Pseudo-random binary sequence closed-loop system identification error with integration control

Z Ren and **G G Zhu***
Mechanical Engineering, Michigan State University, East Lansing, Michigan, USA

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Abstract: This paper studies the closed-loop system identification (ID) error when a dynamic integral controller is used. Pseudo-random binary sequence (PRBS) q-Markov covariance equivalent realization (Cover) is used to identify the closed-loop model, and the open-loop model is obtained based upon the identified closed-loop model. Accurate open-loop models were obtained using PRBS q-Markov Cover system ID directly. For closed-loop system ID, accurate open-loop identified models were obtained with a proportional controller, but when a dynamic controller was used, low-frequency system ID error was found. This study suggests that extra caution is required when a dynamic integral controller is used for closed-loop system identification. The closed-loop identification framework also has significant effects on closed-loop identification error. Both first- and second-order examples are provided in this paper.

Keywords: closed-loop system identification, pseudo-random binary (PRBS), identification error, q-Markov Cover

1 INTRODUCTION

System identification using closed-loop experimental data was developed in the 1970s [**1**] and it has been widely used in engineering practice (see references [**2**] to [**4**]). Closed-loop system identification can be used to obtain the open-loop system models when the open-loop plant cannot be excited at the ideal fixed operational conditions for system identification, for instance, when the open-loop plant is unstable.

There are many approaches to identify a system model in a closed-loop framework as shown in Fig. 1. The closed-loop system identification approach falls into three main groups: direct, indirect, and joint input–output approaches [**3**]. The direct approach ignores the feedback loop and identifies the open-loop system using measurements of plant inputs and outputs; the indirect approach identifies the closed-loop system model and then determines the open-loop system model using the knowledge of the linear controller; and the joint input–output approach regards the input and output jointly as the output and uses certain system identification methods to obtain open-loop models. This study mainly uses the indirect and joint input-output approaches.

In this paper, the q-Markov covariance equivalent realization (q-Markov Cover) system identification [**5–7**] using pseudo-random binary sequence excitation was used to obtain the closed-loop system models. The q-Markov Cover theory was originally developed for model reduction. It guarantees that the reduced-order model preserves the first q-Markov parameters of the original system. The realization of all q-Markov Covers from input and output data of a discrete time system is useful for system identification. q-Markov Cover for system identification uses pulse, white noise, or PRBS as input excitations. It has also been extended for identifying multirate discrete-time systems [**8**].

This paper was motivated by the unexpected low-frequency identification error of an open-loop system model when a proportional and integral (PI) controller was used for closed-loop system identification; while the low-frequency identification error was eliminated when the PI controller was

**Corresponding author: Mechanical Engineering, Michigan State University, 148 ERC South, East Lansing, MI 48824, USA.*
email: zhug@egr.msu.edu

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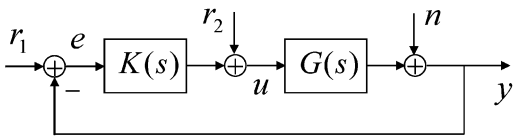

Fig. 1 Closed-loop system identification framework

replaced by a proportional controller. For both cases, the PRBS q-Markov Cover was used to obtain the closed-loop system transfer function and the open-loop system models were obtained using an indirect approach. Note that the bandwidth of a PRBS signal is bounded by its sample period at high frequency and its period (or order) at low frequency. It was also observed that the system identification error occurs around the PRBS signal bandwidth at low frequency. For this study system identification was applied to both a first-order system and a second-order system using open-loop system identification and closed-loop system identification with proportional, first-order dynamic, and PI controllers. The system identification results are compared and analysed at the end of the paper. The effects of different system identification frameworks, as shown in Fig. 1, was also studied.

The paper is organized as follows: section 2 provides the definition of the PRBS signal used in the system identification process; section 3 describes the framework and formulation of closed-loop identification; system identification results for the first- and second-order systems are provided in section 4, along with discussions of the simulation results; conclusions are provided in section 5.

2 INVERSE PRBS SIGNALS

The most commonly used PRBS signal is based on maximum length sequences (called m-sequences) [**9**] for which the length of the PRBS signal is $m = 2^n - 1$, where n is an integer (order of PRBS). Let z^{-1} represent a delay operator, and define $\hat{p}(z^{-1})$ and $p(z^{-1})$ to be polynomials

$$p(z^{-1}) = a_n z^{-n+1} \oplus \cdots \oplus a_2 z^{-1} \oplus a_1 = \hat{p}(z^{-1}) z^{-1} \oplus 1 \tag{1}$$

where a_i is either 1 or 0, and $\oplus$ obeys binary addition, i.e.

$$1 \oplus 1 = 0 = 0 \oplus 0 \quad \text{and} \quad 0 \oplus 1 = 1 = 1 \oplus 1 \tag{2}$$

and the non-zero coefficients a_i of the polynomial are defined in Table 1. Then the PRBS can be generated by

Table 1 Non-zero coefficients of PRBS polynomial

Polynomial order (n)	Period of sequence (m)	Non-zero coefficient
2	3	a_1, a_2
3	7	a_2, a_3
4	15	a_3, a_4
5	31	a_3, a_5
6	63	a_5, a_6
7	127	a_4, a_7
8	255	a_2, a_3, a_4, a_8
9	511	a_5, a_9
10	1023	a_7, a_{10}
11	2047	a_9, a_{11}

$$\hat{u}(k+1) = \hat{p}(z^{-1})\hat{u}(k), \quad k = 0, 1, 2, \ldots, \tag{3}$$

where $\hat{u}(0) = 1$ and $\hat{u}(-1) = \hat{u}(-2) = \cdots = \hat{u}(-n) = 0$. The following sequence is defined

$$s(k) = \begin{cases} a; & \text{if } k \text{ is even} \\ -a; & \text{if } k \text{ is odd} \end{cases} \tag{4}$$

Then, the signal

$$u(k) = s(k) \otimes [-a + 2a\hat{u}(k)] \tag{5}$$

is called the *inverse* PRBS, where $\otimes$ obeys

$$\begin{aligned} &a \otimes a = -a = -a \otimes -a \\ &\text{and} \quad a \otimes -a = a = -a \otimes a \end{aligned} \tag{6}$$

It is clear after some analysis that u has a period $2m$ and $u(k) = -u(k+m)$. The mean of the inverse PRBS is

$$m_u = E_{2m} u(k) \equiv \frac{1}{2m} \sum_{k=0}^{2m-1} u(k) = 0 \tag{7}$$

and the autocorrelation ($R_{uu}(\tau) = E_{2m} u(k+\tau)u(k)$) of u is

$$R_{uu}(\tau) \equiv \frac{1}{2m} \sum_{k=0}^{2m-1} u(k+\tau)u(k) = \begin{cases} a^2, & \tau = 0 \\ -a^2, & \tau = m \\ -a^2/m, & \tau \text{ even} \\ a^2/m, & \tau \text{ odd} \end{cases} \tag{8}$$

The inverse PRBS is used in the q-Markov Cover identification algorithm. For convenience, in the rest of this paper, the term 'PRBS' is used to represent the inverse PRBS.

3 CLOSED-LOOP SYSTEM IDENTIFICATION FRAMEWORK

Consider a general form of linear time-invariant closed-loop system shown in Fig. 1 (see references

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[3], [4] and [10]), where r_1 is the reference signal, r_2 is an extra input, n is the measurement noise, and u and y are input and output respectively. As discussed in the Introduction, there are many approaches to closed-loop identification, and these are categorized as direct, indirect, and joint input–output approaches. The indirect approach utilizes the knowledge of the closed-loop controller [11] while joint input–output approach obtains the open-loop system model using only the identified closed-loop model. In this paper, both indirect and joint input–output approaches are used. Noise is ignored in the discussion.

The input and output relationship of the generalized closed-loop system, shown in Fig. 1, can be expressed as

$$\mathbf{y} = [\mathbf{H}_1 \quad \mathbf{H}_2]\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2\end{bmatrix} = \begin{bmatrix}\left(\mathbf{GK}(\mathbf{I}+\mathbf{GK})^{-1}\right)^{\mathrm{T}}\\ \left(\mathbf{G}(\mathbf{I}+\mathbf{GK})^{-1}\right)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2\end{bmatrix} \quad (9)$$

Let $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ be identified closed-loop transfer functions from r_1 and r_2 to y respectively. When both excitations r_1 and r_2 are used, the open-loop system model $\mathbf{G}_{\mathrm{ID}}$ can be calculated using identified transfer functions $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ (joint input–output approach)

$$\mathbf{G}_{\mathrm{ID}} = \hat{\mathbf{H}}_2\left(\mathbf{I}-\hat{\mathbf{H}}_1\right)^{-1} \quad (10)$$

assuming that $(\mathbf{I}-\hat{\mathbf{H}}_1)^{-1}$ is invertible. Two special cases are also considered, i.e. when $r_1 = 0$ or $r_2 = 0$. For both cases the indirect approach will be used, i.e. the closed-loop controller transfer function is used to solve for the open-loop system models. For the case that $r_2 = 0$

$$\mathbf{G}_{\mathrm{ID}} = \hat{\mathbf{H}}_1(\mathbf{I}-\hat{\mathbf{H}}_1)^{-1}\mathbf{K}^{-1} \quad (11)$$

and for the case that $r_1 = 0$

$$\mathbf{G}_{\mathrm{ID}} = \hat{\mathbf{H}}_2(\mathbf{I}-\mathbf{K}\hat{\mathbf{H}}_2)^{-1} \quad (12)$$

The system identification is defined using equation (10) as the general set-up, equation (11) as the controller set-up, and equation (12) as the compensator set-up.

4 SIMULATION RESULTS

In order to study the identification error due to integral control, the following first- and second-order systems were selected

$$G_1(s) = \frac{1}{0.3s+1} \quad \text{and} \quad G_2(s) = \frac{1}{s^2+s+1} \quad (13)$$

In order to conduct both open- and closed-loop system identification, all models were discretized with a sample period of 5 ms. Discrete closed-loop models were obtained through system identification. There are two approaches used to obtain a continuous time plant model: (a) solving the plant model in a discrete-time domain first then converting it to a continuous-time domain or (b) converting the identified closed-loop model into a continuous model first and then solving for the plant model in continuous time. In the present study, it was found that both approaches provided almost identical models. For the rest of paper, only the results associated with approach (b) are presented.

PRBS q-Markov Cover was used to identify the open-loop transfer functions using r_2 as excitation by setting $K(s) = 0$, where the orders of the PRBS are $m = 8$ for the first-order plant and $m = 11$ for the second-order plant. The magnitude of PRBS is $a = 1$ (see Table 1). The identified open-loop transfer functions

$$\hat{G}_1(s) = \frac{-6 \times 10^{-5}s + 0.984}{0.298s+1}$$

and

$$\hat{G}_2(s) = \frac{-1.7\times 10^{-5}s^2 - 0.0037s + 0.993}{s^2+0.98s+1} \quad (14)$$

match the original plants, equation (13), very well, see Figs 2 and 3 for the frequency responses. Both the identified and original frequency responses are almost identical.

4.1 Closed-loop controllers

Three closed-loop controllers are considered for this study. For the first-order system, these are the proportional ($K_{\mathrm{P1}}(s)$), dynamic ($K_1(s)$), and PI ($K_{\mathrm{PI1}}(s)$) controllers shown in Table 2. The corresponding closed-loop system can then be expressed as

$$\mathbf{y} = [\mathbf{H}_1 \quad \mathbf{H}_2]\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2\end{bmatrix} = \mathbf{H}\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2\end{bmatrix} \quad (15)$$

where the closed-loop transfer functions (H) are provided in Table 2.

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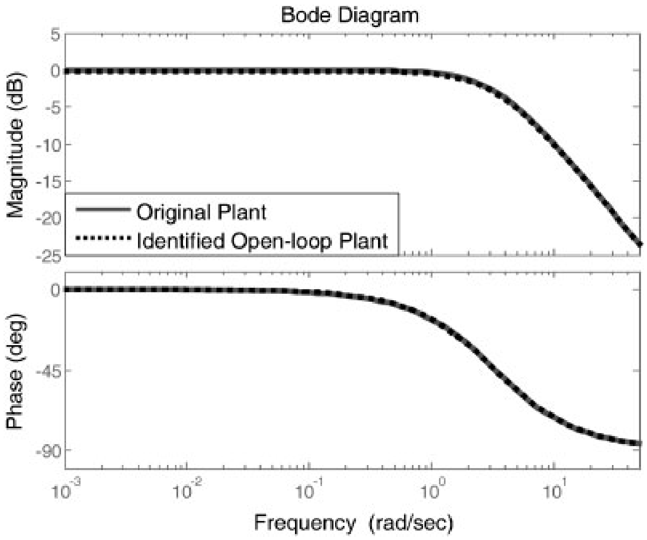

Fig. 2 Frequency response of identified and original first-order models

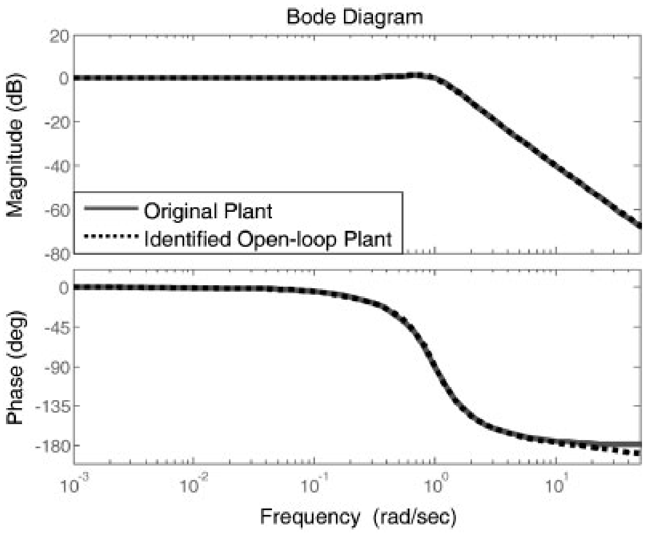

Fig. 3 Frequency response of identified and original second-order models

For the second-order system, the proportional ($K_{P2}(s)$), dynamic ($K_2(s)$), and PI ($K_{PI2}(s)$) controllers are used and these are defined in Table 2. The associated closed-loop system can be written in the form of equation (15), where the closed-loop transfer functions (H) are provided in Table 3.

4.2 PRBS q-Markov Cover system identification

The closed-loop system models are identified in discrete time domain using a PRBS graphical user interface (GUI) [**8**] developed for PRBS q-Markov Cover. The identification parameters used for this study are listed in Table 4. When general set-up is used, both $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ are identified. $\hat{\mathbf{H}}_1$ is identified in the controller set-up and $\hat{\mathbf{H}}_2$ is identified in the compensator set-up.

4.3 Closed-loop ID for the first-order system

For the first-order plant, identified closed-loop models of proportional, first-order dynamic, and PI controllers, and their corresponding open-loop models are listed in Table 5. Figure 4 shows the frequency responses of the calculated plant when a proportional controller is used in closed-loop identification. The calculated plant models have almost identical frequency responses to the original system in all three set-ups. Similarly, it can be found from Fig. 5 that the open-loop models calculated from identified closed-loop models with the first-order dynamic controller also have frequency responses very close to these of the original systems. There is only a very small deviation of direct current (d.c.) gain. However, open-loop models obtained from identified closed-loop models with PI controllers have significant d.c. gain drop at low frequency (see Fig. 6). The figure also shows that compensator set-up generates the worst calculated plant model.

Therefore, for the first-order system, the closed-loop system identifications with the proportional controller are accurate for all three set-ups. For all set-ups with the first-order controller, the calculated plant models are similar to the original plant. With the PI controller, plant models calculated from all set-ups have significant gain drop at low frequency. The compensator set-up with the PI controller provides the worst calculated plant model.

Table 2 Closed-loop controllers and transfer functions of the first-order plant

Closed-loop controllers $K_{P1}(s)=0.2$	$K_1(s)=\dfrac{s+0.5}{s+1}$	$K_{PI1}(s)=\dfrac{0.2s+5}{s}$
Corresponding closed-loop transfer functions H^T		
$\begin{bmatrix}\dfrac{0.2}{0.3s+1.2} \\ \dfrac{1}{0.3s+1.2}\end{bmatrix}$	$\begin{bmatrix}\dfrac{s+0.5}{0.3s^2+2.3s+1.5} \\ \dfrac{s+1}{0.3s^2+2.3s+1.5}\end{bmatrix}$	$\begin{bmatrix}\dfrac{0.2s+5}{0.3s^2+1.2s+5} \\ \dfrac{s}{0.3s^2+1.2s+5}\end{bmatrix}$

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Table 3 Closed-loop controllers and transfer functions of the second-order plant

Closed-loop controllers $K_{P2}(s) = 0.2$	$K_2(s) = \frac{s+0.5}{s+1}$	$K_{PI2}(s) = \frac{0.5s+0.2}{s}$
Corresponding closed-loop transfer functions H^T		
$\begin{bmatrix} \frac{0.2}{s^2+s+1.2} \\ \frac{1}{s^2+s+1.2} \end{bmatrix}$	$\begin{bmatrix} \frac{s+0.5}{s^3+2s^2+3s+1.5} \\ \frac{s+1}{s^3+2s^2+3s+1.5} \end{bmatrix}$	$\begin{bmatrix} \frac{0.5s+0.2}{s^3+s^2+1.5s+0.2} \\ \frac{s}{s^3+s^2+1.5s+0.2} \end{bmatrix}$

Table 4 PRBS q-Markov Cover system ID parameters

	Open-loop	Proportional	PI	First order
Firts-order system identification settings controller				
Sample rate (s)	0.005	0.005	0.005	0.005
PRBS order	8	8	9	10
Signal length (s)	25.6	25.6	51.2	102.4
Number of Markov parameters	10	10	60	60
ID model order	1	1	2	2
Second-order system identification settings				
Sample rate (s)	0.005	0.005	0.005	0.005
PRBS order	11	11	13	11
Signal length (s)	204.8	204.8	819.2	204.8
Number of Markov parameters	60	60	60	60
ID model order	2	2	3	3

Table 5 Identified closed-open-loop models for the first-order plant

Set-ups	Controller	Proportional controller	First-order controller	PI controller
General set-up	Identified closed-loop models	$\begin{bmatrix} \frac{-1.40\times10^{-5}s + 0.665}{s + 4.02} \\ \frac{-6.97\times10^{-5}s + 3.33}{s + 4.02} \end{bmatrix}$	$\begin{bmatrix} \frac{2.26\times10^{-5}s^2+3.34s+1.99}{s^2+7.78s+5.65} \\ \frac{-1.03\times10^{-5}s^2+3.33s+3.67}{s^2+7.78s+5.65} \end{bmatrix}$	$\begin{bmatrix} \frac{-2.18\times10^{-5}s^2+0.657s+16.9}{s^2+4.03s+16.7} \\ \frac{0.000\,132s^2+3.37s+0.0460}{s^2+4.03s+16.7} \end{bmatrix}$
	Calculated plant (Equation (10))	$\frac{0.992}{0.298s+1}$	$\frac{-1.03s^2+3.33s+3.67}{s^2+4.43s+3.65}$	$\frac{1.3\times10^{-4}s^2+3.34s+0.046}{s^2+3.37s-0.19}$
Controller set-up	Identified closed-loop models	$\frac{-1.395\times10^{-5}s + 0.6651}{s + 4.019}$	$\frac{2.263\times10^{-5}s^2+3.339s+1.775}{s^2+7.689s+5.185}$	$\frac{-2.18\times10^{-5}s^2+0.662s+16.7}{s^2+3.95s+16.8}$
	Calculated plant (Equation (11))	$\frac{-2.1\times10^{-5}s+0.99}{0.298s+1}$	$\frac{-1.03\times10^{-5}s^3+3.33s^2+6.38s+3.05}{s^3+5.21s^2+7.49s+3.14}$	$\frac{-2.2\times10^{-5}s^3+0.662s^2+16.7s}{0.2s^3+5.66s^2+16.5s+0.4}$
Compensator set-up	Identified closed-loop models	$\frac{-6.97\times10^{-5}s + 3.33}{s + 4.02}$	$\frac{-1.03\times10^{-5}s^2+3.33s+3.05}{s^2+7.54s+4.67}$	$\frac{0.000\,132s^2+3.38s-0.188}{s^2+4.03s+15.9}$
	Calculated plant (Equation (12))	$\frac{-2.1\times10^{-5}s+0.992}{0.298s+1}$	$\frac{-1.03\times10^{-5}s^3+3.33s^2+6.38s+3.05}{s^3+5.21s^2+7.49s+3.14}$	$\frac{1.3\times10^{-4}s^3+3.38s^2-0.188s}{s^3+3.38s^2-0.433s-0.939}$

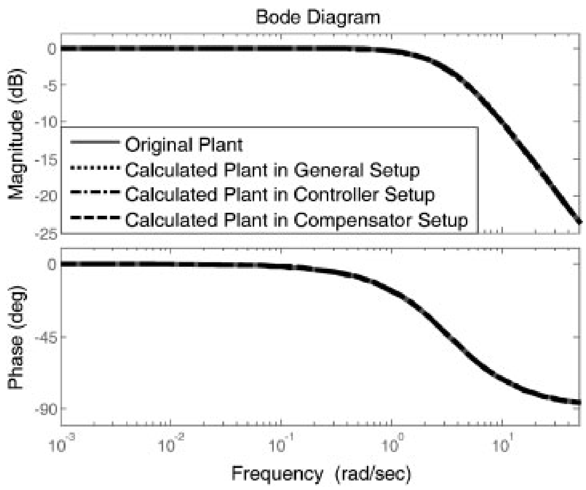

Fig. 4 First-order open-loop models with a proportional controller in three set-ups

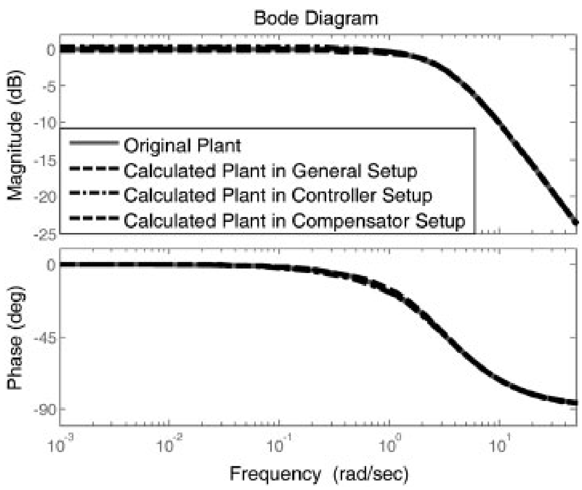

Fig. 5 First-order open-loop models with a first-order controller in three set-ups

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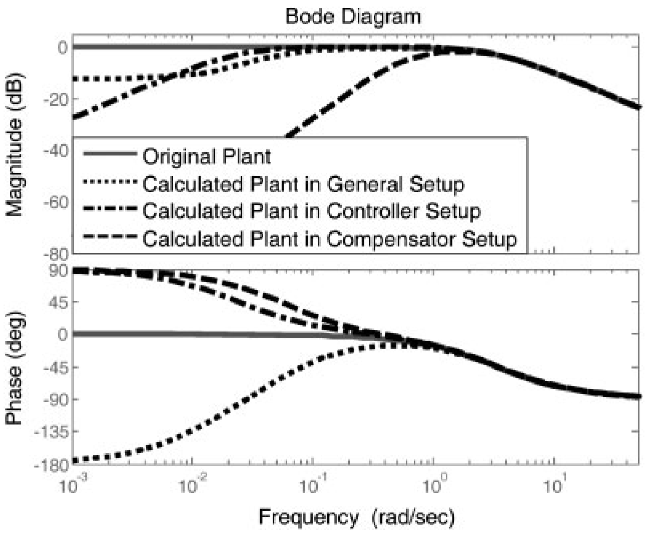

Fig. 6 First-order open-loop models with a PI controller in three set-ups

4.4 Closed-loop ID for the second-order system

For the second-order plant, identified closed-loop models and corresponding calculated plants are listed in Table 6. From Fig. 7, the calculated plant models with proportional controllers have almost identical frequency response to the original system in all three identification set-ups. However, there is a certain phase shift beyond the PRBS frequency bandwidth (at around 20 Hz), indicating that high-frequency PRBS excitation might be needed. Similarly, it can be found from Fig. 8 that the identified open-loop models with first-order controllers have similar frequency response to the original system. Plant models obtained from closed-loop systems using controller and compensator set-ups have slight error in d.c. gain, and phase shifts near corner frequency.

Figure 9 shows the frequency response for the identified open-loop plants when the PI controller is used. The identified open-loop model using the general set-up provides the closest frequency response to the original one. Plant models obtained from the compensator set-up had the worst accuracy. Therefore, similar to the first-order system case, for the second-order system, the closed-loop system identifications for all three set-ups are accurate with the proportional controller. Plant models obtained from closed-loop systems with the first-order controller provide similar frequency responses to the original plant. For the PI controller, however, only the general set-up provides an accurate identified model.

Table 6 Identified closed-open-loop models for the second-order plant

	Controller	Proportional controller	First-order controller	PI controller
General set-up	Identified closed-loop models	$\begin{bmatrix}\frac{3.2\times10^{-6}s^2+0.000\,64s+0.20}{s^2+1.0s+1.2}\\ \frac{1.6\times10^{-5}s^2+0.0032s+1.0}{s^2+1.0s+1.2}\end{bmatrix}$	$\begin{bmatrix}\frac{-3.7\times10^{-6}s^3-0.000\,74s^2+0.99s+0.35}{s^3+1.8s^2+2.8s+1.2}\\ \frac{-3.0\times10^{-6}s^3-0.000\,61s^2+0.99s+0.84}{s^3+1.8s^2+2.8s+1.2}\end{bmatrix}$	$\begin{bmatrix}\frac{-1.3\times10^{-6}s^3-0.000\,26s^2+0.50s+0.20}{s^3+1.0s^2+1.5s+0.20}\\ \frac{1.5\times10^{-6}s^3+0.0003s^2+s+0.0022}{s^3+1.0s^2+1.5s+0.20}\end{bmatrix}$
	Calculated plant (Equation (10))	$\frac{1.6\times10^{-5}s^2+0.0032s+1.0}{s^2+1.0s+0.99}$	$\frac{-3\times10^{-6}s^3-0.000\,61s^2+0.99s+0.84}{s^3+1.8s^2+1.99+0.84}$	$\frac{1.5^{-6}s^3-0.0003s^2+s+0.0022}{s^3+1.0s^2+0.99s+0.0017}$
Controller set-up	Identified closed-loop models	$\frac{3.2\times10^{-6}s^2+0.000\,64s+0.20}{s^2+1.0s+1.2}$	$\frac{-3.7\times10^{-6}s^3-0.000\,75s^2+0.99s+0.21}{s^3+1.7s^2+2.6s+0.78}$	$\frac{-1.3\times10^{-6}s^3-0.000\,25s^2+0.50s+0.20}{s^3+1.0s^2+1.5s+0.20}$
	Calculated plant (Equation (11))	$\frac{1.6\times10^{-5}s^2+0.0032s+1.0}{s^2+1.0s+0.99}$	$\frac{-3.7\times10^{-6}s^4-0.000\,75s^3+s^2+1.2s+0.2}{s^4+2.3s^3+2.5s^2+1.4s+0.28}$	$\frac{-1.3\times10^{-6}s^4-0.000\,25s^3+0.50s^2+0.20s}{0.5s^4+0.70s^3+0.70s^2+0.20s+0.000\,34}$
Compensator set-up	Identified closed-loop models	$\frac{1.62\times10^{-5}s^2+0.0032s+1.0}{s^2+1.0s+1.2}$	$\frac{-3.0\times10^{-6}s^3-0.000\,62s^2+0.99s+0.52}{s^3+1.5s^2+2.5s+0.68}$	$\frac{1.5\times10^{-6}s^3+0.0003s^2+s+0.0023}{s^3+1.0s^2+1.5s+0.20}$
	Calculated plant (Equation (12))	$\frac{1.6\times10^{-5}s^2+0.0032s+1.0}{s^2+1.0s+0.99}$	$\frac{-3\times10^{-6}s^4-0.0006s^3+s^2+1.5s+0.5}{s^4+2.5s^3+3s^2+2.1s+0.42}$	$\frac{-1.3\times10^{-6}s^4-0.0003s^3+s^2+0.0023s}{s^4+s^3+0.99s^2+0.0006s-0.000\,45}$

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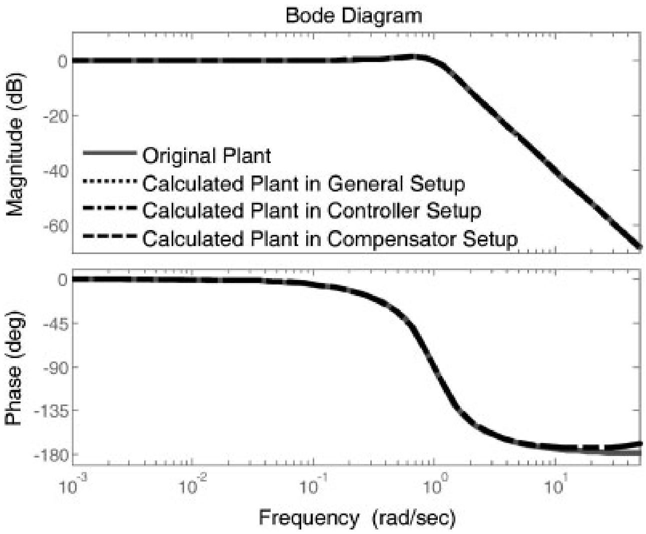

Fig. 7 Second-order open-loop models with a proportional controller in three set-ups

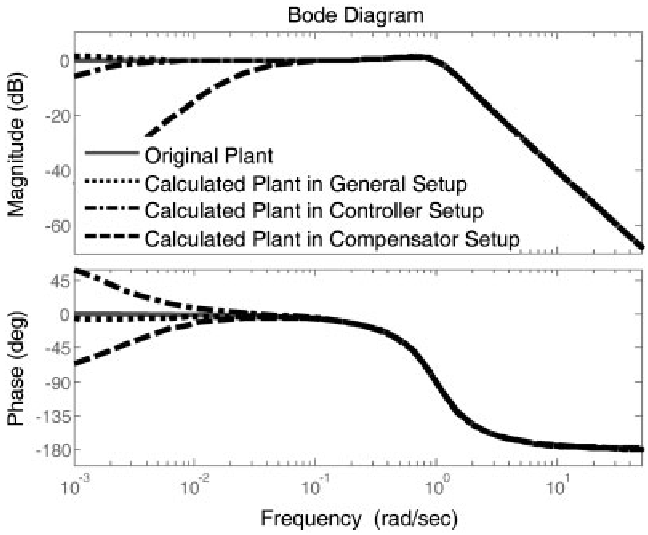

Fig. 9 Second-order open-loop models with a PI controller in three set-ups

4.5 Effect of the PRBS signal order

The following example is used to investigate the effect of PRBS order associated with identification error at low frequency. Increasing PRBS order decreases identification excitation signal bandwidth at low frequency. Using the first-order system in the compensator set-up with a PI controller, an eleventh-order PRBS signal was used with other parameters defined in Table 4, the identified closed-loop system model is

$$\hat{H}_1 = \frac{4.071\times 10^{-6}s^2 + 3.335s + 0.000\,129\,6}{s^2 + 4.003s + 16.66} \quad (16)$$

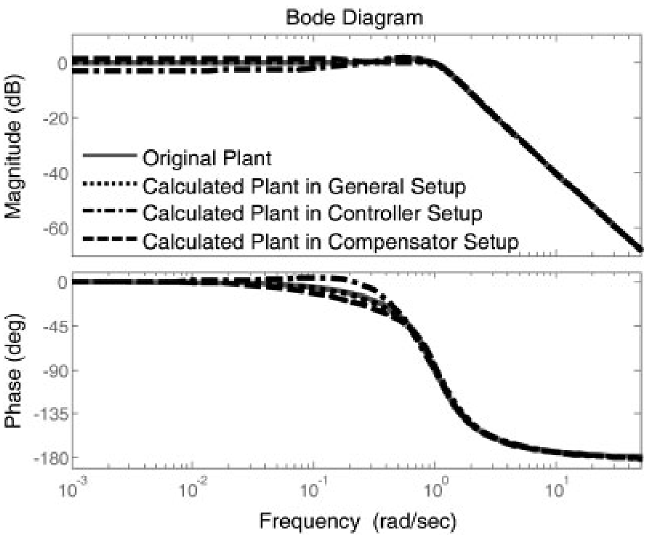

Fig. 8 Second-order open-loop models with a first-order controller in three set-ups

and the corresponding open-loop model is

$$G_{\text{cal}} = \frac{4.07\times 10^{-6}s^3 + 3.335s^2 + 0.000\,13s}{s^3 + 3.336s^2 - 0.015s - 0.000\,648} \quad (17)$$

As expected from Fig. 10, comparing it with the result of using ninth-order PRBS in section 4.3, the low-frequency identification error decreases as PRBS order increases.

5 CONCLUSION

A low-frequency system identification error was discovered when an integral controller was used for

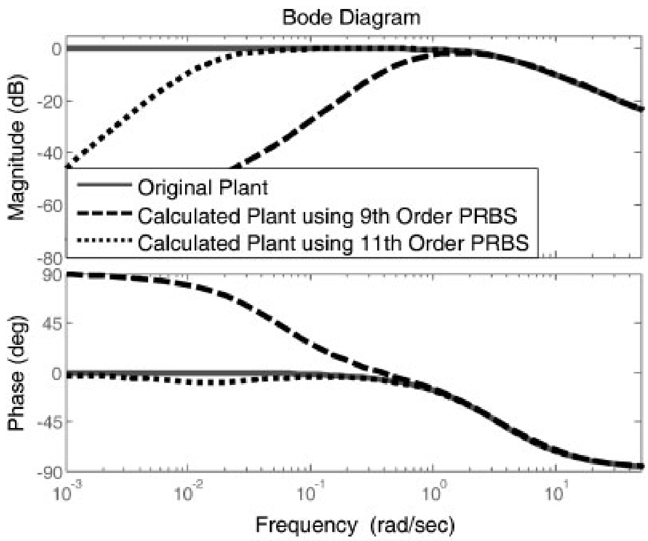

Fig. 10 Effect of PRBS order

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PRBS q-Markov Cover closed-loop system identification using an indirect approach. Significant low-frequency gain of the identified open-loop model was observed when the dynamic integral controller was used. Increasing the order of the PRBS signal decreases the error at low frequencies. The general set-up of the closed-loop system identification set-up provides fairly accurate identified open-loop models even with the dynamic integral controller. Using the proportional controller in the closed-loop system identification leads to the most accurate plant model. If feasible, it is recommended not to use a dynamic integral controller in closed-loop system identification when PRBS is used for excitation. Future work will extend the study to the case for multiple input–multiple output (MIMO) plants using other system identification approaches.

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APPENDIX

Notation

e	error in closed-loop system
m	length of PRBS
n	order of PRBS
$\mathbf{H}$	closed-loop system transfer function
$\hat{\mathbf{H}}$	identified closed-loop system transfer function
G	open-loop plant transfer function
$\mathbf{G}_{\mathrm{ID}}$	identified open-loop plant transfer function
$\hat{G}$	calculated plant transfer function

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