernel function, \( \phi(\theta, \tau) \), whether it is possible to recover the signal from the time-frequency distribution.

d) [2] Does this distribution satisfy the marginals?

2. [15] Consider the following signal, \( s(t) = 0.4 e^{j10t} + e^{j20t} \).

a) [15] Find an expression for the instantaneous frequency of this signal. Plot your result for \( 0 < t < 5 \). Does this result match your intuition? Why or why not?

b) [10] (Extra Credit) Ideally you would like the instantaneous frequency to show you the frequencies that are present in the signal at the given time. Design a new way of computing instantaneous frequency such that instead of giving you a single frequency value at a given time, it will give you a vector of frequencies that are present at the given time. Design it specifically for sum of complex exponentials. Explain your method clearly.

Hint: Cohen defines local averages in terms of conditional distributions over all time or all frequency. Thus, the average frequency at a given time is defined as: \( < \omega >_t = \frac{\int_{\omega} w_P(t, \omega) d\omega}{P(t)} \).

We can talk about the average frequency within a region of the time-frequency surface as \( < \omega h(\omega - \omega_0) >_t \) where \( h \) is an even window function that is concentrated around the origin, e.g. Hamming, Hanning, rectangular, etc.

3. [25] S-method is a time-frequency representation based on the short-time Fourier transform defined as:

\[
SM(t, \omega) = \int P(\theta) S(t, \omega + \theta) S^*(t, \omega - \theta) d\theta
\]  

(1)

where \( S(t, \omega) = \int x(t + \tau) w(\tau) e^{-j\omega \tau} d\tau \), where \( w(\tau) \) is a real-valued even window.

a) [5] For what type of functions \( P(\theta) \), will you get the spectrogram?

b) [10] What type of distribution do you obtain when \( P(\theta) = 1 \)?

c) [5] Does the S-method belong to the general class of distributions defined by Cohen? Justify your answer.
d) [5] Discuss the advantages of this distribution compared to the Wigner distribution in terms of cross-terms.

4. [30] a) [10] Show that the spectrogram is not a scale invariant distribution.
b) [5] State a necessary condition on the window function to make the spectrogram scale covariant.
c) [5] Find a window function that satisfies the condition in part (b).
d) [10] Using the window function in part c), compute the spectrogram of a gabor logon function (time shifted and frequency modulated Gaussian function) and its scaled version. Include plots of the spectrogram for both cases and discuss your results. (The two signals are saved in logons.mat and can be downloaded from the website.)

Hints: You can define your own window in MATLAB and then use it in 'specgram' command. Make 'NOVERLAP' equal to window length minus one.