Optimal Profile Tracking for an Electro-Hydraulic Variable Valve Actuator using Trajectory Linearization

Huan Li, Guoming G. Zhu, Ying Huang, and Donghao Hao

Abstract—The camless valve is able to provide flexible engine valve profiles (timing, duration, lift, etc.) to improve the performance of internal combustion engines. To provide a precise valve profile of an electro-hydraulic variable valve actuator (EHVVA) for the desired engine performance, an optimal tracking controller for the valve rising duration and profile is designed in this paper. A nonlinear model, elaborating the system pressure dynamics determining the valve rising duration, is developed and linearized along the desired rising valve trajectory. Based on the trajectory linearization, a linear quadratic tracking (LQT) controller is designed with Kalman optimal state estimation. The equilibrium control resulted from the trajectory linearization is used as the LQT feedforward control. The control performance is compared with that of baseline controllers through both simulation study and bench tests. The transient and steady-state validation results confirm the effectiveness of proposed control scheme.

I. INTRODUCTION

The increasing concerns of air pollution and energy concerns lead to electrification of the vehicle powertrain system in recent years. For internal combustion engines (ICE), which have been the dominant vehicle power source for more than a century, it becomes necessary to employ advanced technologies to replace traditional mechanical systems with mechatronic systems to meet the ever-increasing demand of continuous improving efficiency and reduced emissions.

The camless VVA (variable valve actuation) is one of the promising technologies that is able to freely adjust the valve profile (timing, duration and lift, etc.) and thus is able to provide additional degree of freedoms to further improve the engine combustion efficiency [1], [2]. Without the help of camshaft for valve timing control, the VVA depends on active electronic control of valve profile to guarantee the desired engine performance. The valve profile tracking problem includes four basic control objectives for most camless VVAs: 1) valve timing (opening and closing) control for optimizing combustion phase and preventing valve collision with the engine piston, 2) valve lift control, 3) profile area (integration of valve displacement over time) control for accurate air charge and exhaust quantities, and 4) soft seating control for reducing valve noise and improving valve durability. Note that the overall valve duration control and the valve transition response (rising/falling) control studied in literature can be classified into valve timing and profile area control, respectively. For the traditional cam-based engine valves, above properties are automatically guaranteed by proper designing cam profile.

The profile tracking problem for camless VVAs has been studied intensively and the four control objectives discussed can be achieved either individually or simultaneously, depending on a specific VVA system design and its application. Adaptive peak lift control was employed in [3] and [4] for repeatable valve lift. Feedforward control is commonly used for valve timing control to compensate for the valve opening or closing delay [5], [6]. Soft seating control is a challenge for the electro-magnetic VVA due to the nonlinear magnetic force and was addressed in [7] with guaranteed valve response using extremum seeking control, and reference [8] used the combination of feedforward LQR (linear quadratic regulator) and repetitive learning control to reduce cycle-to-cycle variations. These control designs were conducted for single or multiple control objectives and some other studies dealt with the overall valve profile control as a single tracking problem. For instance, in [9] the model reference control combined with repetitive control was used for the VVA to achieve asymptotic profile tracking performance. The sliding mode control was employed in [10] to achieve repeatable tracking performance with guaranteed seating velocity. The robust repetitive control in [11] and time-varying internal model-based control in [12] were proved to be very effective to track the desired flexible valve profile both in steady state and transient engine operations.

In this paper, the profile tracking problem is studied for an electro-hydraulic variable valve actuator (EHVVA) system (see [13]) that has dual-lift and build-in soft-landing feature guaranteed mechanically. As a result, complicated control scheme is not required for lift and seating velocity control. However, the valve timing and profile area control are challenging for the studied EHVVA system due to the nonlinear and time-varying nature of the hydraulic system. These nonlinearities include nonlinear flow dynamics and temperature-sensitive fluid viscosity; see previous publications [14] and [15]. Therefore, this paper studies the profile area tracking control that is not studied for this EHVVA.

An event-by-event optimal tracking control scheme is proposed for the valve rising duration and profile. A control-oriented valve profile model was constructed based on the system supply pressure affected by the fluid supply system and engine speed. Due to the nonlinear relationship between the valve rising duration and supply pressure, the system dynamic model was linearized along the desired valve rising trajectory and a discrete-time event-by-event model
was developed accordingly. The equilibrium control input obtained during the model linearization process was used for feedforward control and a finite horizon linear quadratic tracking (LQT) controller was designed for tracking control in a closed-loop based on the linearized event-by-event model.

The remaining of the paper is organized as follows. Section II provides the overview and control problem formulation for the target EHVVA system. Section III presents the control-oriented model, model linearization along the desired trajectory, and the discrete-time event-by-event model. The finite horizon LQT controller is designed based on the discrete-time event-by-event model in Section IV. The simulation study is presented in Section V and the experimental validation results are presented in Section VI. Conclusions are drawn in Section VII.

II. SYSTEM OVERVIEW AND CONTROL OBJECTIVE

A. EHVVA System

Fig. 1 shows the schematic of the studied EHVVA system with two discrete lifts. The engine valve is driven by the supply pressure \( P_h \) when the valve actuator is activated and closed by the return spring force when the valve actuator is deactivated. The dual-lift (high or low lift) is realized by a solenoid lift valve by opening or closing the lift porter to change the lift control sleeve position. The system supply pressure \( P_h \) is regulated by a hydraulic pump driven using a DC motor. A predetermined back pressure \( P_l \) is set higher than the fluid tank pressure but lower than \( P_h \) to make the lift control sleeve rest steadily on its lower and upper positions, respectively, to guarantee accurate lift control. Precise build-in structure designs including notches, undercuts and orifices inside the actuation cylinder are used to achieve soft landing. The details of the EHVVA system can be found in [14].

B. Control Problem Description

Fig. 2 shows the test data under different supply pressure \( P_h \), and illustrates the control objectives of the EHVVA system as discussed in the introduction section. With mechanically guaranteed lift control, soft-seating control, and previously addressed valve timing control [15], the remaining task for the EHVVA system is the tracking control of valve profile area, which is dominated by the valve rising and falling durations (defined as the valve transition time between 10% and 90% lift), where the dwell and peak lift are determined by valve timing (opening and closing) and lift valve, respectively. From Fig. 2, the falling profile and duration \( (t_{vfd}) \) are almost fixed due to the fixed back pressure and return spring stiffness, while the rising profile and duration \( (t_{vrd}) \) vary significantly with the supply pressure caused by varying engine speed; see discussions in the Sec. III A. The relationship between the supply pressure \( P_h \) and valve rising duration \( t_{vrd} \) has been studied using bench test data and a curve-fitted model is developed using a second-order polynomial over the typical operational range; see Fig. 3. It can be seen from both Figs. 2 and 3 that it becomes very difficult to increase the valve rising response (i.e. reduce the rising duration) within higher supply pressure region. Therefore, since the valve profile is dominated by the rising duration (supply pressure), a tracking controller can be used to precisely regulate the supply pressure to achieve desired valve rising duration/profile.

III. SYSTEM MODELING

A. System Dynamics

The system supply pressure is determined by the system inlet flow rate regulated by the DC motor (see Fig. 1) using an analog voltage reference signal \( V_m \) (0~5V) and the system outlet flow rate is affected by the periodic valve
event as a function of engine speed $N_e$ due to the pulse-flow loss. The system supply pressure can be modelled as follows.

$$P_h = k_1 N_e \frac{1}{1 + \tau_1 s} + k_2 V_m \frac{1}{1 + \tau_2 s} + c_0$$

where $k_1$, $k_2$, and $c_0$ are calibration coefficients; $\tau_1$ and $\tau_2$ are the time constants for the outlet and inlet flow rates, respectively. The model is calibrated by comparing the test data and the model results in Simulink.

Let $x_1 = k_1 N_e \frac{1}{1 + \tau_1}$ and $x_2 = P_h - c_0$ denote the pressure component determined by engine speed and the system supply pressure with an offset $c_0$, respectively. Then the 2nd order polynomial in Fig. 3 can be expressed as $y = ax_2^2 + bx_2 + c$, where $y$ denotes the valve rising duration $t_{rd}$; and $a$, $b$, and $c$ are identified calibration coefficients. This leads to a 2nd order nonlinear model for the system.

$$\begin{align*}
    x_1 &= \frac{1}{\tau_1} x_1 + \frac{k_1}{\tau_1} d \\
    x_2 &= \frac{1}{\tau_2} x_1 - \frac{k_2}{\tau_2} x_2 + \frac{k_1}{\tau_1} d \\
    y &= ax_2^2 + bx_2 + c
\end{align*}$$

where $x = [x_1, x_2]^T$ represents the system state vector and $y$ is the output; $d$ represents the engine speed $N_e$ considered as an exogenous input to the system; and $u$ represents the control input $V_m$.

B. Trajectory Linearization

It can be seen that (2) is a nonlinear model due to the exogenous input $d$ and the nonlinear output equation $y$. A linearized model is needed for the optimal linear quadratic tracking control design. At each operation point along the desired tracking trajectory, the system model can be linearized using the classical Jacobian linearization, i.e., model (2) can be transformed into the linearized form:

$$\begin{align*}
    \Delta x &= A \Delta x + B_u \Delta u + B_d \Delta d \\
    \Delta y &= C \Delta x
\end{align*}$$

where $\Delta x = x - x_0$, $\Delta y = y - y_0$, $\Delta u = u - u_0$, and $\Delta d = d - d_0$ are the variation variables and the system coefficient matrices are determined as follows.

$$\begin{align*}
    A &= \frac{\partial f}{\partial x} (x_0, u_0, d_0) \\
    B_u &= \frac{\partial f}{\partial u} (x_0, u_0, d_0) \\
    B_d &= \frac{\partial f}{\partial d} (x_0, u_0, d_0) \\
    C &= \frac{\partial g}{\partial x} (x_0, u_0, d_0)
\end{align*}$$

Note that $(x_0, y_0, u_0, d_0)$ denotes the equilibrium point that can be solved by (2) at the steady state ($x_1 = 0, x_2 = 0$) along the given tracking trajectory $r$ over the valve rising duration (i.e., $y = y_0 = r$) and given the measurable exogenous input $d$ (i.e., $d = d_0$, $\Delta d = 0$). The determined equilibrium point will be used to calculate the system coefficient matrices and the solved equilibrium point control $u_0$ will be used as a nominal (feedforward) control. A closed-loop controller for the variation control input $\Delta u$ need to be designed to compensate for the deviation from the desired trajectory with the presence of system noise and disturbance. Therefore, the control input for the system can be written as

$$u = \Delta u + u_0$$

C. Discrete-Time Event-by-Event Model

The linearized continuous-time model (3) with $\Delta t = 0$ can be discretized using the forward Euler approximation. Considering the system input noise $w$ and output measurement noise $v$, the discrete-time event-by-event model is obtained as:

$$\begin{align*}
    \Delta x(k+1) &= A(k) \Delta x(k) + B(k) \Delta u(k) + w(k) \\
    \Delta y(k) &= C(k) \Delta x(k) + v(k)
\end{align*}$$

where,

$$\begin{align*}
    A(k) &= \begin{bmatrix} 1 - \frac{T_s}{\tau_1} & 0 \\ \frac{1}{\tau_2} - \frac{T_s}{\tau_1} & 1 - \frac{T_s}{\tau_2} \end{bmatrix}, \\
    B(k) &= \begin{bmatrix} 0 \\ T_s k_d \end{bmatrix}, \\
    C(k) &= \begin{bmatrix} 0 & 2ax_2 + b \end{bmatrix},
\end{align*}$$

and $T_s$ is the valve event period. Note that the input noise $w$ and measurement noise $v$ are assumed to be zero mean and independent random vectors such that

$$\begin{align*}
    E\{w(k)\} &= 0, \\
    W &= E\{w(k)w^T(k)\} > 0 \\
    E\{v(k)\} &= 0, \\
    V &= E\{v(k)v^T(k)\} > 0
\end{align*}$$

where $W$ and $V$ are corresponding covariance matrices. Table I shows the identified parameters for the event-by-event model.

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$\tau_1$</th>
<th>$\tau_2$</th>
<th>$c_0$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
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<td>$-1.2 \times 10^{-5}$</td>
<td>2.05</td>
<td>0.19</td>
<td>0.5</td>
<td>-3.49</td>
<td>0.18</td>
<td>-4.7</td>
<td>32.41</td>
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IV. Linear Quadratic Tracking Control

In this section, a finite horizon LQT controller is designed to make the system output $y(k)$ track the reference $r(k)$. More specifically, since the control design will be based on the variation model (5) to provide a variation control input $\Delta u(k)$ at each operation point, the control target is to make the variation output $\Delta y(k)$ track the variation reference $\Delta r(k)$. Since the state $\Delta x_1$ used for state-feedback cannot be measured, a Kalman filter is used as optimal states observer with the presence of system noise.

A. Finite Horizon LQT Control

The control objective of the finite horizon LQT control is to minimize the tracking error $e(k)$ defined in (7) with the feasible control effort $V_m$ along a predefined tracking trajectory. Note that under transient engine operations, the tracking reference of the valve profile is often predefined or determined for a few engine cycles to achieve desired performance, e.g., a fast and smooth valve time and lift transition is required for the SI-HCCI (spark ignition-homogeneous...
charge compression ignition) combustion mode transition control [16]. Tracking error $e(k)$ is defined as

$$e(k) = \Delta y(k) - \Delta r(k) = C(k)\Delta x(k) - \Delta r(k) \quad (7)$$

To simplify notations, $\Delta x_k$, $\Delta y_k$, $\Delta u_k$, $\Delta r_k$, $e_k$, $A_k$, $B_k$ and $C_k$ are used to denote $\Delta x(k)$, $\Delta y(k)$, $\Delta u(k)$, $\Delta r(k)$, $e(k)$, $A(k)$, $B(k)$, and $C(k)$ at current time step $k$ in the rest of the paper. The performance cost function of the LQT controller at each control step (valve event) is defined as

$$J(k) = \frac{1}{2} e_k^T F e_k + \frac{1}{2} \sum_{k_0=k}^{k_f} [e_{k_0}^T Q e_{k_0} + \Delta u_{k_0}^T R \Delta u_{k_0}] \quad (8)$$

where $F = F^T \geq 0$, $Q = Q^T \geq 0$, and $R = R^T > 0$ are given weighing matrices. $k_0 \sim k_f$ is a finite horizon moving window at each control step $k$ for predefined $N$-step $(N = k_f - k_0)$ tracking reference. That is, an $N$-step optimal controller is designed at each control step for the given $N$-step tracking trajectory and only the first control step will be used, which is the so-called moving optimization problem in MPC (model predictive control). Since the LQT controller is designed based on the variation model (5) linearized at the equilibrium point (current operation point) $k = k_0$, the tracking reference for the variation output $\Delta y_k$ is

$$\Delta r_k = r(k) - r(k_0), \quad k = k_0, \ldots, k_f.$$

The optimal tracking problem can be solved by following the minimum principle approach in [17]. The variation control $\Delta u_k$ can be obtained as

$$\Delta u_k = -\Delta u_{FB,k} + \Delta u_{FF,r,k} = -L_{FB,k} \Delta x_k + L_{FF,k} \Delta y_{k+1} \quad (9)$$

$$L_{FB,k} = [R + B_k^T K_{k+1} B_k]^{-1} B_k^T K_{k+1} A_k$$

$$L_{FF,k} = [R + B_k^T K_{k+1} B_k]^{-1} B_k^T$$

where $\Delta u_{FB}$ and $\Delta u_{FF}$, are the state feedback and reference feedforward control, respectively. The matrix $K$ in the control gains $L_{FB,k}$ and $L_{FF,k}$ and the vector $g$ can be obtained by backwards solving the Riccati equation (11) offline and vector equation (12) online using the boundary conditions in (13), respectively.

$$K_k = A_k^T K_{k+1} [I + B_k R^{-1} B_k^T K_{k+1}]^{-1} A_k + C_k^T Q C_k \quad (11)$$

$$g_k = A_k^T [I - K_{k+1} [I + B_k R^{-1} B_k^T K_{k+1}]^{-1}]$$

$$B_k R^{-1} B_k^T$$

$$g_{k+1} = C_k^T Q \Delta r_k$$

Note that in (9) the system state $\Delta x_k$ in $\Delta x_{k+1}$ used for feedback control cannot be measured and is estimated online using the Kalman filter addressed in the next subsection.

B. Kalman State Estimation

The Kalman state estimation is a stochastic filter that minimizes the covariance of the estimation error to provide an optimal state estimation subject to Gaussian noise inputs. For a given initial state $\hat{x}_0$, it uses the current output measurement $\Delta y_k$ and control $\Delta u_k$ to estimate the next step state $\Delta r_{k+1}$ in the following form.

$$\Delta \hat{x}_{k+1} = A_k \Delta \hat{x}_k + B_K \Delta u_k + H_k (\Delta y_k - C_k \Delta \hat{x}_k) \quad (14)$$

Note that the initial state is calculated as follows

$$\Delta \hat{x}_0 = \hat{x}_{k-1} - x_{k,0} \quad (15)$$

The subscript "0" denotes the equilibrium point at valve event $k$. It shows that the initial state is updated once the equilibrium point $x_{k,0}$ is switched, i.e., a new tracking reference $r_k$ or exogenous input $u_k$ is observed. The Kalman filter gain $H_k$ is obtained as follows

$$H_k = A_k \Sigma_k C_k^T (C_k \Sigma_k C_k^T + V)^{-1} \quad (16)$$

where the estimation error covariance matrix $\Sigma_k$ is calculated recursively by the following difference Riccati equation using the initial condition $\Sigma_0 = E(\Delta \hat{x}_0 \Delta \hat{x}_0^T)$.

$$\Sigma_{k+1} = A_k \Sigma_k A_k^T + W - H_k C_k \Sigma_k A_k^T \quad (17)$$

Using the estimated state vector in (9) and substituting it into (4), the system control input can be obtained as

$$u_k = \Delta u_k + u_{k,0} = -L_{FB,k} \Delta \hat{x}_k + L_{FF,k} \Delta y_{k+1} + u_{k,0} \quad (18)$$

V. SIMULATION VALIDATION

The discrete-time event-by-event model and the LQT controller based on the trajectory linearization (LQTFF) are implemented in Simulink and validated through simulation study. For the purpose of comparison, the open-loop control that only uses the equilibrium point control $u_0$ as feedforward (FF) control, and the PID (proportional-integral-derivative) control with and without the feedforward $u_0$ (PID and PIDFF) are also implemented. Fig.4 illustrates the four control schemes.

![Control Schemes](image_url)

**TABLE II**

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$u_0$</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$F$</th>
<th>$Q$</th>
<th>$R$</th>
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<tr>
<td>FF</td>
<td>√</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>PID(SmG)</td>
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<td>18.4</td>
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<td>-</td>
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<td>18.4</td>
<td>12</td>
<td>12</td>
<td>3</td>
<td>-</td>
<td>-</td>
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<tr>
<td>PID(SmG)</td>
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<td>3</td>
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<td>PIDFF(SmG)</td>
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<td>3</td>
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<tr>
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<td>-</td>
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<td>$10^3$</td>
<td>100</td>
<td>1</td>
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The control parameters, the weighting matrices ($F$, $Q$, and $R$) for the LQT controller and the control gains for the PID controllers, are tuned in simulations based upon
the transient tracking performance and system stability. The PID controller are tuned with fixed small $k_i$ gain (SmG), fixed large $k_i$ gain (LaG), and $k_i$ gain scheduled (ScG) with supply pressure to show the effect of system nonlinearity. The control parameters for each control scheme are listed in Table II. Note that $k_{il}$ and $k_{ih}$ denote the scheduled $k_i$ gain at the lower pressure bound (6 MPa) and upper pressure bound (9 MPa) of the fluid supply system, respectively.

![Fig. 5. Simulation results of step reference tracking at 1000 rpm](image1)

Fig. 5 shows the simulation results of step reference tracking for the valve rising duration at 1000 rpm engine speed. The tracking performances, corresponding supply pressure responses and control inputs $u$ for six control schemes are presented in the upper three plots of Fig. 5, respectively. The control input components ($u_0$, $\Delta u_{FB}$, $\Delta u_{FFr}$) of the LQT controller are analyzed in the bottom plot. It can be seen from the first plot that the open-loop control with equilibrium point feedforward (FF) can have good transient response and steady-state accuracy if the model (2) used for $u_0$ calculation is accurate, otherwise steady-state error may exist. Compared with the FF scheme, the PID control with fixed small gain (SmG) has similar response performance in low pressure region while poor performance in high pressure region. Using large gain (LaG) can increase the transient performance with large overshoot in low pressure region. This trade-off relationship coincides with the system nonlinearity (recall Fig. 3) that it consumes more control effort in the high pressure region to achieve the same rising duration increment than that consumed in the low pressure region. Therefore, control gain scheduled with supply pressure is necessary for the PID controller to achieve good transient response in both low and high pressure regions as shown by the PID control scheme with scheduled gain (ScG). Combining the equilibrium point feedforward and gain scheduling PID control, the PIDFF(ScG) scheme leads improved transient performance. The supply pressure and control input responses are consistent with the tracking performance for each control scheme. It can be noticed that both the open-loop and PID control schemes result a delayed response to the tracking reference signal. The LQT controller (LQTFf), however, can track the desired trajectory very closely with minimum transient tracking error without tracking delay mainly due to the reference feedforward control $\Delta u_{FFr}$ obtained from the predefined upcoming $N$-step tracking references and dominates the control when the reference changes.

VI. EXPERIMENTAL VALIDATION

The experimental validation was conducted on the EHVVA prototype [14]. Fig. 6 illustrates the test results of the step reference tracking for the valve rising duration at 1000 rpm, and the results agree with the simulation results well with several exceptions. First, small steady-state tracking error appears in the open-loop feedforward (FF) control at low pressure region due to the modeling error and is eliminated by the closed-loop controllers. Second, the responses of PID control schemes (SmG, LaG, and ScG) are very slow, compared with the FF control scheme mainly due to the presented system noise limited PID gains for closed-loop stability. The control performance can be improved using the feedforward PID control (PIDFF(ScG)).

![Fig. 6. Experiment results of step reference tracking at 1000 rpm](image2)
With the help of the Kalman state estimation, the LQT controller (LQTFF) can track the transient trajectory in 5 events with minimum transient tracking error among all control schemes. The transient response is 4 events faster than that of the PIDFF controller which is the best among the other controllers. The bottom plot in Fig. 6 shows the Kalman state estimation results. Note that the state $\Delta x_1/x_1$ is not measurable and it remains at zero ($x_1$ keeps unchanged) because $x_1$ is excited by the engine speed which is unchanged under this condition.

The steady-state tracking performance (constant $r$) for the valve rising duration is evaluated for 200 engine cycles at 1000 rpm and the statistic distribution of the tracking error is illustrated in Fig. 7. Since it is very important to regulate the valve rising duration within a small crank angle (CA) deviation from the target to guarantee repeatable air charge and exhaust quantity for each engine cycle, the tracking error in time domain is converted into the crank angle domain at engine speed of 5000 rpm to evaluate the worst-case tracking performance. The probabilities of deviation distribution within $\pm 1^\circ$CA for the FF, PID, PIDFF, and LQTFF control schemes are 44%, 58.5%, 65.5%, and 74.5%, respectively.

**Fig. 7.** Histogram of steady-state tracking deviation in 200 engine cycles

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### VII. CONCLUSIONS

In this paper, an LQT (linear quadratic tracking) controller with Kalman state estimation is designed based on trajectory linearization for a developed nonlinear model of the EHVVA (electro-hydraulic variable valve actuator) system. The equilibrium point control obtained from the trajectory linearization is used as the feedforward for the designed LQT controller. Both simulation and experimental validations are conducted and the results match well. In the experimental study, the proposed LQT controller shows significant improvement under both transient and steady-state operations, compared with the open-loop feedforward, PID, and feedforward PID controllers. The LQT controller is able to track the transient trajectory of the valve rising duration in 5 valve events with minimal tracking error, which is 4 events faster than the feedforward PID controller (the best controller among the studied controllers). The steady-state tracking tests show 30%, 16%, and 9% improvements for the $\pm 1^\circ$CA tracking error distribution, compared with the open-loop feedforward, PID, and feedforward PID controllers, respectively. Future work is to study and validate the valve profile tracking control under transient engine speed.

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### REFERENCES


