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H_∞ Filtering for Cloud-Aided Semi-active Suspension with Delayed Road Information

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Abstract: The paper considers a cloud-aided semi-active vehicle suspension. In this system, road profile information is downloaded from a cloud database to facilitate the onboard state estimation. Time-varying delays are considered during the data transmission. An H_∞ filter is designed that exploits both onboard sensor measurements and delayed road profile information from the cloud. Disturbances due to GPS localization error and time delay inaccuracies are treated. The H_∞ performance analysis is presented and sufficient conditions for the existence of the filter are derived as linear matrix inequalities (LMIs). Filter design is then developed with the Projection Lemma. Numerical simulations are presented to illustrate the effectiveness of the designed filter in estimating the suspension states.

Keywords: Time–delay systems, H_∞ filtering, Semi-active suspension, Cloud computing, Linear matrix inequalities.

1. INTRODUCTION

The paper treats a state estimation problem for a cloud-aided actively or semi-actively controlled vehicle suspension considered in Li et al. (2014b). The estimator (filter) is designed within an H-infinity state estimation framework. It combines vehicle sensor measurements, which are instantaneous, and road profile information communicated from the cloud to the vehicle with a time-varying delay.

Cloud computing is an emerging paradigm for implementation of advanced automotive control and optimization computations, see Filev et al. (2013); Mangharam (2012); Ozatay et al. (2014). Numerous automotive functions have been identified as candidates for Vehicle-to-Cloud-to-Vehicle (V2C2V) implementations, see Filev et al. (2013). In Li et al. (2014a), a cloud-aided safety-based route planning system has been proposed that exploits road risk index database and real-time factors like traffic and weather, and generates a route optimized for safety and travel time. In Li et al. (2014b), the cloud-aided vehicle semi-active suspension control system is developed, in which road profile information is used as a preview for suspension control.

The cloud in V2C2V implementations can be viewed as a source of unlimited computing power and up-to-date database of information. There are two primary V2C2V control architectures, which we refer to as the computation-based and the information-based (abbrev. info-based), respectively. For computation-based implementation, in-vehicle sensor data is sent to the cloud and optimization is performed on the cloud by combining available stored and real-time data. Control signals are then sent to the vehicle. Alternatively, for info-based implementation (See Figure 1), the stored information on the cloud are sent to the vehicle to be used for onboard control. The latter implementation of semi-active suspension control is considered in this paper with a specific focus on handling the communication delays.

In suspension control problems, the road profile input is typically treated as a white noise (e.g., see Giorgetti et al. (2005); Miller (1988)). With Vehicle-to-Cloud-to-Vehicle (V2C2V) implementation, up-to-date cloud databases are maintained and can provide road profile information to vehicles if requested. Real-time information can also be gathered from the internet and crowdsourced from V2C2V-implemented vehicles.

However, the information transmission is hindered by time delays. The architecture of information-based V2C2V system with time delays is shown in Figure. 1. When needed, the vehicle can send a data request together with its GPS coordinates to the cloud. Then the cloud will send the requested data to the vehicle. The messages are exchanged via a wireless communication channel in which a vehicle-to-cloud delay (τ_{v2c}) and cloud-to-vehicle delay (τ_{c2v}) may occur due to bandwidth limitations or temporary communication unavailability.

The H_∞ filtering techniques have been previously developed for systems with time delays, see e.g., Fridman (2006). In this paper, we develop an H_∞ filter for info-based V2C2V semi-active suspension with time-varying information delays, and we demonstrate that disturbances

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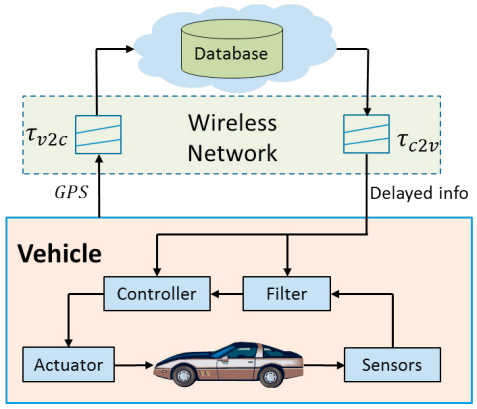

Fig. 1. Info-based V2V2V vehicle control with time delays.

due to GPS inaccuracy, delay uncertainty and measurement noises can be attenuated. The filter design is reduced to linear matrix inequalities (LMIs), which are solved numerically.

This paper is organized as follows. In Section 2 we present the problem formulation. In Section 3, several background results which facilitate our analysis are reviewed. In Section 4, stability and H_∞ performance analyses are presented and LMIs are derived that guarantee a prescribed estimation error H_∞ norm bound. Section 5 presents the design of the filter gain that exploits the Projection method. The results of numerical simulations are reported in Section 6. Section 7 concludes the paper.

Notation: The following notations are used throughout the paper. Superscript "T" and "-1"denotes matrix transpose and inverse, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $L_2[0,\infty)$ is the space of square-integrable functions on $[0,\infty)$, and for $w(t) \in L_2[0,\infty)$, $\|w\|_2^2 = \int_0^\infty w(t)^{\mathrm{T}} w(t)dt$; In symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdot\ \cdot\ \cdot\}$ stands for a block-diagonal matrix. $Sym(A)$ is a shorthand notation for $A + A^{\mathrm{T}}$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. For a symmetric matrix, $P > 0\,(P \geq 0)$ means that P is positive-(semi)definite. I and 0 represent, respectively, the identity matrix and zero matrix.

2. PROBLEM FORMULATION

In this paper, we consider a filtering problem for cloud-aided semi-active suspension system introduced in Li et al. (2014b). Quarter-car models are frequently used for suspension control designs, see e.g., Giorgetti et al. (2005); Miller (1988); Giua et al. (1998), because they are simple yet capture many important characteristics of the full-car model. A quarter-car model, with 2 degrees of freedom (DOF), as shown in Fig. 2, is used. The M_s and M_{us} represent the car body (sprung) mass and the tire and axles (unsprung mass), respectively. The spring and shock absorber with adjustable damping ratio constitute the suspension system, connecting sprung (body) and unsprung (wheel assembly) masses. The tire is modeled as a spring with stiffness k_{us} and its damping ratio is assumed to be negligible. From Fig. 2, we have the following equations of motion:

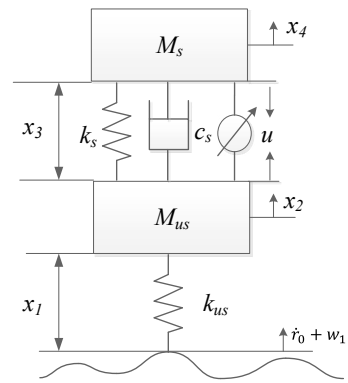

Fig. 2. Semi-active suspension dynamics.

$$\begin{aligned} \dot{x}_1 &= x_2 - w_1 - \dot{r}_0, \\ M_{us}\dot{x}_2 &= -k_{us}x_1 + k_s x_3 + c_s(x_4 - x_2) + u, \\ \dot{x}_3 &= x_4 - x_2, \\ M_s\dot{x}_4 &= -k_s x_3 - c_s(x_4 - x_2) - u, \end{aligned} \tag{1}$$

where x_1 is the tire deflection from equilibrium; x_2 is the unsprung mass velocity; x_3 is the suspension deflection from equilibrium; x_4 is the sprung mass velocity; $\dot{r}_0 + w_1$ represents the *velocity* disturbance with $\dot{r}_0$ being the nominal road profile from the cloud and w_1 being the unknown disturbance due to GPS localization uncertainties; c_s is the constant damping and u is adjustable damper force; k_s and k_{us} are suspension stiffness and tire stiffness, respectively.

Define,

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T.$$

The suspension system model is then,

$$\dot{x} = Ax + Bu + B_r\dot{r}_o + B_r w_1, \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{us}}{M_{us}} & -\frac{c_s}{M_{us}} & \frac{k_s}{M_{us}} & \frac{c_s}{M_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s}{M_s} & -\frac{k_s}{M_s} & -\frac{c_s}{M_s} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{M_{us}} \\ 0 \\ -\frac{1}{M_s} \end{bmatrix}, \tag{3}$$

$$B_r = [-1 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}.$$

For vehicles equipped with semi-active suspension, measurements of suspension deflection (x_3) and body velocity (x_4) are typically available, while tire deflection (x_1) and vertical wheel velocity (x_2) are not measured. Let y_0 denote the vector of measurements and z denote the objective signal to be estimated,

$$\begin{aligned} \dot{x} &= Ax + Bu + B_r\dot{r}_o + B_r w_1, \\ y_0 &= [x_3 \quad x_4]^{\mathrm{T}} = C_0 x + D_0 w_2, \\ z &= x, \end{aligned} \tag{4}$$

where $C_0 = [0_{2\times 2} \quad I_2]$; $w_2 \in \mathbb{R}^2$ is the measurement disturbance and D_0 is a scaling factor.

Figure 3 illustrates the developed cloud-based vehicle software agent that has access to stored vehicle parameters (M_{us}, M_s, k_{us}, k_s, c_s), receives vehicle state estimate, $\hat{x}$, vehicle longitudinal velocity, v_{car}, wheel speed, and GPS coordinates, and sends nominal road profile information, $\dot{r}_0$ for use by on-board vehicle state estimator. The received road profile will be delayed in the wireless communication

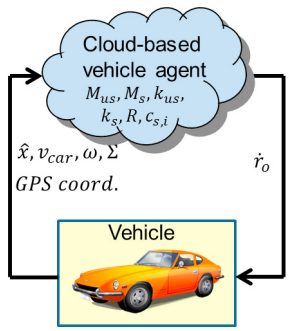

Fig. 3. V2C2V Suspension Control System.

channel. Thus, we will have a delayed measurement of the road profile onboard, that is

$$y_1(t) = \dot{r}_0(t - \tau_0(t)) + D_1 w_3(t), \tag{5}$$

where $\tau_0(t)$ is the nominal time-varying delay and assumed to be known (messages are time stamped and clocks are synchronized). $w_3(t)$ represents the unknown disturbance due to delay uncertainties and D_1 is a scaling factor. Note that the true time delay $\tau(t)$ may not be perfectly known, i.e., $\tau(t) = \tau_0(t) + \Delta t$ with $\tau(t)$ being the actual time delay and Δt being the disparity. When Δt is small, it follows that

$$\dot{r}_0(t - \tau(t)) \approx \dot{r}_0(t - \tau_0(t)) + \ddot{r}_0(t - \tau_0(t)) \cdot \Delta t, \tag{6}$$

where the last term can be incorporated in the uncertain disturbance $w_3(t)$ in (5), which means the formulation in (5) can account for the case that the time delay which is not perfectly known.

Assumption 1. We assume no packet dropouts during the transmission. The time delay $\tau(t)$ is time varying and $\underline{\tau} \le \tau(t) \le \bar{\tau}$, where $\underline{\tau}$ and $\bar{\tau}$ are known lower and upper bound of the delay. We also assume that $\dot{\tau}(t) \le d < \infty$, where d is a known bound.

Remark 1. In the paper we assume that the road profile is received based on real-time GPS coordinates. If the route is known, the road profile can be preloaded into a buffer and accessed onboard. Implementing such a buffer can obviously compensate the effects of the delay provided the buffer can be filled before information is needed. In practice, the communication bandwidth may be variable as the vehicle travels and the buffer may not be filled in time. In this case, the time delay still exists and Assumption 1 is still applicable with $\underline{\tau} = 0$.

For the filter design, we assume that $\ddot{r}_0 = D_r w_4$, where w_4 is a process disturbance. Defining the augmented state as $x_a = [x^{\mathrm{T}} \quad \dot{r}_0^{\mathrm{T}}]^{\mathrm{T}}$, augmented measurement output $y_a = \begin{bmatrix} y_0^{\mathrm{T}} & y_1^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, and augmented disturbance $w = \begin{bmatrix} w_1^{\mathrm{T}} & w_2^{\mathrm{T}} & w_3^{\mathrm{T}} & w_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, we have

$$\begin{aligned} \dot{x}_a(t) &= A_a x_a(t) + B_a w(t) + B_{au} u(t), \\ y_a(t) &= C_{a0} x_a(t) + C_{a1} x_a(t - \tau_0(t)) + D_a w(t), \end{aligned} \tag{7}$$

where

$$A_a = \begin{bmatrix} A & B_r \\ 0_{1\times 4} & 0_{1\times 1} \end{bmatrix}, \quad B_a = \begin{bmatrix} B_r & 0 & 0 & 0 \\ 0 & 0 & 0 & D_r \end{bmatrix},$$

$$B_{au} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad D_a = \begin{bmatrix} 0 & D_0 & 0 & 0 \\ 0 & 0 & D_1 & 0 \end{bmatrix}$$

$$C_{a0} = diag\{C_0, 0\}, \quad C_{a1} = diag\{0_{2\times 4}, 1\}.$$

A linear time invariant Luenberger filter for system (7) has the following form:

$$\begin{aligned} \dot{\hat{x}}_a(t) &= A_a \hat{x}_a(t) + L\big[y_a(t) - C_{a0}\hat{x}_a(t) \\ &\qquad - C_{a1}\hat{x}_a\big(t - \tau(t)\big)\big] + B_{au} u(t), \\ \hat{x}_a(s) &= 0, \quad \forall s \in [-\tau, 0]. \end{aligned} \tag{8}$$

Let $e(t) \triangleq x_a(t) - \hat{x}_a(t)$ denote the estimation error. Then the error dynamics have the following error form

$$\dot{e}(t) = \bar{A} e(t) + \bar{A}_d e(t - \tau(t)) + \bar{B} w(t), \tag{9}$$

with

$$\bar{A} = A_a - LC_{a0}, \quad \bar{A}_d = -Lc_{a1}, \quad \bar{B} = B_a - LD_a.$$

The problem addressed in this paper is formulated as follows: given system (7) and a prescribed level of noise attenuation $\gamma > 0$, determine a linear filter in the form (8) such that the origin of (9) with $w(t) = 0$ is asymptotically stable and

$$\sup_{\omega \in L_2[0,\infty)} \frac{\|e(t)\|_2^2}{\|w\|_2^2} < \gamma^2. \tag{10}$$

3. PRELIMINARIES

In this section, we present the following lemmas which will be used in the proofs of subsequent sections.

Lemma 1. Boyd et al. (1994): The linear matrix inequality

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix} < 0$$

where $S_{11} = S_{11}^{\mathrm{T}}$ and $S_{22} = S_{22}^{\mathrm{T}}$ is equivalent to

$$S_{11} < 0, \quad S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$$

or

$$S_{22} < 0, \quad S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0.$$

Lemma 1 is the well-known Schur complement lemma.

Lemma 2. Boukas and Liu (2002): Let X, Y be real constant matrices of compatible dimensions. Then the inequality

$$X^{\mathrm{T}} Y + Y^{\mathrm{T}} X \le \epsilon X^{\mathrm{T}} X + \frac{1}{\epsilon} Y^{\mathrm{T}} Y$$

holds for any $\epsilon > 0$.

Lemma 3. El Farissi and Latreuch (2012) If f, g: $[a, b] \to \mathbb{R}^n$ are similarly ordered, that is,

$$(f(x) - f(y))^{\mathrm{T}}(g(x) - g(y)) \ge 0, \quad \forall x, y \in [a, b], \tag{11}$$

then,

$$\begin{aligned} &\frac{1}{b-a}\int_a^b f(x)g(x)dx \\ &\ge \Big[\frac{1}{b-a}\int_a^b f(x)dx\Big]\Big[\frac{1}{b-a}\int_a^b f(x)dx\Big]. \end{aligned} \tag{12}$$

Lemma 3 is Chebyshev's inequality under similarly-ordered conditions.

Lemma 4. Boyd et al. (1994) Let $W = W^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, $U \in \mathbb{R}^{m\times n}$, and $V \in \mathbb{R}^{k\times n}$ be given matrices, and suppose that $rank(U) < n$ and $rank(V) < n$. The following LMI problem:

$$W + U^{\mathrm{T}} \mathcal{X}^{\mathrm{T}} V + V^{\mathrm{T}} \mathcal{X} U < 0. \tag{13}$$

is solvable for $\mathcal{X}$ if and only if

$$\begin{aligned} U_\perp^{\mathrm{T}} W U_\perp < 0 \quad &\text{if } V_\perp = 0, \quad U_\perp \ne 0, \\ V_\perp^{\mathrm{T}} W V_\perp < 0 \quad &\text{if } U_\perp = 0, \quad V_\perp \ne 0, \end{aligned}$$

$$U_\perp^{\mathrm{T}} W U_\perp < 0 \quad V_\perp^{\mathrm{T}} W V_\perp < 0, \quad \text{if } V_\perp \neq 0, \quad U_\perp \neq 0,$$

where $U_\perp$ and $V_\perp$ represent the right null spaces of U and V, respectively. Lemma 4 is referred to as the Projection Lemma.

4. H_∞ PERFORMANCE ANALYSIS

In this section, sufficient conditions for existence of the H_∞ filter are given as LMIs in the following theorem.

Theorem 1. Let L be a given matrix and γ a given positive scalar. The error system in (9) is stable and (10) holds if there exist symmetric matrices $P > 0$, $Q_i > 0$, $R_j > 0$, $i = 1, 2, 3$, $j = 1, 2$, satisfying

$$\mathcal{T}^{\mathrm{T}} Z \mathcal{T} + Sym(\mathcal{T}^{\mathrm{T}} P H_0) + \Psi < 0 \tag{14}$$

with

$$\mathcal{T} = [\bar{A} \quad \bar{A}_d \quad \bar{B} \quad 0_{5\times5} \quad 0_{5\times5}],$$
$$Z = \underline{\tau}^2 R_1 + (\bar{\tau} - \underline{\tau})^2 R_2,$$
$$\Psi = diag\Big\{ \sum_{i=1}^{3} Q_i + I_5,\ -(1-d)Q_1,\ -\gamma^2,\ -Q_2,\ -Q_3 \Big\} - H_1^{\mathrm{T}} R_1 H_1 - H_2^{\mathrm{T}} R_2 H_2,$$
$$H_0 = [I_5 \quad 0_{5\times5} \quad 0_{5\times1} \quad 0_{5\times5} \quad 0_{5\times5}],$$
$$H_1 = [I_5 \quad 0_{5\times5} \quad 0_{5\times1} \quad -I_5 \quad 0_{5\times5}],$$
$$H_2 = [0_{5\times5} \quad 0_{5\times5} \quad 0_{5\times1} \quad I_5 \quad -I_5],$$

Proof. To prove the result, similar to Fridman (2006); Yin et al. (2015), we construct a Lyapunov–Krasovskii functional (LKF) V such that $\dot{V} + e^{\mathrm{T}}(t)e(t) - \gamma^2\omega^2(t) < 0$. Then the error system in (9) is asymptotically stable and the H_∞ performance bound in (10) is satisfied. Consider the following LKF

$$V(e(t), t) = \sum_{i=1}^{3} V_i(e(t), t),$$

with

$$V_1(e(t), t) = e^{\mathrm{T}}(t) P e(t),$$

$$V_2(e(t), t) = \int_{t-\tau(t)}^{t} e^{\mathrm{T}}(s) Q_1 e(s) ds + \int_{t-\underline{\tau}}^{t} e^{\mathrm{T}}(s) Q_2 e(s) ds + \int_{t-\bar{\tau}}^{t} e^{\mathrm{T}}(s) Q_3 e(s) ds,$$

$$V_3(e(t), t) = \underline{\tau} \int_{-\underline{\tau}}^{0} \int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s) R_1 \dot{e}(s) ds d\theta + (\bar{\tau} - \underline{\tau}) \int_{-\bar{\tau}}^{-\underline{\tau}} \int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s) R_2 \dot{e}(s) ds d\theta,$$

Direct computations yield

$$\dot{V}_1(e(t), t) = 2\dot{e}^{\mathrm{T}}(t) P e(t), \tag{15}$$

$$\begin{aligned}\dot{V}_2(e(t), t) &= e^{\mathrm{T}}(t) Q_1 e(t) - (1 - \dot{\tau}(t)) e^{\mathrm{T}}(t - \tau(t)) Q_1 e(t - \tau(t)) \\ &+ e^{\mathrm{T}}(t) Q_2 e(t) - e^{\mathrm{T}}(t - \underline{\tau}) Q_2 e(t - \underline{\tau}) \\ &+ e^{\mathrm{T}}(t) Q_3 e(t) - e^{\mathrm{T}}(t - \bar{\tau}) Q_3 e(t - \bar{\tau}),\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{V}_3(e(t), t) &= \underline{\tau}^2 \dot{e}^{\mathrm{T}}(t) R_1 \dot{e}(t) - \underline{\tau} \int_{t-\underline{\tau}}^{t} \dot{e}^{\mathrm{T}}(s) R_1 \dot{e}(s) ds \\ &+ (\bar{\tau} - \underline{\tau})^2 \dot{e}^{\mathrm{T}}(t) R_2 \dot{e}(t) - (\bar{\tau} - \underline{\tau}) \int_{t-\bar{\tau}}^{t-\underline{\tau}} \dot{e}^{\mathrm{T}}(s) R_2 \dot{e}(s) ds\end{aligned} \tag{17}$$

From Lemma 3, it follows that

$$\begin{aligned}&\int_{t-\underline{\tau}}^{t} \dot{e}^{\mathrm{T}}(s) R_1 \dot{e}(s) ds \\ &\geq \frac{1}{\underline{\tau}} \int_{t-\underline{\tau}}^{t} \dot{e}^{\mathrm{T}}(s) ds R_1 \int_{t-\underline{\tau}}^{t} \dot{e}(s) ds \\ &= \frac{1}{\underline{\tau}} \big[e(t) - e(t-\underline{\tau})\big]^{\mathrm{T}} R_1 \big[e(t) - e(t-\underline{\tau})\big].\end{aligned} \tag{18}$$

Similarly, we have

$$\begin{aligned}&\int_{t-\bar{\tau}}^{t-\underline{\tau}} \dot{e}^{\mathrm{T}}(s) R_2 \dot{e}(s) ds \\ &\geq \frac{1}{\bar{\tau} - \underline{\tau}} \big[e(t-\underline{\tau}) - e(t-\bar{\tau})\big]^{\mathrm{T}} R_2 \big[e(t-\underline{\tau}) - e(t-\bar{\tau})\big].\end{aligned} \tag{19}$$

Combining (17)–(19), it follows that

$$\begin{aligned}&\dot{V}_3(\eta(t), t) \\ &\leq \underline{\tau}^2 \dot{e}^{\mathrm{T}}(t) R_1 \dot{e}(t) + (\bar{\tau} - \underline{\tau})^2 \dot{e}^{\mathrm{T}}(t) R_2 \dot{e}(t) \\ &- \big[e(t) - e(t-\underline{\tau})\big]^{\mathrm{T}} R_1 \big[e(t) - e(t-\underline{\tau})\big] \\ &- \big[e(t-\underline{\tau}) - e(t-\bar{\tau})\big]^{\mathrm{T}} R_2 \big[e(t-\underline{\tau}) - e(t-\bar{\tau})\big].\end{aligned} \tag{20}$$

Noting that as in (9),

$$\dot{e}(t) = \bar{A} e(t) + \bar{A}_d e(t - \tau(t)) = \mathcal{T} \xi(t),$$

with

$$\xi(t) = \big[e^{\mathrm{T}}(t)\ e^{\mathrm{T}}(t - \tau(t))\ w(t)\ e^{\mathrm{T}}(t - \underline{\tau})\ e^{\mathrm{T}}(t - \bar{\tau})\big]^{\mathrm{T}},$$

and $\mathcal{T}$ defined in the statement of Theorem 1. Also noting that

$$e(t) = H_0 \xi(t), \quad e(t) - e(t - \underline{\tau}) = H_1 \xi(t),$$
$$e(t - \underline{\tau}) - e(t - \bar{\tau}) = H_2 \xi(t),$$

we have

$$\dot{V}(e(t), t) + e^{\mathrm{T}}(t) e(t) - \gamma^2 w^{\mathrm{T}}(t) w(t) \leq \xi^{\mathrm{T}}(t) \Lambda \xi(t), \tag{21}$$

where

$$\Lambda = \mathcal{T}^{\mathrm{T}} Z \mathcal{T} + Sym(\mathcal{T}^{\mathrm{T}} P H_0) + \Psi. \tag{22}$$

From (14), $\Lambda < 0$. This completes the proof of stability and H_∞ performance.

5. FILTER DESIGN

In this section, we solve the H_∞ filtering problem for system (7), that is, finding filter gain L in (8) such that the error system in (9) is asymptotically stable with a guaranteed attenuation level as in (10). The following theorem gives sufficient conditions for the existence of such a filter.

Theorem 2. Let $\gamma > 0$ be a given constant representing desired attenuation level. With time-varying delays, there exists an H_∞ filter in form of (8) such that the error system in (9) is stable and $\|e(t)\|_2 < \gamma \|w(t)\|_2$ for all $w(t) \in L_2[0, \infty)$, if there exist positive-definite matrices

P, Q_1, Q_2, Q_3, R_1, R_2, G, positive scalar κ, and a matrix K such that

$$\Xi + Sym(\Pi) < 0, \tag{23}$$

where

$$\Xi = \begin{bmatrix} Z & P & 0_{5\times5} & 0_{5\times1} & 0_{5\times5} & 0_{5\times5} \\ * & \Xi_{22} & 0_{5\times5} & 0_{5\times1} & R_1 & 0_{5\times5} \\ * & * & -(1-d)Q_1 & 0_{5\times1} & 0_{5\times5} & 0_{5\times5} \\ * & * & * & -\kappa & 0_{5\times5} & 0_{5\times5} \\ * & * & * & * & \Xi_{55} & R_2 \\ * & * & * & * & * & \Xi_{66} \end{bmatrix},$$

$$\Xi_{22} = \sum_{i=1}^{3} Q_i + I_5 - R_1,$$

$$\Xi_{55} = -Q_2 - R_1 - R_2, \quad \Xi_{66} = -Q_3 - R_2,$$

$$\Pi = \begin{bmatrix} -G & GA_a - KC_{a0} & -KC_{a1} & GB_a - KD_a & 0_{5\times5} & 0_{5\times5} \\ -G & GA_a - KC_{a0} & -KC_{a1} & GB_a - KD_a & 0_{5\times5} & 0_{5\times5} \\ -G & GA_a - KC_{a0} & -KC_{a1} & GB_a - KD_a & 0_{5\times5} & 0_{5\times5} \\ 0_{5\times5} & 0_{5\times5} & 0_{5\times5} & 0_{5\times1} & 0_{5\times5} & 0_{5\times5} \\ -G & GA_a - KC_{a0} & -KC_{a1} & GB_a - KD_a & 0_{5\times5} & 0_{5\times5} \\ -G & GA_a - KC_{a0} & -KC_{a1} & GB_a - KD_a & 0_{5\times5} & 0_{5\times5} \end{bmatrix}.$$

Furthermore, if (23) is feasible, then the filter gain can be obtained as $L = G^{-1}K$.

Proof. From Theorem 1, we know if we can show (14), then the error system in (9) is stable and the H_∞ performance (10) is guaranteed. From (22), it is easy to see that matrices, P, Q_1, R_1 etc., are coupled with filter gain L in Λ and this complicates the filter design. To resolve this issue, we next introduce the Projection Lemma to linearize the nonlinear matrix inequalities in (14).
From (22), it follows that

$$\Lambda = \begin{bmatrix} \mathcal{T} \\ I_{21} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z & PH_0 \\ * & \Psi \end{bmatrix} \begin{bmatrix} \mathcal{T} \\ I_{21} \end{bmatrix}, \tag{24}$$

It is easy to check that

$$\begin{bmatrix} -I_5 & \mathcal{T} \end{bmatrix}_\perp = \begin{bmatrix} \mathcal{T} \\ I_{21} \end{bmatrix},$$

Now assign

$$\begin{bmatrix} Z & PH_0 \\ * & \Psi \end{bmatrix} \to W, \quad \tilde{G} \to X,$$

$$\begin{bmatrix} -I_5 & \mathcal{T} \end{bmatrix} \to U, \quad \begin{bmatrix} \mathcal{T} \\ I_{21} \end{bmatrix} \to U_\perp,$$

$$I_{26} \to V, \quad 0 \to V_\perp.$$

Using Lemma 4, it can be shown that (14) is equivalent to

$$\begin{bmatrix} Z & PH_0 \\ * & \Psi \end{bmatrix} + Sym\left\{\tilde{G}[-I_5 \quad \mathcal{T}]\right\} < 0. \tag{25}$$

For simplicity of filter synthesis, we specify $\tilde{G}$ as follows

$$\tilde{G} = [G \quad G \quad G \quad 0_{5\times1} \quad G \quad G]^{\mathrm{T}}. \tag{26}$$

Plugging (26) in (25) and defining $K = GL$, (23) is obtained. We also note that if (23) is feasible, the filter gain can be obtained as $L = G^{-1}K$. This completes the proof.

6. SIMULATIONS

In this section, we present the results of numerical simulation. The parameters in (1) are specified in Table 1. For simulations, a road segment over a 10 sec horizon is modeled as follows,

Table 1. Simulation parameters

m_s	m_{us}	k_s	k_{us}	c_s
290 kg	60 kg	16800 N/m	19000 N/m	200 $N \cdot s/m$

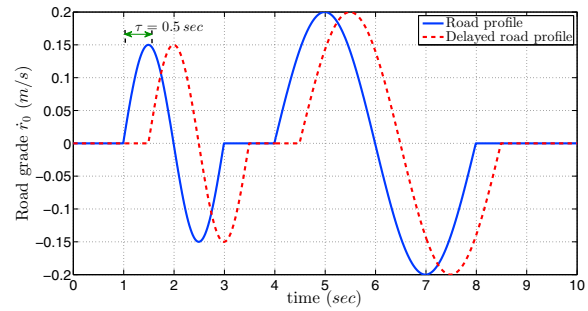

Fig. 4. Road grade profile ($\dot{r}_0$).

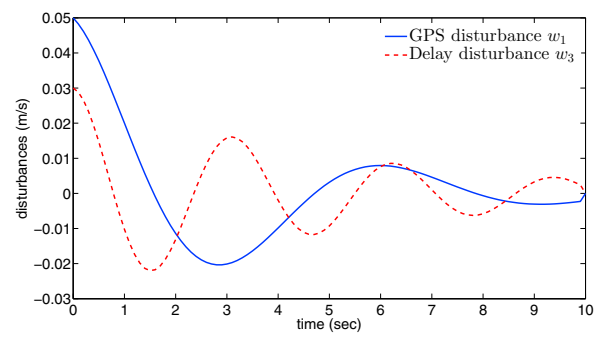

Fig. 5. GPS inaccuracy and time delay uncertainty disturbances.

Table 2. Attenuation level comparison

Kalman	H_∞, $\tau = 0.2s$	H_∞, $\tau = 0.5s$	H_∞, $\tau = 1s$
0.4256	0.2261	0.3321	0.5331

$$\dot{r}_o(t) = \begin{cases} 0.15 \cdot \sin \pi(t-1) & 1s \le t \le 3s, \\ 0.2 \cdot \sin \pi/2t & 4s \le t \le 8s, \\ 0 & \text{otherwise.} \end{cases}$$

See Figure 4. Let $\gamma = 0.9$, $\underline{\tau} = 0.1$ sec, $\bar{\tau} = 0.5$ sec and $d = 0.4$. To simulate the effects of GPS localization error and time delay uncertainty, we specify $w_1 = 0.05\cos t \cdot e^{-0.3t}$ and $w_3 = 0.03\cos 2t \cdot e^{-0.2t}$ as in Figure 5, while w_2 and w_4 are specified with unit intensity white noises. We aim at a filter in the form of (8) such that the H_∞ performance in (10) is satisfied. Applying Theorem 2 and with the help of Matlab LMI toolbox, simulation results for different delays and the performance comparison with a Kalman filter without the road profile information are reported Fig. 6–9. Note that the last case $\tau = 1$ sec violates the specified upper bound $\bar{\tau}$.

Based on the simulation results, we compute the actual attenuation $\gamma = \frac{\|e\|_2}{\|w\|_2}$ over the 10-sec horizon. See Table 2. It is clear that with the delayed road information, the H_∞ filter has better performance than the Kalman filter with no road information when the delay is small. When the delay is large, the H_∞ filter performance is degraded.

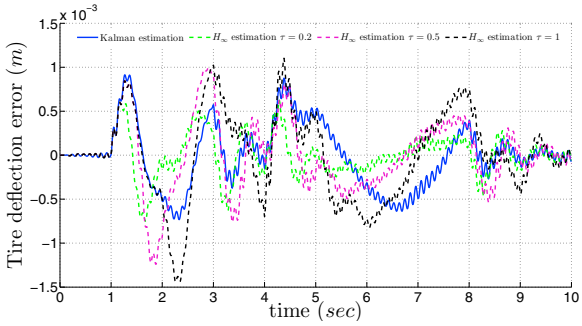

Fig. 6. Tire deflection x_1 estimation error.

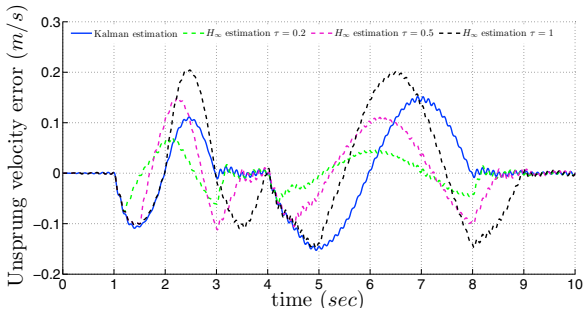

Fig. 7. Unsprung mass velocity x_2 estimation error.

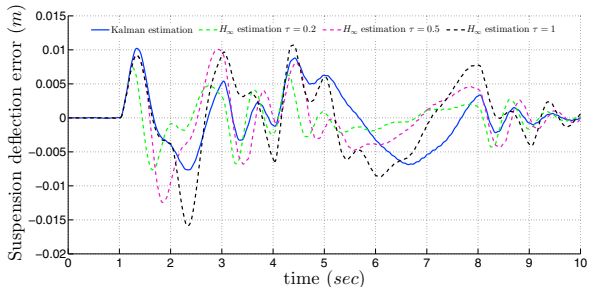

Fig. 8. Suspension deflection x_3 estimation error.

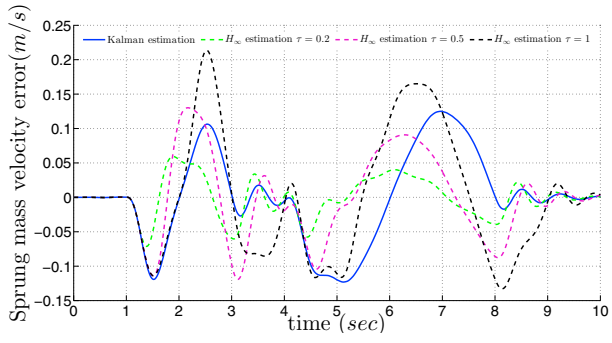

Fig. 9. Sprung mass velocity x_4 estimation error.

7. CONCLUSIONS

In this paper we studied an H_∞ filtering problem which is to estimate states of a cloud-aided semi-active suspension system. The filter exploits vehicle sensor measurements and road profile information sent from the cloud to the vehicle with a time delay. Disturbances due to GPS localization error, measurement noise and time-delay uncertainty are treated with the designed H_∞ filter. Sufficient conditions for existence of the H_∞ filter are derived with Lyapunov–Krasovskii methods and lead to linear matrix inequalities. The explicit expressions for the filter parameters were derived using the projection lemma. Numerical simulations were presented to show that better estimation performance is obtained in comparison with the Kalman filter when the delay is small.

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