PET Statistical Reconstruction with Modeling of Axial Effects of FORE

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***Abstract*— Rebinning methods are often used to simplify the reconstruction of fully 3D PET data. These methods change the first and second order statistics of the data. Currently, most 2D reconstruction techniques do not cater to the new statics of the rebinned data. In this work, we explore the influence of the commonly used Fourier rebinnning (FORE) method on the statistics in the axial direction. FORE essentially interpolates the 3D data into the rebinned planes based on frequency content naturally causing a blurring and correlating effect amongst planes. We propose an approximation for this effect and describe the resulting conditional mean and covariance of the data . We also present novel methods for incorporating the modified mean and covariance into an iterative reconstruction technique. The method with just the improved second order effect results in little improvement, while the technique with an improved axial mean resulted in strong contrast gains in reconstructions.**

I. Introduction

Fourier rebinning (FORE) [4] reduces fully 3D PET data into a series of 2D transaxial slices. This process changes the statistics of the data in both the transaxial and axial direction. Past work explored the transaxial influence of FORE [1] and suggested some methods for applying this influence in iterative reconstruction. This current research looks at FORE's modification of the axial statistics.

In normal application, each slice of FORE rebinned data is treated as independent, with no relationship with neighboring slices. In actuality, FORE introduces significant correlation amongst planes because it interpolates the fully 3D data among rebinned slices based on frequency content. We propose an approximation of this interpolating influence and review its effect on the first and second order statistics of the data. Then, we use these modified statistics in a novel iterative reconstruction algorithm .

II. Justification for Modified Statistics

Fully 3D PET data is well modeled as conditionally independent Poisson variables. After FORE rebinning, the data is no longer Poisson or independent because the data must be corrected for several multiplicative effects (attenuation, efficiencies, etc.) in 3D and the rebinning process consists of a scaled linear combination of the data. In order to gain insight into the influence of the rebinning process on the statistics, we use the FORE kernel to express the FORE algorithm in the space domain. In [1], the FORE kernel was utilized to find the transaxial effect of FORE on the conditional mean and covariance of the data. This work seeks to find and use the axial effect of FORE.

FORE can be written in the space domain as the matrix multiplication

$$y_{2D} = \mathbf{A} y_{3D} \tag{1}$$

where y_{3D} is the 3D data, y_{2D} is the rebinned data, and $\mathbf{A}$ is the matrix form of convolving by the FORE kernels, which fully describe the rebinning transformation.

This notation aids the discussion of FORE's influence on the conditional mean of the data. When performing 2D image reconstruction, the mean, $\tilde{\mu}_{2D}$, of all of the rebinned data conditioned on the image is usually defined as the 2D forward projection, $\mathbf{P}_{2D}$, of the current image estimate, x:

$$\tilde{\mu}_{2D} = \mathbf{P}_{2D} x \tag{2}$$

In other words, $\mathbf{P}_{2D}$ transforms the 3D image into the rebinned data space. A more accurate representation of the conditional mean of FORE rebinned data is

$$\mu_{2D} = \mathbf{A} \mathbf{P}_{3D} x \tag{3}$$

where the mean is the FORE rebinned version of the 3D forward projection of the current image. Even though we are trying to avoid $\mathbf{P}_{3D}$ (due to its size), it should be noted that $\mathbf{P}_{2D} \neq \mathbf{A}\mathbf{P}_{3D}$. This leads to the conclusion that a mean estimated with a projection matrix which includes some effect of FORE would better approximate the true mean, μ_{2D}.

FORE also affects the second order statistics of the data. With $\mathbf{K}_{y_{3D}}$ representing the covariance of the 3D data, the covariance of the rebinned data, $\mathbf{K}_{y_{2D}}$ can be expressed as

$$\mathbf{K}_{y_{2D}} = \mathbf{A} \mathbf{K}_{y_{3D}} \mathbf{A}' \tag{4}$$

Since we assume that the 3D data is conditionally independent, $\mathbf{K}_{y_{3D}}$ is a diagonal matrix. After FORE, the data is no longer independent and $\mathbf{K}_{y_{2D}}$ is not diagonal. Current reconstruction methods treat the data as independent and ignore the information available in non-diagonal entries. It is difficult to compute (4) directly because $\mathbf{A}$ involves a convolution in two dimensions for all of the oblique planes. In order to simplify the computation of a modified conditional mean (3) and covariance (4) an approximation needs to be adopted.

III. Axial Modifications

The FORE algorithm interpolates frequency information into rebinned planes based on their location in the frequency space. Therefore, the interpolation effect on the data, and consequently the influence on the first and second order statistics, would be best understood in the frequency space. Incorporating a frequency domain approach into an iterative reconstruction algorithm appears infeasible so we propose a more simple method for approximating the influence of FORE in the axial dimension.

The axial approximation states that all entries in a single rebinned plane z_a contribute the same percentage of their values to all entries in rebinned plane z_b as expressed in the relationship

$$y_{2D}(s,\phi,z_b) \propto \sum_{z_a} H(z_b,z_a) y_{2D}(s,\phi,z_a) \tag{5}$$

where s is the radial position and ϕ is the angle in the rebinned sinogram y_{2D}. This matrix, $\mathbf{H}$, could also be viewed as a description of the blurring extent of FORE in the axial direction. In this work, we use FORE kernels to approximate $\mathbf{H}$ by adding contributing FORE kernels to find the percentage of a given direct plane from neighboring planes. Figure 1 displays $\mathbf{H}$ for an 18 ring scanner.

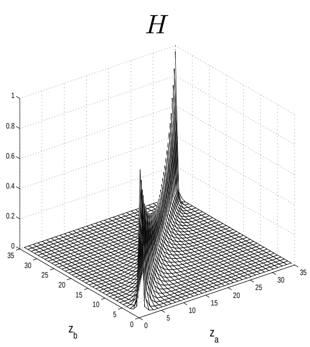

Fig. 1. Axial relationship for 18 ring scanner with a geometry similar to the GE Advance.

Since we have a relationship linking rebinned plane z_a with rebinned plane z_b, we also have a relationship linking the ideal forward projection, $\mathbf{P}_{2D}$, with the axially blurred projection matrix, $\hat{\mathbf{P}}_{\mathbf{2D}}$, of the rebinned planes as shown below

$$\hat{\mathbf{P}}_{2D}(z_b) = \sum_{z_a} H(z_b,z_a)\mathbf{P}_{2D}(z_a) \tag{6}$$

This new projection matrix will be used to compute the conditional mean for each image estimate.

The modified covariance matrix can be approximated from the idealized weighting matrix of the 2D data, $\tilde{\mathbf{K}}_{y_{2D}}$. The matrix $\tilde{\mathbf{K}}_{y_{2D}}$ is diagonal with ith entry σ_i, an estimate of the standard deviation of the ith data entry, y_{2D_i}. Many values have been proposed for this weighting matrix for conventional statistical reconstruction methods [5], [3], [2]. With some manipulation of matrix indexing, the modified weighting matrix can be expressed in the form

$$\hat{\mathbf{K}}_{y_{2D}} = \mathbf{H}_b \tilde{\mathbf{K}}_{y_{2D}} \mathbf{H}_b' \tag{7}$$

where $\mathbf{H}_b$ is a block diagonal matrix of the form

$$\mathbf{H}_b = \begin{bmatrix} \mathbf{H} & 0 & 0 \\ 0 & \ddots & \\ 0 & 0 & \mathbf{H} \end{bmatrix}$$

Note that when $\mathbf{H}$ is diagonal, $\hat{\mathbf{K}}_{y_{2D}}$ is diagonal and equals the idealized covariance matrix of the 2D data.

IV. Implementation

We incorporate the modified mean and covariance matrix into the penalized weighted least-square (PWLS) objective function:

$$\Phi(x) = \frac{1}{2}(y_{2D} - \hat{\mathbf{P}}_{2D}x)'\hat{\mathbf{K}}_{2D}^{-1}(y_{2D} - \hat{\mathbf{P}}_{2D}x) + \beta G(x) \tag{8}$$

where β is the smoothing parameter which weights the regularization term. The generalized Gaussian Markov random field (GGMRF) model, denoted as $G(x)$, is the regularization term applied to all of the methods reviewed here.

While the computation of the new projection matrix is straightforward (6), the inverse of the non-diagonal weighting/covariance matrix often poses a challenge. In this case, the utility of the approximation allows the inverse to be written as

$$\hat{\mathbf{K}}_{y_{2D}}^{-1} = (\mathbf{H}_b')^{-1}\tilde{\mathbf{K}}_{y_{2D}}^{-1}\mathbf{H}_b^{-1}. \tag{9}$$

Since $\mathbf{H}$ is symmetric, $\mathbf{H}_b' = \mathbf{H}_b$. The inverse of $\tilde{\mathbf{K}}_{y_{2D}}^{-1}$ is trivial because it is diagonal. The inverse of $\mathbf{H}_b^{-1}$ is simply

$$\mathbf{H_b}^{-1} = \begin{bmatrix} \mathbf{H}^{-1} & 0 & 0 \\ \vdots & \ddots & \\ 0 & 0 & \mathbf{H}^{-1} \end{bmatrix} \tag{10}$$

Consequently, $\hat{\mathbf{K}}_{y_{2D}}^{-1}$ reduces to the inverse of a diagonal matrix and a block diagonal matrix, resulting in the need to simply compute one inverse of a single small matrix, $\mathbf{H}$. The objective function (8) with the modified features was maximized with ICD/Newton-Raphson optimization [7]. The new mean and covariance terms require significant modifications to the original application of ICD. In short, first and second derivatives used in the optimization require values from neighboring planes.

Table I compares the computational load of the proposed algorithms with conventional two dimensional reconstruction algorithms. The time column presents the average time for one iteration of one plane of a 128×128 image using a 279×360 rebinned data set on a Sun Blade1000 Workstation with 512Mb of RAM. It should be stressed that the software implementing the new algorithms is not optimized and the time is only offered to give a sense of the computational load. $\mathbf{P}$ is a sparse matrix containing M_0N nonzero entries. The new mean method increases the number of entries in the sparse matrix. Specifically, the number of non-zero entries for the Improved Axial-Mean case is adjustable based on the number of neighboring plane relationships, B_{pl}, included in the mean.

TABLE I

COMPUTATIONAL COMPARISON OF 2D RECONSTRUCTION METHODS, LISTING MEASURES OF COMPUTATIONAL TIME PER FULL ITERATION OF A SINGLE SLICE. M_* IS THE NUMBER OF NONZERO PROJECTIONS ASSOCIATED WITH EACH OF THE N PIXELS. FOR THE TIME STUDY, $B_{pl} = 9$.

Algorithm	# of multi. & div.	# of matrix reads	time (sec.)
FBP	M_0N	1	Full Recon: 10
PWLS	$4M_0N$	2	19
Improved Mean	$4M_{\text{axial}}N$	2	99
Improved Cov	$4B_{pl}M_0N$	2	115

All of the methods used the attenuation weighting (AW) scheme introduced by Comtat *et al*[3]. Basically, the data sets are corrected for attenuation in the 3D domain. Then, they are rebinned and decorrected with the 2D attenuation correction factors (ACFs). Finally, the ACFs are incorporated into the system matrix used in the iterative reconstruction.

V. RESULTS

The axial improved methods were first tested with simulated fully 3D data of a torso phantom with 8 elliptical features, 4 hot and 4 cold of varying sizes. The 3D data set consisted of 152 oblique planes of 170x160 sinograms, which were subsequently rebinned into 35 direct and cross planes. The 3D data included the influence of attenuation. 20 realizations of simulated data with features and 20 realizations of just background levels were reconstructed at varying smoothing levels to compare the contrast of the different methods. Noise was computed as the standard deviation of the 20 means of volume of interests (placed in the location of the 8 features) in the background reconstrucions.

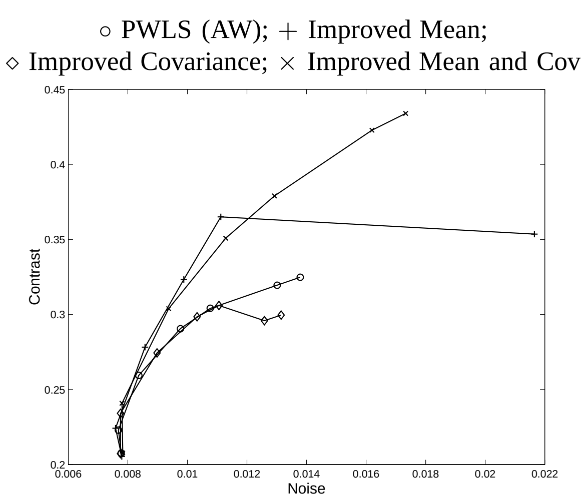

Fig. 2. Contrast versus noise curves for reconstructions of FORE rebinned torso phantom simulations. Points from left to right correspond to a decreasing smoothing parameter and the contrast of the true features is 0.5.

Figure 2 displays the contrast versus noise curves for PWLS(AW) and PWLS(AW) methods with combinations of improved statistics. The modified covariance matrix method

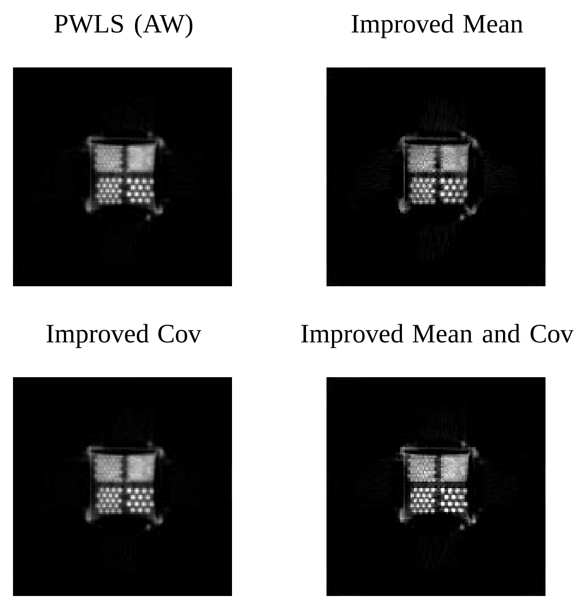

Fig. 3. One transaxial slice of 128 × 128 reconstructions of Derenzo phantom

offers only a minor improvement in contrast and noise over regular PWLS for some smoothing levels. The modified mean slightly increases the noise for the same smoothing parameter, but also greatly improves the contrast. With an adjustment in smoothing parameter, the modified mean method provides superior results in terms of contrast. The improved mean and covariance method has similar performance as the improved mean only method.

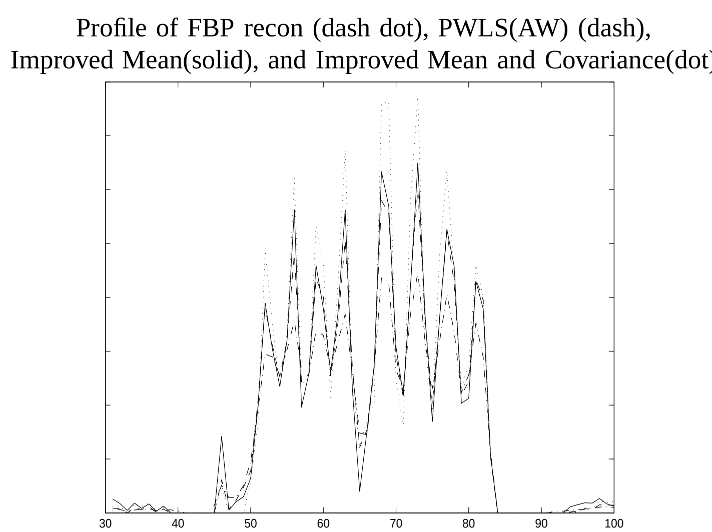

Fig. 4. Profiles from horizontal line 67 of 128x128 reconstructions shows the improved contrast for the modified mean methods.

This approach was also applied to real 3D data from the IndyPET-II small animal scanner [6]. We imaged a Derenzo-like, hot phantom with varying size holes (4.0mm to 1.6 mm diameter) filled with the FDG. The fully 3D data sets from this phantom consisted of 222 oblique planes of 279 × 360 sinograms. Then, the data were FORE rebinned into 47 direct planes and reconstructed with the methods discussed here.

A transaxial slice of reconstructions of this phantom appear in figure 3. Inspection of printed versions of the reconstructions reveals little differences among the methods, although one could argue that the improved axial mean and covariance method offers improved contrast. Figure 4 shows profiles from the reconstructions along a single horizontal line in a transaxial plane. The new covariance matrix approach offers no discernible improvement over PWLS, but the new mean clearly has improved contrast. The combination of both using the new mean and covariance results in significant performance benefits. The noise in the zero emission regions remain low while the hot spots have more extreme edges

VI. Conclusion

We approximated the axial influence of FORE on the mean and covariance of the data. Furthermore, we applied the new statisics into a PWLS algorithm. The improved covariance method resulted in little to no improvement in terms of contrast. This result can be expected due to the fact that this is only a second order effect. On the other hand, the improved mean method resulted in strong gains in contrast. Moreover, using both the new mean and covariance had promising results with the real data set.

These positive initial results offer a convincing argument to improve the axial mean method. While the improved methods remain significantly faster than fully 3D reconstruction, we still need to optimize the algorithm for it to become competitive with the widely used FORE+OSEM approach. We also hope to find a better approximation of the axial effect of FORE.

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