A control-oriented hybrid combustion model of a homogeneous charge compression ignition capable spark ignition engine

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Abstract
To implement the homogeneous charge compression ignition combustion mode in a spark ignition engine, it is necessary to have smooth mode transition between the spark ignition and homogeneous charge compression ignition combustions. The spark ignition–homogeneous charge compression ignition hybrid combustion mode modeled in this paper describes the combustion mode that starts with the spark ignition combustion and ends with the homogeneous charge compression ignition combustion. The main motivation of studying the hybrid combustion mode is that the percentage of the homogeneous charge compression ignition combustion is a good parameter for combustion mode transition control when the hybrid combustion mode is used during the transition. This paper presents a control oriented model of the spark ignition–homogeneous charge compression ignition hybrid combustion mode, where the spark ignition combustion phase is modeled under the two-zone assumption and the homogeneous charge compression ignition combustion phase under the one-zone assumption. Note that the spark ignition and homogeneous charge compression ignition combustions are special cases in this combustion model. The developed model is capable of simulating engine combustion over the entire operating range, and it was implemented in a real-time hardware-in-the-loop simulation environment. The simulation results were compared with those of the corresponding GT-Power model, and good correlations were found for both spark ignition and homogeneous charge compression ignition combustions.

Keywords
Spark ignition combustion, homogeneous charge compression ignition combustion, combustion modeling, combustion control

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Introduction
The continuing pursuit of improving fuel economy and the increasingly stringent emission regulations has rekindled the research interest in the homogeneous charge compression ignition (HCCI) combustion in recent years. The flameless nature of the HCCI combustion and its high dilution operation capability lead to low combustion temperature. As a result, the formation of NOx (nitrogen oxides) can be significantly reduced. Furthermore, an HCCI capable engine is capable of unthrottled operation that greatly reduces pumping loss and improves fuel economy.\(^1\)

On the other hand, HCCI combustion has its own limitations. The HCCI combustion is limited at high engine load due to the audible engine knock,\(^2\) and at low load due to misfire caused by the lack of sufficient thermal energy to trigger the auto-ignition of the gas–fuel mixture late in the compression stroke.\(^3\) In fact, the HCCI combustion can be regarded as a type of engine operating mode rather than a type of engine.\(^4\) In order to take advantage of the HCCI combustion mode in an internal combustion engine, an other combustion mode, such as the SI (spark ignited) combustion, is required at high load, at ultra-low load (such as idle), and at certain operational conditions such as cold start, and at high engine speed. It is fairly challenging to operate the engine in two distinct combustion modes, and it is even more difficult to have smooth combustion mode

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transition between SI and HCCI combustions modes, since the favorable thermo conditions for one combustion mode are always adverse to the other. Due to the significant response delay of the hydraulic and electric variable valve timing (VVT) systems, cycle-to-cycle residual gas dynamics, and the response delay in the engine air handling system, it is almost impossible to achieve desired thermo conditions for the combustion mode transition in one engine cycle.\textsuperscript{3,6} To address this issue, multi-cycle combustion mode transition strategies were proposed.\textsuperscript{7,8} In Zhang et al.,\textsuperscript{9} experimental data show that a hybrid (or spark assisted) combustion mode occurred during the mode transition. It starts in the SI combustion mode with a relatively low heat release rate, and once the thermo and chemical conditions of the unburned gas satisfy the start of HCCI (SOHCCI) combustion criteria, the combustion continues in the HCCI combustion mode. The hybrid combustion process is illustrated by the dashed curves of mass fraction burned (MFB) shown in Figure 1. During an ideal SI to HCCI combustion transition process, as the charge temperature, defined as in-cylinder gas mixture temperature at intake valve closing, increases gradually cycle-after-cycle, the percentage of SI combustion, defined as the MFB percentage at SOHCCI (see Figure 1), decreases gradually over each combustion cycle while the HCCI combustion percentage, MFB between SOHCCI and the end of combustion, increases gradually. For the HCCI to SI combustion transition, the process is reversed. This hybrid combustion mode makes it feasible to use the conventional valvetrain for the HCCI capable SI engine since it allows the engine cam phase to change gradually.\textsuperscript{9} In the rest of this paper, the term hybrid combustion refers to the SI-HCCI hybrid combustion mode.

Smooth combustion mode transition can be realized by utilizing appropriate control strategies to regulate the SI-HCCI hybrid combustions properties during the mode transition, as demonstrated.\textsuperscript{3,5,7,9} Yang and Zhu\textsuperscript{10} also demonstrated the effectiveness of using spark assisted HCCI (or SI-HCCI combustion in this paper) to avoid the misfire by controlling the in-cylinder air-to-fuel ratio in the proper range such that SI is possible. Also, the SI to HCCI transition mode has been known as more difficult to control than the HCCI to SI transition mode since the residual gas management or in-cylinder temperature control is completely different in terms of thermal energy introduction and thermal inertia aspects. To develop and validate the combustion mode transition control strategies, a precise control oriented combustion model is a necessity. It should be able to simulate the engine combustion process in real-time with enough modeling complexity for combustion feedback control. Multi-zone, three-dimensional computational fluid dynamics (CFD) models with detailed chemical kinetics can be found\textsuperscript{4,10,11} and they are capable of describing the engine thermal and flow dynamics, heat transfer, and pollutant formation phenomena of both SI and HCCI combustions with reasonable accuracy. Similar combustion models have been implemented into commercial codes such as GT-Power\textsuperscript{12} and Wave.\textsuperscript{13} However, these high-fidelity models cannot be used for control strategy development and validation since they can only be operated in the offline simulation mode, but they can be used as reference models for the control-oriented combustion model presented in this paper.

In this paper, the zero-dimensional mean-value modeling method\textsuperscript{14,15} is used to model average chemical kinetics and thermodynamic properties of the combustion process, due to its low computational throughput required for real-time simulations. The modeled combustion variables, such as in-cylinder gas mixture pressure and temperature, are updated every crank degree.\textsuperscript{16} The nonlinear equations describing the chemical kinetics and thermodynamics of the combustion process are solved by discretizing the continuous nonlinear equations. Furthermore, the combustion model presented in this paper for the SI-HCCI hybrid combustion treats both SI and HCCI combustion modes as special cases of the hybrid combustion. Therefore, the SI-HCCI hybrid combustion model contains all possible combustion modes during the mode transition. In the SI-HCCI hybrid combustion model, the SI combustion phase from ST to SOHCCI (see Figure 1) is modeled under the two-zone assumption, and the thermodynamic and chemical properties of the unburned zone gas mixture are modeled to accurately estimate the start of HCCI combustion phase; whereas the HCCI combustion phase from the SOHCCI to the end of combustion is modeled under the one-zone assumption to reduce the computational throughput.

The developed SI-HCCI hybrid combustion model was implemented into a dSPACE based hardware-in-the-loop (HIL) engine simulation environment for real-time simulations and model validations. The simulation results were also compared with those obtained from the corresponding high-fidelity GT-Power model, and fairly good correlations were observed between these two combustion models. This shows that the proposed
combustion model can be used to approximate the high-fidelity combustion model and implemented in real-time control at the same time.

The paper is organized as follows. In the second section the target engine configuration is described and the SI-HCCI hybrid combustion model architecture is presented according to the engine configuration. The third and fourth sections provide the governing equations of the combustion and gas exchange phases. The fifth section addresses the implementation of the developed combustion model into the real-time HIL simulation environment. The model calibration and validation results are presented in the sixth section. Conclusions are drawn in the seventh section.

SI-HCCI hybrid combustion model architecture

This section discusses the target engine configuration capable of the SI, HCCI, and SI-HCCI hybrid combinations. The model architecture is presented to combine the three combustion modes into a single combustion model.

Target engine configuration

Figure 2 shows the configuration of the target HCCI capable SI engine, and the engine specifications are listed in Table 1. The key feature of the engine is its valvetrain system that has two-stage valve lifts for both intake and exhaust valves. The high lift is 9 mm for the SI combustion mode; and the low lift is 5 mm for the HCCI combustion mode. Note that the cam profiles for the high and low lift are different, which provides additional freedom to optimize both SI and HCCI combustions. Furthermore, both intake and exhaust valve timings can be adjusted within 6-40 crank degrees to improve the controllability of the internal exhaust gas recirculation (EGR) fraction, the effective compression ratio, and the engine volumetric efficiency during the combustion mode transition and steady state SI and HCCI operations. Note that using the two-step valve system with electrical VVT for HCCI combustion has been demonstrated.17 For the engine considered, the valve lift and its profile were optimized using GT-Power simulations. In addition to the modification of the engine valvetrain, external cooled EGR is used to enable high dilution charge with low charge mixture temperature, which leads to 7% and 19% external EGR rate during the SI to HCCI combustion mode transition. The engine throttle can be electronically controlled to obtain the desired engine charge in both SI and unthrottled HCCI operations. To enable the closed-loop combustion control of each engine cylinder, each engine cylinder is equipped with a piezoelectric pressure transducer and each exhaust port is fitted with a fast response UEGO (universal exhaust gas oxygen) sensor. Accordingly, the proposed control oriented combustion model is required to simulate the corresponding in-cylinder gas pressure and air-to-fuel ratio signals.

The target engine is capable of operating at the SI combustion mode with high intake and exhaust valve lifts, low EGR fraction, and with SI, at the SI-HCCI hybrid combustion mode with low intake and exhaust valve lifts, medium EGR rate and with SI, and at the HCCI combustion mode with low intake and exhaust valve lifts, high EGR rate, and without SI. During the combustion mode transition between the SI and HCCI combustions, the engine will be operated at the three combustion modes from SI to SI-HCCI to HCCI combustion. Thereby, the proposed combustion model is required to model all three combustion modes. At the same time, it also needs to have computational throughput low enough for real-time simulations, since the developed combustion model is to be used for the control strategy development and validation of the combustion mode transition.

This paper focuses mainly on the combustion model development, therefore the EGR and EGR cooler models were not presented. For the integrated engine model, the EGR valve was model based upon the valve model...
described by Fiveland and Assanis, and the EGR cooler was modeled based on the equation (12.2) given on the WOT website. Details about the integrated engine model can be found in Heywood. In the integrated engine model, the air-handling subsystem models, such as the exhaust manifold model and EGR loop model, are coupled with the combustion variables of individual cylinders and the cycle-to-cycle and cylinder-to-cylinder dynamics are shown in these simulations.

**Combustion model architecture**

To model the in-cylinder gas properties, such as mass, pressure, and temperature, with the accuracy over the entire engine cycle to meet the real-time HIL simulation requirements, the proposed combustion model needs to be updated every crank degree (crank-based), which is different from the modeling approaches used by Rausen et al. and Canova and Mohler. There are two additional reasons for using the crank-based modeling approach. The first is due to the fact that most combustion characteristics are usually functions of the crank angle, such as spark timing, burn duration and the crank angle of peak in-cylinder pressure location. The second is that the entire combustion process can be divided into several combustion phases as functions of crank angle. For this paper, the combustion process is divided into six phases separated by six events. They are intake valve closing (IVC), spark timing (ST), SOHCCI, exhaust valve opening (EVO), exhaust valve closing (EVC), and intake valve opening (IVO), as shown in Figure 3. The in-cylinder gas properties are modeled differently for each phase that is defined between two combustion events.

However the crank-based modeling approach has its own challenges. During the real-time simulation the entire combustion model needs to be updated within the period associated with one crank degree. This leads to fairly short computational duration at high engine speed. For example, at 6000 rpm one crank degree corresponds to about 28 micro-seconds. In order to avoid overruns during real-time simulations, the combustion model must be as simple as possible but with satisfactory simulation accuracy. Accordingly, all in-cylinder gas properties are assumed to be in quasi-steady state during the period of each crank degree, and in-cylinder gas flow dynamics are not modeled for any of the combustion phases.

In Figure 3, events ST and SOHCCI are marked in the dashed boxes to distinguish them from other events, since the existences of these two events depend on the engine combustion mode, and the other four events always exist for any 4-stroke internal combustion engine. Note that the main difference among the SI, HCCI, and SI-HCCI combustion modes depends on the events between IVC and EVO. When the SOHCCI event does not occur, the engine is operated at the SI combustion mode; when the SOHCCI event occurs before or at the SI event, the engine is operated at the HCCI combustion mode; while when the SOHCCI occurs after the SI combustion the engine is operated at the SI-HCCI hybrid combustion mode. Note that both SI and HCCI combustion modes are special cases of the SI-HCCI hybrid combustion mode. In fact, there also exists a fourth combustion mode, in which both SI and SOHCCI do not occur in the combustion phase, or SOHCCI happens very late without the SI combustion. The fourth mode is undesired and needs to be avoided since it means misfire or partial burn. Figure 4 illustrates the block diagram of the combustion model.

As shown in Figure 4, the Arrhenius integration (ARI) is used as the criterion of the start of HCCI combustion in the unburned zone, and it is described by

\[
ARI = \int_{\theta_{\text{min}}}^{\theta} AR(\theta) \cdot d\theta
\]
where \(x_f\) and \(x_{ox}\) are unburned fuel and oxidizer concentrations; exponents \(b\) and \(c\) are the influence factors; \(R\) is gas constant; multiplier \(A\) and Arrhenius activation energy \(E_a\) can be obtained by matching the combustion burn rate experimentally. The SOHCCI crank position \(\theta_{IVC}\) is defined as the crank angle at which 1% of the fuel is burned under HCCI combustion. Accordingly, once \(ARI\) exceeds 1 the combustion model switches to the HCCI combustion calculation. The Arrhenius integration in equation (1) starts at \(\theta_{IVC}\), and is reset at \(\theta_{EVO}\). During this period the integration is updated every crank degree. The variable \(T\) in equation (1) is a key parameter and affects the rising rate of the \(ARI\) significantly. If spark ignition never occurs, the variable \(T\) in equation (1) equals the in-cylinder gas temperature, while during the SI combustion phase \(T\) equals the unburned gas temperature \(T_U\), which is calculated by the two-zone SI combustion model that is presented in the third section. This is also the main motivation of developing the two-zone SI combustion model, since the one-zone SI combustion model proposed by Yang and colleagues\(^{21,22}\) does not provide this required information.

In Figure 3, the phase between IVC and EVO is defined as combustion phase. It is modeled by three sub-models: the gas compression model, the one-zone HCCI combustion model, and the two-zone SI combustion model, as shown in Figure 4. The governing equations of the combustion phase are presented in the third section.

The gas exchange phase starts at EVO and ends at IVO of the next engine cycle as illustrated in Figure 3. Two valve timing strategies are modeled in the gas exchange phase of the HCCI capable SI engine. They are negative valve overlap (NVO) with low valve lift and positive valve overlap (PVO) with low valve lift. PVO is mainly used for SI combustion mode, in which IVO occurs before EVC. On the contrary NVO is mainly used for the HCCI and SI-HCCI hybrid combustion modes, in which EVC comes earlier than IVO. Figure 3 illustrates the NVO operation, and one can see the pressure increment of the in-cylinder gas during the NVO operation. This is due to the recompression applied to the trapped gas. The fourth section describes the governing equations for the gas exchange phase.

### Combustion phase models

As illustrated in Figure 3, the combustion phase consists of three sub-phases: the compression phase between IVO and ST, the SI combustion phase between ST and SOHCCI, and the HCCI combustion phase between SOHCCI and EVO. This section discusses the mathematical models for the three phases. For this control oriented combustion model, the in-cylinder gas fuel mixture was assumed to be completely homogeneous before the start of combustion.

### SI combustion phase model

During the SI combustion it is assumed that the spark ignited flame front divides the in-cylinder gas mixture into two zones: the burned and unburned zones as shown in Figure 5. To simplify the two-zone combustion model, the shape of the burned zone is assumed to be a circle centered at the cylinder (see Figure 5). For the SI combustion the temperature of the unburned zone is quite different from that of the burned zone as seen in Figure 11 and Figure 12 (see later). However, this difference is neglected in the one-zone SI combustion model discussed by Yang and Zhu\(^{21}\), where an average temperature is used for both zones. Although the in-cylinder gas temperature stratification affects the local combustion speed and emission formation, the developed control oriented model is not capable of estimating the temperature distribution due to the real-time implementation requirement. It turns out that the simulation error of the in-cylinder pressure using the one-zone model is relatively small. However, for the SI-HCCI hybrid combustion, the unburned zone temperature is an important parameter for predicting the start of HCCI combustion. The expansion of the burned...
zone during the initial SI combustion process applies work to the unburned zone. As a result it makes the unburned zone temperature rise quickly, leading to the start of HCCI combustion in the unburned zone. On the other hand, the two-zone SI combustion model approximates the SI combustion better than the one-zone model. These are the main motivations for developing this two-zone SI combustion model.

To simplify the modeling process, the following assumptions were made in the two-zone SI combustion model:

- The fuel, air, and residual gas charges are uniformly premixed at IVC.
- The pressure of the in-cylinder gas mixture is assumed to be evenly distributed throughout both burned and unburned zones, while the temperatures of the two zones are different.
- At each combustion simulation step (crank degree), it is assumed that a portion of gas–fuel mixture is transferred from the unburned to burned zone, and it is assumed that a portion of gas–fuel mixture is premixed at IVC.

After SI, the fuel mass fraction burned is approximated by the following Wiebe function

\[ x(\theta) = 1 - \exp \left[ -a \left( \frac{\theta - \theta_{ST}}{\Delta \theta} \right)^{m + 1} \right] \]  

(2)

where \( x \) is the MFB of fuel; \( \theta \) is the current crank position; the predicted burn duration \( \Delta \theta \) and the Wiebe exponent \( m \) are calibration parameters as functions (implemented as lookup table) of engine speed, load, and coolant temperature. Coefficient \( a \) depends on how burn duration \( \Delta \theta \) is defined. When \( \Delta \theta \) is specified as the crank angle between 10\% and 90\% MFB, \( a \) can be calculated by

\[ a = \left\{ \left[ -\ln (1 - 0.9) \right]^{m+1} - \left[ -\ln (1 - 0.1) \right]^{m+1} \right\}^{m+1} \]  

(3)

Within the SI combustion phase, the mass of the burned zone gas is calculated based upon the mass of burned fuel that can be calculated by (2). According to the first law of thermodynamics,23 the energy balance of the burned zone is represented by

\[ \frac{d(M_u e_u)}{d\theta} + P \frac{dV_u}{d\theta} + Q_u = \eta_{SI} h_{LHV} M_f \frac{dx}{d\theta} + \frac{dM_u}{d\theta} h_U \]  

(4)

where \( M_u, V_u, e_u \) are the mass, volume, and internal energy of the burned zone, respectively; \( Q_u \) is the heat transfer from the burned zone, where \( Q_u = xQ \) and \( Q \) is given by equation (9); \( M_f \) is the total fuel mass trapped in the cylinder for the given engine cycle; \( P \) is the gas pressure of both zones; \( h_{LHV} \) is the low heating value of fuel; \( h_U \) is the specific enthalpy of the unburned zone; \( \eta_{SI} \) is the combustion efficiency due to incomplete combustion.

The energy balance equation of unburned zone is

\[ \frac{d(M_u e_u)}{d\theta} + P \frac{dV_u}{d\theta} + Q_u = \frac{dM_u}{d\theta} h_U \]  

(5)

where \( M_u, V_u, e_u \) are the mass, volume, and internal energy of unburned zone, respectively; \( Q_u \) is the heat transfer from the unburned zone and \( Q_u = (1-x)Q \) is the portion of total heat transfer to the unburned zone.

Moreover, the gases of both burned and unburned zones can be considered as ideal gases,18,24 and therefore ideal gas law holds for both zones. For the burned zone, we have

\[ \frac{PV_B}{RT_B} = M_B = xM_t \]  

(6)

where \( M_t \) is the total gas mass of both zones; \( T_B \) is the burned zone gas temperature; \( R \) is gas constant.

For the unburned zone, the ideal gas law can be expressed by

\[ \frac{PV_U}{RT_U} = M_U = (1-x)M_t \]  

(7)

where \( T_U \) is the unburned zone gas temperature.

Additionally, the cylinder geometry yields the following equation

\[ V_B + V_U = V \]  

(8)

where \( V \) is the instant cylinder volume.

For an SI gasoline engine the heat transfer due to radiation is relatively small, compared to the convective heat transfer.19 Therefore only convective heat transfer \( Q \) is computed in the energy balance equations. The Woschni correlation model19,20 is used to calculate the heat transfer quantity

\[ Q(\theta) = A_c h_c [T(\theta - 1) - T_w] \]  

(9)

and

\[ h_c = q B^{l-1} P w^l T^{0.75-1.62l} / N_e \]  

(10)

where \( B \) is the cylinder bore; \( w \) is the gas flow velocity that is a function of engine speed \( N_e \); \( A_c \) is the contact area between gas and cylinder wall; \( T_w \) is the average temperature of the cylinder wall; coefficient \( q \) and exponent \( l \) are used as model calibration parameters, where \( q = 0.54 \) and \( l = 0.8 \) provide the best correlation for the SI combustion mode.

Note that the gas temperature \( T \) in equations (9) and (10) is the average temperature of both burned and unburned zones that can be calculated by

\[ T = \frac{\chi C_{vB} T_B + (1-x)\chi C_{vU} T_U}{\chi C_{vB} + (1-x)\chi C_{vU}} \]  

(11)

where \( C_v \) is the specific heat for constant volume.
**HCCI combustion phase model**

The HCCI combustion is modeled under the one-zone assumption due to its flameless nature. Fuel and air are assumed to be mixed homogeneously throughout the unburned gas mixture; thermodynamic characteristics such as pressure and temperature are uniformly distributed in the cylinder. Accordingly, only the mean values of in-cylinder gas pressure and temperature are modeled.

Unlike the SI combustion, there is no direct control (such as spark for SI combustion) to initiate the HCCI combustion. The fast heat release rate of the HCCI combustion is actually triggered by a very slow chemical reaction of the gas–fuel mixture during the compression phase between the IVC and start of HCCI combustion, where it takes certain crank degrees for one percent fuel be burned. The most commonly used approach in control-oriented modeling of the HCCI combustion is to assume that the chemical reaction process is governed by a single rate Arrhenius equation.

\[
AR = A_0 e^{rac{Ea}{RT}}
\]

where \(AR\) is the rate of disappearance of unburned fuel and other parameters have been discussed in equation (1). As mentioned at the beginning of the second section the integration of the Arrhenius function is used to estimate the SOHCCI timing, where the SOHCCI timing separates the slow chemical reaction from the fast one. During the fast combustion phase the fuel MFB can also be approximated by the following Wiebe function

\[
x(\theta_i) = 1 - \exp\left[-a \left(\frac{\theta_i - \theta_{SOHCCI}}{\Delta \theta_{HCCI}}\right)^m + 1\right]
\]

where the coefficients \(a\), the exponential \(m\) and the combustion duration \(\Delta \theta_{HCCI}\) are functions of engine speed, load (fuel quantity for HCCI combustion), and coolant temperature. They are different from those used in the SI combustion model (see equation (2)). Based on the calculated MFB, the in-cylinder gas pressure and temperature are calculated by

\[
T(\theta_i) = T(\theta_{i-1}) \left(\frac{V(\theta_{i-1})}{V(\theta_i)}\right)^{(k-1)}
+ \eta_{HCCI} M_{\text{ciHCCI}} \left[x(\theta_i) - x(\theta_{i-1})\right] - Q(\theta_i)
\]

and

\[
P(\theta_i) = P(\theta_{i-1}) \left(\frac{V(\theta_{i-1})}{V(\theta_i)}\right) \cdot \frac{T(\theta_i)}{T(\theta_{i-1})}
\]

where \(k\) is the average heat capacity ratio of the gas charged into the cylinder; \(\eta_{HCCI}\) is a function of engine speed and fuel mass, and it is a calibrated to match the simulated Indicated mean effective pressure (IMEP) is provided by the GT-Power model. The in-cylinder gas temperature is derived based on isentropic assumption and energy balance of a closed system.

There are two terms in the right hand of equation (14). The first term represents an isentropic compression or expanding process, and the second calculates the temperature increase due to the heat released during the combustion. Therefore, the complicated combustion thermodynamics is simplified into an isentropic volume change process without heat exchange in one crank degree period and a heat exchange process without volume change in an infinitely small time period.

**Compression phase model**

The governing equations of the gas compression phase between IVC and ST are the same as equations (14) and (15) with \(x = 0\) throughout the phase.

**Gas exchange phase models**

As shown in Figure 3, the gas exchange process occurs between EVO and IVC. During this process exhaust gas leaves the cylinder, and fresh charge enters the cylinder and mixes with residual gas from the last engine cycle. The gas exchange process is normally very complicated and difficult to model for real-time simulations. A CFD model is required to model it accurately, but it is too complicated to be used in the control oriented model due to the real-time simulation requirement. This paper combines both correlation and physical modeling approaches to model the gas exchange process.

**Gas exhaust process model**

The gas exhaust phase corresponds to the phase between EVO and IVC and the phase between EVO and EVC for NVO. In this phase, the in-cylinder gas mixture expands in the cylinder, exhaust runner, and exhaust manifold. The in-cylinder pressure drops quickly, but not instantaneously, down to the level of the exhaust manifold pressure, see Figure 6. It normally takes a few crank degrees for the in-cylinder pressure to reach the exhaust manifold pressure. It is quite difficult to model this process accurately using simple dynamic equations for real-time simulations. For simplicity, a first-order transfer function is used to approximate this process as follows

\[
P(z) = \frac{1 - \tau_{EVO}}{1 - \tau_{EVO} \cdot z^{-1}} P_{EM}(z)
\]

where \(z\) is the unit delay operator for one crank degree; \(\tau_{EVO}\) is the transition time constant as a function of engine speed and load for exhaust valve opening; \(P_{EM}\) is the exhaust manifold absolute pressure.

The in-cylinder gas temperature is calculated as a function of the pressure drop. That is

\[
T(\theta_i) = T(\theta_{i-1}) \left[\frac{P(\theta_i)}{P(\theta_{i-1})}\right]^{\frac{z_{EM}}{T}} - \frac{Q(\theta_{i-1})}{M(\theta_{i-1})C_v}
\]
By investigating the pressure signal from GT-Power simulation in Figure 6, one can find that during the late stage of the gas exhaust phase the in-cylinder pressure increases significantly, and it finally reaches 1.9 bar at EVC, which is much bigger than the exhaust manifold pressure (1.0 bar in this case). There are two possible causes. One is that the exhaust valve is almost closed during the late stage, which prevents gas from leaving the cylinder; and the other is that the EVC timing is before gas exchange TDC. This phenomenon cannot be ignored since \( p(\theta_{EVC}) \) is used to calculate residual gas mass \( M_r \), which affects HCCI combustion timing and duration significantly. The dynamics of the pressure increase can also be modeled using a first-order transfer function

\[
P(z) = \frac{1 - \tau_{EVC}}{1 - \tau_{EVC} \cdot z^{-1}} \cdot P_E(z) \eta_{EVC}
\]

where \( \eta_{EVC} \) is the predicted pressure ratio at EVC, and it is mainly affected by engine speed, EVC timing and exhaust valve lift; \( \tau_{EVC} \) is the transition time constant of EVC and it is a function of engine speed and load. As shown in Figure 6, this model yields a good match between the pressure signals provided by simulation results of both GT-Power and the control oriented combustion models, especially at the EVC timing.

**Residual gas recompression during NVO operation**

NVO is often used to regulate the HCCI combustion properties such as combustion timing. There are two main advantages. One is to reform the trapped pilot fuel in this phase (EVC-IVO in Figure 3);\(^{25}\) and the other is to adjust the residual gas temperature. As a result the in-cylinder gas temperature at IVC can be optimized for the desired SOHCCI timing. The first effect is hard to model using governing equations. In Ravi et al.,\(^{25}\) the Arrhenius threshold is correlated based on the experimental data. It is well known that pilot fuel reforming due to recompression can effectively affect HCCI combustion timing and the pilot fuel injection timing and quantity can be used as control parameters in a few HCCI control strategies. However, during the SI to HCCI mode transition of a multi-cylinder engine equipped with electrical VVT and a two-step valve system, the reforming effect is less significant than that in the steady HCCI mode due to the electrical VVT response dynamics, which reduces the effect of negative valve overlap and impairs the recompression of residue gas and fuel. Therefore, this paper ignores this effect due to the lack of the experimental data. Although the recompression has less effect on combustion timing control during the combustion mode transition than during the steady state HCCI operation, reforming the pilot fuel during the recompression could still be useful. It can be used as a quick control variable to micro-tune the combustion timing for the consequent combustion event.

The second effect can be approximated by a combination of an isentropic volume change process of ideal gas in a closed system and a heat transfer process. Temperature and pressure are calculated by

\[
T(\theta) = T(\theta_{\mu-1}) \cdot \left( \frac{V(\theta_{\mu-1})}{V(\theta_{\mu})} \right)^{\frac{(x-1)}{x}} - \frac{Q(\theta_{\mu-1})}{M(\theta_{\mu-1}) C_v}
\]

and

\[
P(\theta) = P(\theta_{\mu-1}) \cdot \left( \frac{V(\theta_{\mu-1})}{V(\theta_{\mu})} \right)^x
\]

Another important parameter calculated in this phase is the residual gas mass. It is calculated based upon the ideal gas law, and updated once per engine cycle at EVC for NVO

\[
M_r = M(\theta_{EVC}) = \frac{P(\theta_{EVC}) \cdot V(\theta_{EVC})}{R \cdot T(\theta_{EVC})}
\]

**Gas exchange behavior during PVO**

The gas exchange behavior during the phase between IVO and EVC for the PVO is also very complicated. Since both intake and exhaust valves are partially opened in this phase, residual gas and fresh charge can flow in many ways depending on the pressure ratio across each valve. For simplification, it is assumed that gas exchange across the exhaust valve is terminated in this phase and part of the residual gas can flow to the intake port but it flows back into the cylinder completely during the remaining intake process. Based upon this assumption, the mass of residual gas for PVO can be calculated at IVO by

![Figure 6. In-cylinder pressures in gas exchange process. RPM: revolutions per minute; MAP: intake manifold air pressure; EMP: exhaust manifold pressure; EVC: exhaust valve closing; IVO: intake valve opening; IVC: intake valve closing; ACTDC: after combustion top dead center.](image-url)
This modeling approach may result in slight error in \( M_r \) calculation. Fortunately, the PVO only occurs during the SI combustion process, in which the influence of residual gas to the entire engine performance is much less than that during the SI-HCCI and HCCI combustion processes that usually involve NVO.

For the cooled EGR, the EGR valve was modeled according to the valve model described in Fiveland and Assanis, and the EGR cooler was modeled based on the equation (12.2) of the ‘WOT’ website.

Air intake process model

The air intake process from IVO to IVC is also a process of in-cylinder gas mixing. During this phase, the fresh charge air, injected fuel vapor, and residual gas are assumed to be mixed homogeneously. At the same time, the in-cylinder pressure approaches the intake manifold pressure. A first-order transfer function is also used in this model as follows:

\[
P(z) = \frac{1 - \tau_{IVO}}{1 - \tau_{IVO} \cdot z} \cdot P_{IM}(z) \tag{23}
\]

and

\[
T(\theta_i) = \frac{M_f \cdot C_{sf} \cdot T_i + M_a \cdot C_{sa} \cdot T_{IM} + M_r \cdot C_{sr} \cdot T_r}{M_f \cdot C_{sf} + M_a \cdot C_{sa} + M_r \cdot C_{sr}} \tag{24}
\]

where \( \tau_{IVO} \) is the transition time constant for intake valve opening, which is a function of engine speed and load; \( P_{IM} \) is the intake manifold absolute pressure; \( M_a \) is the fresh charge mass (note that it could be the mixture of fresh air and external EGR gas); \( T_r \) is the temperature of residual gas and it can be calculated by the approach described in equation (17).

During the late stage of the air intake phase, the in-cylinder pressure may also deviate from the intake manifold pressure, see Figure 6. This phenomenon is similar to the case of the exhaust phase. The same approach is used to model the transition as follows:

\[
P(z) = \frac{1 - \tau_{IVC}}{1 - \tau_{IVC} \cdot z} P_{IM}(z)\eta_{IVC} \tag{25}
\]

where \( \eta_{IVC} \) is the predicted pressure ratio at IVC, which is a function of engine speed, IVC timing and exhaust valve lift; and \( \tau_{IVC} \) is the transition time constant for intake valve closing and it is a function of engine speed and load. At IVC, the total in-cylinder gas mass \( M_t \) is calculated by

\[
M_t = M(\theta_{IVC}) = \frac{P(\theta_{IVC}) \cdot V(\theta_{IVC})}{R \cdot T(\theta_{IVC})} \tag{26}
\]

Model implementation in HIL simulation environment

This section describes the dSPACE based real-time HIL engine simulation environment and the implementation of the mathematical models presented in the third and fourth sections into the HIL simulation environment.

Combustion model solution

The entire engine model consists of numerous mathematical equations. Some of them are nonlinear differential equations. To solve these equations analytically in real-time is almost impossible. During the HIL simulations, these equations are solved numerically. This sub-section discusses the numeric solution of the combustion model presented in this paper.

For most of the mathematical models presented in the third and fourth sections, the combustion variables of present crank angle degree are directly calculated based upon those of the previous crank angle degree. Whereas in the two-zone SI combustion model, the governing equations (4) to (8), are nonlinear differential equations. To solve them, the equations are discretized and converted into the following format:

\[
\begin{align*}
\mathbf{a}_1 V_{\text{g}}(\theta_i) + b_1 T_{\text{g}}(\theta_i) &= c_1 \\
\mathbf{a}_2 V_{\text{f}}(\theta_i) + b_2 T_{\text{f}}(\theta_i) &= c_2 \\
V_{\text{g}}(\theta_i) P(\theta_i)/T_{\text{g}}(\theta_i) &= c_3 \\
V_{\text{f}}(\theta_i) P(\theta_i)/T_{\text{f}}(\theta_i) &= c_4 \\
V_{\text{g}}(\theta_i) + V_{\text{f}}(\theta_i) &= c_5
\end{align*} \tag{27}
\]

where

\[
\begin{align*}
\mathbf{a}_1 &= P(\theta_{i-1}) \\
b_1 &= x(\theta_i) M_c C_v \\
\mathbf{a}_2 &= P(\theta_{i-1}) \\
b_2 &= [1 - x(\theta_i)] M_c C_v \\
c_1 &= \eta_{IVC} M_f x(\theta_i) - x(\theta_{i-1}) + P(\theta_{i-1}) V_{\text{g}}(\theta_{i-1}) - x(\theta_i) Q(\theta_i) \\
&+ M_f x(\theta_{i-1}) C_v T_{\text{g}}(\theta_{i-1}) + M_f x(\theta_i) - x(\theta_{i-1}) C_p T_{\text{f}}(\theta_{i-1}) \\
c_2 &= P(\theta_{i-1}) V_{\text{f}}(\theta_{i-1}) - M_f x(\theta_i) - x(\theta_{i-1}) C_p T_{\text{f}}(\theta_{i-1}) \\
&+ M_f [1 - x(\theta_{i-1})] C_v T_{\text{g}}(\theta_{i-1}) - [1 - x(\theta_i)] Q(\theta_i) \\
c_3 &= x(\theta_i) M_c R \\
c_4 &= [1 - x(\theta_i)] M_c R \\
c_5 &= V(\theta_i)
\end{align*}
\]

By solving the algebraic equations set (27), the five unknowns \( [V_{\text{g}}(\theta_i) V_{\text{f}}(\theta_i) T_{\text{g}}(\theta_i) T_{\text{f}}(\theta_i) P(\theta_i)]^T \) in (27) can be represented as functions of the information computed in the last crank angle degree \( [V_{\text{g}}(\theta_{i-1}) V_{\text{f}}(\theta_{i-1}) T_{\text{g}}(\theta_{i-1}) T_{\text{f}}(\theta_{i-1}) P(\theta_{i-1}) x(\theta_{i-1})]^T \) and the MFB of current crank angle degree \( x(\theta_i) \) is calculated by the Wiebe function (2). Equation (27) was solved analytically using Matlab Symbolic Math Toolbox, and the rational solution, see Appendix 3, was used for numerical solution of the algebraic equations set (27) in the HIL simulations.

Note that due to the nonlinearity of equations (6) and (7) the solutions of equation set (27) are not
unique. After implementing the model into the dSPACE HIL simulation environment for real-time simulation, the two-zone engine model can be simulated in real-time at engine speeds up to 5000 rpm without overrun. Fortunately, the possible operating range of the two-zone model (SI-HCCI hybrid mode) is between 1000 rpm and 3500 rpm.

For the SI-HCCI combustion simulations, the in-cylinder gas temperatures of the one-zone HCCI model and the two-zone SI model are updated every crank degree during the SI combustion. The SI unburned zone temperature is mainly used to accurately estimate the start of HCCI combustion. Once the start of HCCI combustion criterion is satisfied, updating of both burned and unburned zone temperatures stops, and the overall in-cylinder gas temperature is updated till the end of the current engine cycle due to the continued heat release of HCCI combustion and volumetric change in the cylinder. In-cylinder gas pressure is assumed to be identical over both zones for the two-zone SI model.

**HIL simulation system architecture**

Figure 7 shows the system architecture of the HIL simulation environment. For simplicity, not all the signals and subsystems are included in the diagram. It is only used to demonstrate the implementation of the engine model into the HIL simulation environment. In Figure 7, the ECU (engine control unit) on top of the diagram is used for implementing the combustion mode transition control strategies that are presented by Yang and Zhu, and it is not the focus of this paper. All blocks below the ECU block are to be implemented in the HIL engine simulator. Within the combustion block, the mathematical models proposed in Sections 3 and 4 are updated every crank angle degree. The wall wetting dynamic model of PFI (port fuel injection) fueling as represented by the fuel inject block is updated every engine cycle for each injector. The wall wetting dynamics describes the PFI fuel injection dynamics where part of the injected fuel flows into the cylinder directly and part of it remains on the back of the intake valve, and can be found in Fiveland and Assanis. The rest of the blocks are the time-based (1 millisecond) models for crankshaft torque balance, air intake/exhaust and EGR flow dynamics, etc. Accordingly, there are three parallel tasks executed in the engine simulator processor (dSPACE DS-1006 CPU). Note that the mathematical models for all blocks other than the combustion model block are presented by Yang and Zhu. They are well developed mean-value engine models that are widely utilized in numerous technique papers.

During the HIL simulations, engine crank signal (720 pulses per cycle), TDC signal (2 pulses per cycle) and cam signal (1 pulse per cycle) are generated based on the modeled engine speed $N_e$ using the dSPACE hardware (DS-2211 APU board). These signals are sent to the ECU to synchronize the engine simulator and the ECU.

At the same time, crank and cam signals are looped back to the HIL simulation CPU board to generate interruption signals for crank and cycle based simulations, and to trigger crank based and cycle based model calculations. The interruption triggered by crank signal has the highest priority in the interruption queue, since the crank based routine needs to be executed within a specific crank angle. The cycle based interruption has the second highest priority. On the output side (right) of the engine simulation diagram, the signal conditioning block converts all computed engine variables to actual engine signals that the ECU can utilize. On the input side (left) of the engine simulation diagram, the signal preprocessing block translates all ECU control signals back to the parameters that the engine model can use.

A diagram of the entire HIL engine simulation platform is shown in Figure 8. Two host computers are used to interact with the dSPACE based real-time engine simulator and the Opra-RT based engine prototype controller, respectively. They are used to set the simulation parameters and to display the simulation results. An oscilloscope is also adopted to display the real-time HIL simulation results. The nonlinear equations (4) to (8) are solved using the solution provided in Appendix 3.

**Model calibration and validation**

To calibrate and validate the developed SI-HCCI hybrid combustion model, a GT-Power single cylinder engine model was also developed for the engine configuration shown in Figure 2 with specifications shown in Table 1. GT-Power is an engine and powertrain modeling tool widely used in the automotive industry.
Comparing with the combustion model developed in this paper, the 1D (one dimensional) combustion model employed by the GT-Power software provides high-fidelity in-cylinder combustion information such as pressure and temperature. However, its high computational throughput prevents it from being directly used for the real-time HIL simulations. In this paper, the GT-power model is used as the reference model to calibrate and validate the combustion model developed in this paper.

Model calibration data were generated by GT-Power simulations over the engine speed and load operational ranges. For each evaluation condition, GT-Power simulation results were used as the baseline. Firstly, $h_{EVC}$ and $h_{IVC}$ (look-up tables) in equations (18) and (25) were calibrated as functions of engine speed, valve timing and lift to make the in-cylinder gas pressure at EVC and IVC match with the GT-Power simulation results; secondly, coefficients such as $m$ and $k$ in equations (2) and (14) were calibrated to match the MFB profiles, finally, $h_{SI}$ and $h_{HCCI}$ in equations (4) and (14) were calibrated as functions of engine speed and load to match engine IMEP.

Two-zone SI combustion model validation

The two-zone SI combustion model was validated at 3000 rpm under three different engine load conditions with intake manifold pressure $P_{IM} = 0.4$, 0.7 and 1.0 bar corresponding to low, medium, and full load conditions. The combustion durations under the three conditions are quite different, the associated spark timings were set to the MBT (Minimal advance for the Best Torque) at each load condition, and the valve strategy is PVO. Fairly good agreement of both pressure and temperature signals can be found in simulation results shown in Figure 9 and Figure 10. This demonstrates that the developed two-zone SI combustion model is capable of providing comparable simulation results to those of the GT-Power model. Note that the in-cylinder temperature of the two-zone SI combustion model, see Figure 10, is the averaged temperature, provided by equation (11), of both burned and unburned zones. Although the modeling error of temperature is fairly large at certain crank angles, especially around the exhaust valve opening timing, the error at the region of interest (for instance, start of combustion) is fairly small.

Figure 11 and Figure 12 present the two temperature profiles (burned and unburned zones) of the two-zone SI combustion model, where the burned zone temperature is shown in Figure 11 and unburned zone temperature in Figure 12. Note that the one-zone model can only provide the averaged temperature of the burned and unburned zones. Both temperature profiles show good agreement with the GT-Power simulation results. This confirms the effectiveness of the developed two-zone model.

In Figure 12 both unburned zone temperatures are also compared with the in-cylinder gas temperature simulated without combustion. It shows that before
sparking \( ST = -17° \) ACTDC in this case, the three temperatures are the same; and after the spark timing, the unburned zone temperature increases much faster than that without combustion. This is due to the work applied by the burned zone gas. The Arrhenius integration in equation (1) is more sensitive to higher unburned zone temperature, which indicates that the slow chemical reaction in the unburned zone before SOHCCI can be accelerated by the rapid increment of the unburned zone temperature. Accordingly if the unburned zone gas temperature can be controlled precisely, so can the SOHCCI timing of both HCCI and SI-HCCI hybrid combustions.

To validate the combustion model over a broad engine operational range, additional simulations were conducted at the engine speed varied between 1000 rpm and 5000 rpm with a 1000 rpm interval and low, medium, and full engine loads. Both PVO and NVO valve strategies were used. The valve timings for PVO are IVO = 340° ACTDC and EVC = 380° ACTDC and for NVO, IVO = 380° ACTDC and EVC = 340° ACTDC. To simplify the presentation, only a few key engine variables were plotted in Figure 13 and Figure 14. Note that these key variables, shown in Figure 13 and Figure 14, outline the agreement of the pressure, temperatures (burned and unburned) of both GT-Power and developed models.

From Figure 13 and Figure 14, one can find that all pressure related variables, such as IMEP, peak cylinder pressure \( P_{\text{max}} \) and the crank position of \( P_{\text{max}} \), have fairly good agreement with those of the GT-Power simulations, while for the temperature related variables, the errors are relatively larger. One possible reason is that the gas property parameters such as \( C_v \) and \( R \) used in the two models are different. In the GT-Power model these parameters are functions of not only the gas temperature but also the gas chemical composition, while in the developed two-zone model the chemical composition is not considered due to the simplified real-time modeling.

**One-zone HCCI combustion model validation**

Figure 15 shows the HCCI combustion simulation results using the developed one-zone and GT-Power models. For this simulation, 16 mg of fuel was injected into each cylinder per engine cycle to generate 5.02 bar IMEP at 2000 rpm, which is close to the load limitation of the HCCI combustion. Note that for HCCI combustion, the engine load is not determined by the engine MAP but by the injected fuel quantity due to the un-throttled operation. The lift profiles of both intake and exhaust valves were switched to the low lift stages, since the low exhaust valve lift makes it possible to trap more internal residual gas with the NVO operation and the low intake valve lift enables un-throttled engine operation.

Figure 15 presents the simulation results of the in-cylinder pressure, temperature, and the normalized HRR (heat release rate) obtained by taking the derivative of the MFB \(^{27}\) (a magnified HRR plot is also provided). From Figure 15, it can be seen that there is significant difference in HRR at the start and end of the combustion. This is mainly due to the different combustion models used, where a combination of three Wiebe functions was used in the GT-Power model to describe the start, intermediate, and end of combustion and the proposed model uses only one Wiebe function. This results in a fairly large difference at the start and end of combustions. The 50% MFB locations for the proposed and GT-Power model are 1.19 and 2.01° after TDC, respectively.

The engine intake valve timing was advanced to reduce the effective compression ratio and to avoid engine knock at the upper load limit of the HCCI combustion. The HCCI combustion timing is also sensitive to both intake and exhaust valve timings. Proper valve timing selection leads to improved combustion.
Figure 13. Correlation of the two-zone and GT-Power SI combustion model: $M_t$, $M_r$, $P_{\text{max}}$, and $P_{\text{max}}$ location. PVO: positive valve overlap; NVO: negative valve overlap.

Figure 14. Correlation of the two-zone SI combustion models: $T_{\text{max}}$, $T_{\text{u-max}}$, $T_{\text{IVC}}$ and IMEP.
efficiency. For the simulation results shown in Figure 16, the intake valve opening (IVO) timing was between 360 and 400°/C176 after TDC, and the exhaust valve closing (EVC) timing was between 310 and 350°/C176 after TDC. The simulation results were also compared with the corresponding GT-Power simulation results with good agreement. Note that the HCCI combustion model used in the GT-Power engine model is a predictive, single-zone model. The chemical reaction properties are defined by the GT-Power reference object “EngCylChemGas”. The HCCI combustion model was not calibrated by test data for the engine configuration described in this paper since the main purpose of this paper is to demonstrate the potential of using the control oriented combustion model to match the complicated, chemical reaction based combustion model.

Due to the unavailability of the GT-Power SI-HCCI combustion model, the simulation results of the SI-HCCI combustion model were not compared with that of the GT-Power model and this will be part of the future work when the experimental data are available. It is worth mentioning that the simulation results of the developed SI-HCCI combustion model have the same trend, compared to the experimental results of the spark assisted HCCI combustion shown by Zhang et al.8 The simulations of the SI-HCCI combustion mode transition control can be found in Yang.28

Conclusions
This paper presents a control oriented SI-HCCI hybrid combustion model of an HCCI (homogeneously charge compression ignition) capable SI (spark ignited) engine. The developed combustion model is capable of simulating the SI-HCCI hybrid combustion that starts with the SI combustion and ends with the HCCI combustion,
where the SI and HCCI combustions are its special cases. The SI and HCCI combustions were modeled under the two-zone and one-zone assumptions, respectively. The developed combustion model was implemented in an HIL (hardware-in-the-loop) real-time engine simulation environment. The HIL simulation results were validated with the data generated by the corresponding GT-Power model. It is concluded that the developed SI-HCCI hybrid combustion model can be used for real-time simulation of the combustion mode transition; the accuracy of the SI and HCCI combustion model is comparable to the relatively high-fidelity GT-Power model; and it is suitable for engine control strategy development and validation.

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References

Appendix I

Abbreviations

ACTDC after combustion TDC
ECU engine control unit
EGR exhaust gas recirculation
EVC exhaust valve closing
EVO exhaust valve opening
Appendix 2

List of simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
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<th>Unit</th>
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<td>Oxidizer concentration</td>
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Appendix 3

Solution of equation (27)

$$V\theta_1 = (b_2c_1c_3 - D^{0.5} + b_1c_2c_4 + a_1b_2c_3c_5 - a_2b_1c_4c_5)/2C$$
$$V\theta_2 = -(b_2c_1c_3 - D^{0.5} + b_1c_2c_4 - a_1b_2c_3c_5 + a_2b_1c_4c_5)/2C$$
$$T\theta_1 = -(a_1c_2c_3 + a_2c_1c_4 - a_2(D^{0.5} + b_2c_1c_3 + b_1c_2c_4 - a_1b_2c_3c_5 - a_2b_1c_4c_5)/2b_1 - a_1a_2c_4c_5)/C$$
$$T\theta_2 = (a_1c_2c_3 + a_2c_1c_4 - a_2(D^{0.5} + b_2c_1c_3 + b_1c_2c_4 - a_1b_2c_3c_5 - a_2b_1c_4c_5)/2b_1 - a_1a_2c_4c_5)/C$$
$$P(\theta_1) = (D^{0.5} + b_2c_1c_3 + b_1c_2c_4 - a_2b_1c_4c_5 + a_2b_1c_4c_5/(2b_1b_2c_3c_5)$$

With

$$C = a_1b_2c_3 - a_2b_1c_4$$
$$D = a_1^2b_2^2c_3^2c_4^2 - 2a_1a_2b_1b_2c_3c_4c_5 + 2a_1b_2b_2c_3c_4c_5 - 2a_1b_2c_1c_3c_4c_5 + a_2b_2^2c_4^2c_5^2$$
$$-2a_2b_2^2c_4^2c_5 + 2a_2b_1b_2c_1c_3c_4c_5 + b_1^2c_3^2c_4^2 + 2b_1b_2c_1c_3c_4 + b_2^2c_4^2c_5$$