ECE 202 EXAM 3
April 13, 2007

- No textbooks, notes or HW solutions.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [25] Answer the following questions briefly.
   a) For a circuit with impulse response \( h(t) = e^{-2t}u(t) \), find the output to the input \( x(t) = u(t) \) using the convolution integral.

      \[
      y(t) = \int_{0}^{t} e^{-2(t-\tau)} d\tau = e^{-2t} \int_{0}^{t} e^{2\tau} d\tau
      \]

      \[
      = e^{-2t} \left[ \frac{e^{2\tau}}{2} \right]_{0}^{t} = e^{-2t} \left[ \frac{e^{2t}}{2} - \frac{1}{2} \right] = \left[ \frac{1}{2} - \frac{1}{2} e^{-2t} \right] u(t)
      \]

   b) For a RLC bandpass circuit, which element would you adjust (and by how much) to double the bandwidth without changing the center frequency?

      \[
      \omega_0 = \frac{1}{\sqrt{LC}}
      \]

      \[
      BW = \frac{\omega_0}{\theta_0} \Rightarrow \text{double BW} \Rightarrow \text{half} \quad \frac{Q_0}{\omega_0} = \frac{1}{R} \sqrt{\frac{L}{C}}
      \]

      \[
      \boxed{\text{Double } R}
      \]

   c) For a circuit with transfer function, \( T(s) = \frac{2000(s + 200)}{(s + 1000)(s - 400)} \), determine whether the circuit is stable or not. Explain why.

      \[
      \text{poles at } -1000 \text{ and } 400
      \]

      \[
      \text{not stable since not all of the poles are in the LHP.}
      \]
d) A voltage source of \( v(t) = 10 \cos(100t) \) is applied to a lowpass filter with a transfer function \( T(s) = \frac{100}{s+10} \). Predict the output voltage \( v_o(t) \).

Hint: Use Bode diagrams to estimate the gain and the phase at the input frequency.

\[
\frac{100}{j\omega+10} = \frac{100}{10} \frac{1}{1+j\frac{\omega}{10}} = 10 \left( 1 + j\omega \right)^{-20}
\]

\[
20 \log_{10} 10 = 20 \text{dB}
\]

Gain at \( \omega = 100 \rightarrow 0 \text{dB} = 1
\]

Phase at \( \omega = 100 \rightarrow -90^\circ
\]

\[
v_o(t) = 10 \cos(100t - 90^\circ)
\]

\[
e) \text{ The first three terms in the Fourier series expansion of a periodic signal is } f(t) = \frac{8}{\pi^2} \cos(100\pi t) + \frac{8}{9\pi^2} \cos(300\pi t) + \frac{8}{25\pi^2} \cos(500\pi t) + \ldots \text{. Determine the fundamental frequency in Hz.}
\]

\[
100\pi = 2\pi f_0
\]

\[
f_0 \approx 50 \text{ Hz}
\]
2. Given the following inverting amplifier:
   a) Find the transfer function, \( T(s) \).
   b) Find the output voltage, \( v_2(t) \), when the input is \( v_1(t) = u(t) \).

\[
\begin{align*}
T(s) &= -\frac{2}{2} = -\frac{100K \cdot \frac{1}{s \cdot 20 \times 10^{-9}}}{50K} \\
&= -\frac{100 \times 10^3}{s \cdot 20 \times 10^{-9}} \\
&= -\frac{100 \times 10^3}{s \cdot 2 \times 10^{-3}} \\
&= -\frac{2}{1 + s \cdot 2 \times 10^{-3}} \\
&= -\frac{1000}{s + 500}
\end{align*}
\]

b) \( v_Z(s) = \left(\frac{1}{s}\right) \left(\frac{-1000}{s + 500}\right) = \frac{k_1}{s} + \frac{k_2}{s + 500} \)

\( k_1 = 2 \)
\( k_2 = 2 \)

\( v_Z(t) = -2u(t) + 2e^{-500t}u(t) \)
3. [25] Given the network function \( T(s) = \frac{1000s}{(s+10)(s+100)} \), construct the Bode plot for \(|T(j\omega)|\) and \(\angle T(j\omega)\) with respect to frequency. The magnitude plot should be in terms of dBs and the phase plot in terms of degrees. Label your axis carefully. Determine whether this is a lowpass, highpass, bandpass or bandstop filter.

\[
T(j\omega) = \frac{1000j\omega}{(j\omega+10)(j\omega+100)} \\
= \frac{1000}{10j(10j)} \cdot \frac{1}{1+j\omega/10} \cdot \frac{1}{1+j\omega/100} \\
= (I) \cdot (II) \cdot (III) \cdot (IV) \cdot (V)
\]

\[
\begin{align*}
|T(j\omega)| & \quad \text{(Magnitude Plot)} \\
\angle T(j\omega) & \quad \text{(Phase Plot)}
\end{align*}
\]
4. [25] For the following rectangular wave with amplitude 1 V and period 1 sec:
   a) [5] Determine whether the signal has odd symmetry or even symmetry.
   b) [20] Determine the Fourier series coefficients for this signal, i.e. find expressions for $a_0, a_n, b_n$.

   \[ f(t) \]

   \[ -1.25 -1.0 -0.75 -0.25 0.25 0.75 1.25 \]

   a) \textbf{Even symmetry.}

   b) $b_n = 0$ since even symmetric

   \[
   a_0 = \int_{-0.25}^{0.25} f(t) \, dt = 0.5 \]

   \[
   T_0 = \frac{1}{1} = 1 \text{ s} \]

   \[
   a_n = 2 \int_{-0.25}^{0.25} \cos(2\pi f_0 t) \, dt = \frac{2 \sin(2\pi f_0 t)}{2\pi f_0} \bigg|_{-0.25}^{0.25}
   \]

   \[
   = 2 \left[ \frac{\sin(0.5\pi)}{2\pi n} - \frac{-\sin(-0.5\pi)}{2\pi n} \right] = 2 \left[ \frac{2\sin(0.5\pi)}{2\pi n} \right]
   \]

   \[
   a_n = \frac{2\sin(0.5\pi n)}{\pi n}
   \]