ABSTRACT
Control-oriented models of automotive turbocharger compressors typically describe the compressor power assuming an isentropic thermodynamic process with a fixed isentropic efficiency and a fixed mechanical efficiency for power transmission between the turbine and compressor. Although these simplifications make the control model tractable, they also introduce additional errors due to unmodeled dynamics, especially when the turbocharger is operated outside its normal operational region. This is also true for map-based approaches, since these supplier-provided maps tend to be sparse or incomplete at the boundary operational regions and often ad-hoc extrapolation is required, leading to large modeling error. Furthermore, these compressor maps are obtained from the steady flow bench tests, which introduce additional errors under pulsating flow conditions in the context of internal combustion engines. In this paper, a physics-based model of compressor power is developed using Euler equations for turbomachinery, where the mass flow rate and compressor rotational speed are used as model inputs. Two new coefficients, speed and power coefficients are defined. This makes it possible to directly estimate the compressor power over the entire compressor operating range based on a single analytic relationship. The proposed modeling approach is validated against test data from standard turbocharger flow bench, steady state engine dynamometer as well as transient simulation tests. The validation results show that the proposed model has adequate accuracy for model-based control design and also reduces the dimension of the parameter space typically needed to model the compressor dynamics.

NOMENCLATURE
\( W \) : Work [J]
\( \dot{m} \) : Mass flow rate [kg/sec]
\( \tau \) : Torque [Nm]
\( \omega \) : Compressor angular velocity [rad/sec]
\( \rho \) : Gas density [kg/m\(^3\)]
\( R \) : Universal gas constant [J kg\(^{-1}\) K\(^{-1}\)]
\( A \) : Geometric area [m\(^2\)]
\( \gamma \) : Isentropic index

Subscripts:
1, in: represent the compressor upstream or inlet condition.
2, out: represent the compressor downstream or outlet condition.
Symbols not in this list are defined locally in the text.

INTRODUCTION
It is common for plant models, used in the air path control of turbocharged (TC) diesel engines, to assume that the power consumed by the compressor is an isentropic thermodynamic process. The actual power is then derived from either a map-based compressor isentropic efficiency [1]-[3] or an empirically fitted isentropic efficiency [6],[8],[9]. A common alternative approach is to define the compressor power as a first order dynamic with an ad-hoc time constant with the turbine power as the source term [4],[5]. Map-based approaches of compressor power, on the other hand, rely on the overall TC system efficiency available as a series (for various vane positions) of supplier provided maps [7] applied to the calculated turbine power. Alternately empirically fitted compressor efficiencies are typically regressions as a 2nd or 3rd order polynomial in the Blade Speed Ratio (BSR). The polynomial coefficients are often dependent on the shaft speed [8]-[11] and are identified against the populated regions of the manufacturer supplied maps. These polynomial models are,
therefore, also subject to extrapolation for operating conditions beyond the manufacturer supplied test points. Note that the typical manufacturer provided compressor performance map is based on hot gas flow bench test data. These tests are performed under steady flow conditions, hence, practical applications could deviate from these maps under pulsating flow when coupled to an Internal Combustion (IC) engine [1], [2], [13]. Another issue, often encountered when operating at light load conditions, is the sparsity of manufacturer provided compressor performance maps. This as is indicated in Figure 1, for a sample turbocharged engine. As a result, operating the compressor outside the available map requires extrapolation under the assumption of a convex hull extension of the supplied map. Several investigations into extrapolation methods, based on these assumptions are reported in open literature as in [14], [15].

The traditional compressor power is computed from the standard isentropic efficiency equation in (1) based on the inputs as indicated in Figure 2.

\[
W_c = \frac{1}{\eta_{C}} m_{out} \left( \frac{P_{out}}{P_{in}} \right)^{\gamma - 1} \left( \frac{T_{in}}{T_{out}} \right) - 1
\]

(1)

Figure 1. Operating range deficit between mapped and desired engine operational ranges

Figure 2. Traditional approach for deriving compressor power

In (1), the compressor isentropic efficiency must be defined. In a review of existing literature, compressor isentropic efficiency is commonly modeled using empirical fits based on manufacturer provided compressor maps. In [9], the efficiency is expressed as a polynomial function of the non-dimensional mass flow rate. The polynomial coefficients are three individual functions of Mach number. In [26] the efficiency is modeled using an elliptical fit based on mass flow rate and pressure ratio. This model depends on the maximum efficiency and the location in terms of corrected mass flow rate and compressor pressure ratio. In [23], the efficiency model form [26] is further modified for pressure ratio variation. In [25] the corrected compressor work is defined as a polynomial function of corrected mass flow rate and the model is further fitted with experimental data. TABLE 1 summarizes these approaches and our assessment of these models for performance in the interior as well as the exterior of the supplier maps.

<table>
<thead>
<tr>
<th>Mapping Method</th>
<th>Performance within the mapped area</th>
<th>Performance outside the mapped area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jensen et al. [9]</td>
<td>Good</td>
<td>No data</td>
</tr>
<tr>
<td>Guzzella-Ammatant [27]</td>
<td>Good</td>
<td></td>
</tr>
<tr>
<td>Kolmanovsky [10]</td>
<td>Good</td>
<td></td>
</tr>
<tr>
<td>Anderson [23]</td>
<td>Good</td>
<td></td>
</tr>
<tr>
<td>Canova et al. [24],[25]</td>
<td>Good</td>
<td></td>
</tr>
<tr>
<td>Eriksson et al. [26]</td>
<td>Good</td>
<td>Subject to extrapolation methods and base map granularity. General performance varied form average to poor.</td>
</tr>
<tr>
<td>Sieros et al.: Simple linear [28]</td>
<td>Good</td>
<td></td>
</tr>
</tbody>
</table>

In this paper, the compressor power model is developed based on the Euler equations for turbomachinery. The model uses the mass flow rate and compressor speed as inputs. It is found that with the proposed approach a quadratic analytic function of two parameters, the speed and power coefficients, is adequate to characterize the compressor performance over the entire operating range. The model is validated using hot gas flow bench test data, steady-state engine dynamometer test data, and transient simulation data. Results indicate that the proposed reduced order model is suitable for both steady state, pulsation flow conditions as well as transient operation.

The paper is organized as follows. Section II discusses the generalized compressor model based on the Euler equations with different slip factors and Section III provides validation results from turbocharger flow bench, turbocharged engine dynamometer and transient simulation data. The last section adds some conclusions.

GENERALIZED COMPRESSOR POWER MODELING

Compressor power model based on turbomachinery Euler equation

A typical centrifugal compressor geometry layout is shown in Figure 3, and the velocity triangles of the gas flow at the compressor impeller inlet and outlet are shown in Figure 4, and are adapted from [1], [17]. Using the Euler equations the compressor power can be expressed as:
\[ \dot{W}_C = \alpha \tau = \alpha m (v_{out} - v_{in}) = \dot{m}(U_2 C_{\theta 2} - U_1 C_{\theta 1}) \] (2)

For the centrifugal impeller, it is assumed that the air enters the impeller eye in the axial direction so that the initial angular momentum of the air at the inlet of the compressor can be assumed to be zero \( (U_1 C_{\theta 1} = 0) \) [1], [17]. The ideal compressor power equation can then be reduced to:

\[ \dot{W}_{C,\text{ideal}} = \dot{m} U_2 C_{\theta 2} \] (3)

or equivalently:

\[ \dot{W}_{C,\text{ideal}} = \dot{m} \omega R_2 C_{\theta 2} \] (4)

Ideally, the flow exits the blade with a back sweep angle \( \beta'_2 \) with some slip. The theoretical absolute flow velocity can be expressed as

\[ C_{\theta 2} = U_2 - C_{\theta 2} \tan \beta'_2 \] (5)

where, \( C_{\theta 2} \) is impeller outlet flow radial velocity. Neglecting slip loss for the radial direction, the ratio between \( C_{\theta 2} \) and \( C_{\theta 1} \) in an impeller is defined as slip factor \( \sigma \) as shown in (6); The slip factor depends on a number of factors such as the number of impeller blades, the passage geometry, the impeller eye tip to impeller exit diameter ratio, the mass flow rate as well as compressor speed [17].

\[ \sigma = \frac{C_{\theta 2}}{C_{\theta 1}} \] (6)

With the slip factor the compressor power equation (4) becomes:

\[ \dot{W}_{C,\text{ideal}} = \dot{m} \sigma R_2 C_{\theta 2} \] (7)

For simplicity, \( C_{\theta 2} = C_{\theta 1} \) is assumed, where \( C_{\theta 1} \) is impeller inlet radial velocity. This leads to (constant inlet density assumption)

\[ C_{\theta 2} = C_{\theta 1} = \frac{\dot{m}}{\rho_2 A_2} = \frac{\dot{m}}{\rho_1 A_1} \] (8)

Substituting (8) into (7), the compressor power can be formulated as

\[ \dot{W}_{C,\text{ideal}} = \sigma R_2^2 \dot{m} \omega^2 - \frac{\sigma R_2^2 \tan \beta'_2}{\rho_1 A_1} \alpha \dot{m}^2 \] (9)

As a result of flow and ‘windage’ losses, the actual required input work is greater than the theoretical value necessary to achieve the target flow rate [33]. To account for this, power loss factor \( \psi \) and friction loss term \( k_f \) are introduced to (10):

\[ \dot{W}_C = \psi \dot{W}_{C,\text{ideal}} + \dot{W}_f \] (10)

In this paper, \( \psi \) is assumed to be constant. Based on the study from [17] and [34], the friction loss for impeller and diffuser are:

\[ \dot{W}_f = k_f \dot{m}^3 = (k_{f1} + k_{f2}) \dot{m}^3 \] (11)
Expected variation of $C_{Power}$ with $C_{Speed}$ based on the relationship in (15) is shown in Figure 5. The range of this variation will be influenced by variation in one or more of the operating/design parameters as indicated in Figure 5. Different compressor designs will result in different impeller wheel diameters, power loss factor, slip factor, and exit area. For a given compressor design and inlet condition, the inlet gas density $\rho_1$ as well as the parameters $\psi$ and $k_f$ may be assumed to be constant. The power coefficient, as in (15), can then be expressed compactly as a function of the speed coefficient and the slip factor for a given operating condition, $i$.

$$C_{Power,i} = f(C_{Speed,i}, \sigma_i), \quad i = 1, 2, \ldots, n$$

(16)

Equation (16) indicates that the power coefficient is a function of speed coefficient and the slip factor for a given compressor design. One of the goals of this work was to investigate the predicton capabilities of the power and speed coefficients in conjunction with an appropriate slip factor model such that the dimension of the parameter space in (16) may be reduced from two to one is indicated in (17).

$$C_{Power} = f(C_{Speed})$$

(17)

Equation (18) shows the compressor power models in terms of the expanded forms of $C_{power}$ and $C_{speed}$. It is clear that in order to achieve a form as in (17) the slip factor must be considered either as a parameter fixed by design or defined in terms of the speed and/or power coefficient to maintain the homogeneity of the compressor power model with respect to $C_{power}$ and $C_{speed}$. The effectiveness of such approximations will therefore need to be evaluated.

$$\frac{W_{C}}{m^3} = \frac{\psi R_2}{\rho_1 A_1} \left( \frac{\omega}{m} \right)^2 - \frac{\nu R_2 \tan \beta_2}{\rho_1 A_1} \left( \frac{\omega}{m} \right) + k_f$$

(18)

Slip factor investigation

Several slip factors models are readily available in open literature [17, 22, 29-32]. The slip factor typically depends on the compressor design parameters as well as the operating conditions, such as: compressor rotational speed and mass flow rate. This led to the development of several empirical slip factor models as in [17, 22, 29-32]. In order to maintain the impact of flow conditions on slip, non-constant slip factor candidate models, as proposed in [22], [29], and [32] are investigated in this work:

Slip 1: $\sigma = 1 + m \tan \beta_2^* \frac{\frac{2Z \sin \beta_2}{Z}}{\rho_1 A_1} \left( \frac{\omega}{m} \right) + 0.5 \left( 1 - 0.5 \left( 1 - \frac{\omega}{m} \right) \right) \left( 0 < \sigma < 1 \right)$ (Reffstrup)

(19)

Slip 2: $\sigma = 1 + \frac{m \tan \beta_2^*}{\rho_1 A_1} \left( \frac{\omega}{m} \right) \left( 0 < \sigma < 1 \right)$ (Stodola)

(20)

Slip 3: $\sigma = 1 - a \frac{m \tan \beta_2^*}{\rho_1 A_1} \left( 0 < \sigma < 1 \right)$ (Stahler)

(21)

where $a$ is a design parameter in (21) related to the flow exit angle and is a constant for a given impeller design. $Z$ is the number of impeller blades. Integrating these three slip models into the proposed compressor power model (15), a generalized compressor power model can be established as follows:

$$C_{Power} = \epsilon_1 (C_{Speed})^\frac{1}{n} - \epsilon_2 (C_{Speed}) + \epsilon_3$$

(22)

where coefficients $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$ depend on the selected slip factor model and are defined in TABLE 2. The model (22) is referred to as the generalized compressor power model in the rest of this paper. Note that the three coefficients in (22) take constant values and are fixed by compressor design under the assumption of constant or slowly varying inlet conditions. Using the definitions of speed and power coefficients, the proposed compressor power model (22) becomes:

$$\frac{W_{C}}{m^3} = \epsilon_1 \left( \frac{\omega}{m} \right)^2 - \epsilon_2 \left( \frac{\omega}{m} \right) + \epsilon_3$$

(23)

Equations (22) and (23) are the proposed analytical models for the compressor power. The proposed model shows that the compressor operation can be represented as an analytic quadratic function rather than a two dimensional map. The proposed model has only two inputs: mass flow rate and TC rotational speed. For practical applications, these two parameters are available as measurements. Alternately the speed can also be obtained by solving turbocharger rotor dynamic differential equations as in [3]-[5], and the compressor mass flow rate could be modeled with shaft speed and pressure ratio as inputs. In this paper, the compressor power model is validated under the former assumption of the two parameters are available as measurements.

| Slipp model 1 | $\epsilon_1$ | $\psi R_2 \tan \beta_2^* \left( 2Z \cos \beta_2' \right)$ |
| Slipp model 2 | $\epsilon_2$ | $\frac{R_2 \tan \beta_2^*}{\rho_1 A_1}$ |
| Slipp model 3 | $\epsilon_3$ | $k_f$ |

TABLE 2. Model coefficients for various slip models

MODEL VALIDATION
Model validation using standard flow bench test data

Data from standard hot gas flow bench tests for three different production compressors was used to verify the proposed model and the relationship between $C_{power}$ and $C_{speed}$. The test setup for generating these data sets can be found in [12]. The inlet conditions of compressors are considered as fixed [21], which agrees with the constant inlet condition assumption used here. The test matrix for the three different compressors is shown in Figure 6. The minimum test speed for Compressor-1 and Compressor-2 are 30,000 RPM, while the lowest speed for Compressor-3 is 46,000 RPM.

Power and speed coefficients are calculated based on (13) and (14) for the test data points as in Figure 6. The values are shown in Fig 7 (labeled as “compressor-i test data”) for the three different compressor designs considered in this work. The $C_{power}$ vs $C_{speed}$ characteristic curves for each compressor can be characterized as a unique quadratic function. This representation is an important and significant order reduction when considering the large two-dimensional efficiency maps typically used for defining compressor power [8],[11],[23],[25],[27]. In order to identify the model (22), a fitting cost function is defined as follows:

$$J = \sum_{i=1}^{n} \left[W_{model}(i) - W_{meas}(i)\right]^2$$

(24)

where $n$ is the number of steady state compressor operating points from hot gas flow bench test results; $W_{model}$ is the calculated compressor power using (23); and $W_{meas}$ is the standard compressor power calculated using (25) with measured inputs:

$$W_{meas} = m_{in}C_{p}(T_{out} - T_{in})$$

(25)

The design parameters for Compressor-1 were available from the supplier. The first step was to identify the power loss coefficient and friction loss power for each slip model candidate for Compressor-1. A Least-Squares optimization was used to identify $\psi$, $k_f$ and $a$ such that the cost function (24) is minimized.

The identified parameters are shown in TABLE 3. The identified power loss parameters ($\psi$, $k_f$, and $a$) are in a reasonable range, indicating that compressor input power necessary to achieve the desired flow rate must be larger than the ideal power calculated in (4). The identified parameter $a$ in Slip Model-3 agrees with the result from [29]. The maximum friction loss power identified was around 2.38kW over the entire compressor operating range. Based on the fitting results in TABLE 3, the Slip Model-3 offers the best fit (least error). The identified model for Compressor-1 using the Slip-Model 3 is shown in Figure 7. Fitting results confirm that the model structure proposed in (22) can be generalized for modeling centrifugal compressor power.

TABLE 3. Identified model coefficient for Compressor-1 with different slip factor models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\psi$</th>
<th>$k_f$</th>
<th>$a$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip model 1</td>
<td>1.295</td>
<td>19040</td>
<td>-</td>
<td>0.968</td>
</tr>
<tr>
<td>Slip model 2</td>
<td>1.323</td>
<td>6374</td>
<td>-</td>
<td>0.968</td>
</tr>
<tr>
<td>Slip model 3</td>
<td>1.351</td>
<td>11390</td>
<td>1334</td>
<td>0.989</td>
</tr>
</tbody>
</table>

Since compressor design parameters were not available for the other two compressors (Compressor-2 and Compressor-3), we used the generalized model structure (22) to identify the 3 parameters $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ directly and the identified values are shown in TABLE 4. It is interesting to note from Figure 7 that the three different compressor designs lead to three distinct characteristic curves. This is expected and reflects the design differences between the different compressors. In fact this representation also allows a quick and easy method to compare different compressor designs. As an example, it is clear that for a given mass flow rate and TC shaft speed the compressor power requirement follows the trend, $Power_{1} > Power_{2} > Power_{3}$, indicating that Design-1 may perhaps have a larger loss relative to the other designs. Also note that it appears that Compressor-3 may offer the widest operating range of the three designs considered here.

TABLE 4. Identified model coefficient for compressor-2 and compressor-3 with model structure (22)

<table>
<thead>
<tr>
<th>Compressor</th>
<th>$\epsilon_1$</th>
<th>$\epsilon_2$</th>
<th>$\epsilon_3$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor 2</td>
<td>0.001762</td>
<td>10.15</td>
<td>15600</td>
<td>0.995</td>
</tr>
<tr>
<td>Compressor 3</td>
<td>0.001179</td>
<td>0.001762</td>
<td>13900</td>
<td>0.996</td>
</tr>
</tbody>
</table>

Logarithmic scale plots offer new insights. The Log scale plots for the three compressor designs are shown in Figure 7, and are represented by three corresponding straight lines. This follows directly from the Log scale representation of the generalized model as (26)

$$\log(C_{power}) = p\log(C_{speed}) + \log(q)$$

(26)

In (26), $p$ is the slope of each characteristic line and $q$ is the y-intercept for each design. Equation (26) can also be formulated as:

$$C_{power} = 10^p (C_{speed})^q$$

(27)
\[ \frac{W_C}{m} = 10^p \left( \frac{\alpha}{\mu} \right)^q \]  

(28)

Using the same fitting method as described in the previous section, identified \( p \) and \( q \) and the associated results are shown in TABLE 5. One important advantage of the straight line model is that (27) can, ideally, be obtained from only two compressor operating conditions (one in high load, one in low load). This method significantly reduces the experimental burden during the early stages of compressor development.

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TABLE 5. Identified model coefficient for three compressor with (27)

<table>
<thead>
<tr>
<th>Compressor</th>
<th>( p )</th>
<th>( q )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor 1</td>
<td>2.51</td>
<td>5.795 \times 10^{-6}</td>
<td>0.994</td>
</tr>
<tr>
<td>Compressor 2</td>
<td>2.50</td>
<td>4.769 \times 10^{-6}</td>
<td>0.989</td>
</tr>
<tr>
<td>Compressor 3</td>
<td>2.36</td>
<td>1.375 \times 10^{-5}</td>
<td>0.996</td>
</tr>
</tbody>
</table>

In summary, model validation results confirm the effectiveness of generalized compressor power structure proposed in this work. These results also demonstrate that for a centrifugal compressor the operating characteristics can be reduced to a single analytic quadratic function in linear scale and a straight line in logarithmic scale.

**Model validation based on steady state engine dynamometer test data**

In this section, Compressor-1, identified earlier, was tested on the engine dynamometer with a heavy duty turbocharged diesel engine. Detailed engine and test setup information can be found in [19] and [20]. The steady state test covers the whole engine operating range (185 testing points). The engine speed changes from 500 to 3,500 RPM. The engine brake torque varies from 10 to 1200 Nm. The compressor operating range (corresponding to each engine operating point) results in a mass flow rate between 0.01 and 0.45 kg/s and a compressor speed between 5.9k and 109k RPM. The compressor operating range (above 30,000 RPM) from the engine dynamometer tests is a subset of the standard turbocharger flow bench test data for the same operating range as shown in Figure 8.

![Figure 7. Model validation for three different compressors](image)

![Figure 8. Test ranges of TC flow bench and engine dyno tests](image)

![Figure 9. Comparison of the standard TC flow bench and engine steady state dynamometer test data for Compressor-1](image)

The test bench data set was limited to 30kRPM TC speed at the low end. Validation results in Figure 9 show the \( C_{power} \) vs \( C_{speed} \) curves for the engine data, the flow bench test data and the fitted model (based on the test bench data). Since the engine data set extended to lower load conditions (TC RPM < 30k), the
characteristic curves for the engine data are plotted in two sets to cover both operating regimes around the 30k RPM TC speed. It is clear from Figure 9 that while the model adequately reproduces the engine performance for the operating condition with TC speed > 30K RPM, there is significant error for operating conditions with TC speed < 30kRPM. These operating conditions are typified by low mass flow rates and investigations reveal that the measurements available from the engine at these low load conditions suffer from severe variability and are therefore quite suspect. An additional source of error could be related to the heat transfer effect from the hot end (turbine) adding a positive bias to the compressor outlet temperature. This effect of heat transfer on compressor efficiency under light load operating conditions was verified by experimentally and reported in [35], [36]. The higher compressor outlet temperature leads to a higher calculated compressor power (based on measurements), which makes the measured power coefficient (below 30,000RPM) to be higher than the model predicted value, as is observed in Figure 9. This is also confirmed when considering transient test data from a GT-Power based engine simulation as discussed below.

![Compressor and GT simulation results](image)

**Figure 10. Transient simulation validation**

**Transient Model Validation through GT-Power simulations**

In order to verify the behavior of the proposed reduced order model, transient engine simulation data is used in this section. The main reason for using data from engine simulation was to avoid measurement errors and biases, as discussed earlier. GT-power engine simulation results used in this section are based on the same engine with Compressor-1 connected to a variable geometry turbocharger (VGT). The GT-power simulation setup and model validation can be found in [20]. In this transient simulation, the engine follows the target torque based on the pedal position. The VGT vane feedback control is used to track the target boost pressure generated from calibrated maps. A US06 driving cycle was studied for this case as shown in Figure 10. Compressor speed varies from 14,000 to 105,000 RPM for this transient simulation. The relationship between the power and speed coefficients shows reasonable agreement between data and model prediction for Compressor-1 with Slip Model 3.

**CONCLUSION**

A compressor power model, based on the Euler turbomachinery equations and realistic assumptions was developed. Two new performance coefficients, the power and speed coefficients were proposed as an alternative to multiple performance maps. The proposed correlation between \( C_{\text{power}} \) and \( C_{\text{speed}} \) is especially useful in defining the compressor power necessary for regulating a desired compressor mass flow rate. This can then be translated into a VGT vane position or the assist demand in assisted boosting systems. This relationship can also be easily used to compare compressor design variants and compressor power requirements. The model is validated against data sets from standard turbocharger flow bench tests, steady-state engine dynamometer tests as well as transient engine simulations. Validation results indicate that the proposed model provides accurate compressor power prediction over a broad range of compressor operating conditions and provides for an easy and reliable extrapolation for operating conditions outside the standard mapping domain. Further, the proposed model reduces the dimensionality of the parameter space typically necessary for such applications. The reduced order, reduced complexity model is especially useful for the control applications. Future work will focus on improving prediction accuracy in the face of measurement noise as previously discussed.

**REFERENCES**


