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Ph.D. Qualifying Exam in
Fluid Mechanics

- Closed book and Notes, Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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**Problem 1:** A steady fluid of constant density $\rho$ flows through a vertical converging-diverging duct with length $L$. The duct cross sectional areas at inlet, throat and outlets are $A_{\text{inlet}}$, $A_{\text{throat}}$, $A_{\text{outlet}}$, respectively. Average pressure at outlet is measured to be $P_{\text{outlet}}$ and average velocity at inlet is measured to be $V_{\text{inlet}}$. Assume the gravity to be opposite of the flow direction and $A_{\text{outlet}}$ to be smaller or equal to $A_{\text{inlet}}$. (a) Neglecting irreversibility such as friction, generate expressions for the average velocity at outlet and average pressure at inlet in terms of given and measured variables. (b) In a real flow (with irreversibilities), do you expect the actual pressure at the inlet to be higher or lower than the one predicted in part (a)? Explain.
**Problem 2:** Consider a class of two-dimensional, incompressible vortical flows in cylindrical coordinates; the tangential velocity component is $v_\theta = \frac{K}{r}$, where $K$ is a constant. (a) Generate an expression of radial velocity $v_r$. (b) By assuming the flow to be steady and $v_r = 0$, the flow become a line vortex whose axis lies along the $z$-coordinate. Use the Navier-Stokes (momentum) equations to calculate the pressure as a function of $r$ in the line vortex flow. Neglect gravity.
**Problem 3:** Consider the two-dimensional potential (inviscid and irrotational) flow around a rotating cylinder, as illustrated below. The rotation of the cylinder results in the generation of a lift force $L$. If the flow velocity components, in a polar coordinate system, are given by:

$$u_r = U_\infty \cos \theta \left(1 - \frac{a^2}{r^2}\right), \quad \text{and} \quad u_\theta = -U_\infty \sin \theta \left(1 + \frac{a^2}{r^2}\right) - \frac{\Gamma}{2\pi r},$$

where $\Gamma$ is the circulation computed around a closed contour surrounding the cylinder, and $a$ is the cylinder radius, determine the lift force per unit span in terms of $\rho$, $U_\infty$ and $\Gamma$ using the *integral momentum equation applied to a suitable control volume*. You may find the following information useful:

\[
\begin{align*}
\int \sin^2(ax) dx &= \frac{x}{2} - \frac{\sin(2ax)}{4a} \\
\int \sin^3(ax) dx &= -\frac{3 \cos(ax)}{4a} + \frac{\cos(3ax)}{12a} \\
\int \cos^2(ax) dx &= \frac{x}{2} + \frac{\sin(2ax)}{4a} \\
\int \cos^3(ax) dx &= \frac{3 \sin(ax)}{4a} + \frac{\sin(3ax)}{12a}
\end{align*}
\]

\[
\rho, \ p_\infty U_\infty
\]

\[
\begin{array}{c}
\rho, \ p_\infty U_\infty \\
\vector{a} \\
\vector{L} \\
\vector{r} \\
\theta, x, y
\end{array}
\]
**Problem 4:** It is desired to build a micro device in which a 10 μm-diameter cylinder oscillates at a frequency of 100 kHz with/against an air flow of uniform velocity \((U_o)\) of 10 m/s. The amplitude of oscillation \((\delta)\) is anticipated to be 1 μm. For the purposes of designing the device, it is desired to estimate the unsteady flow force acting on the cylinder (which may be assumed infinitely long). Because of the difficulty and cost in fabrication of the micro device as well as the difficulty in measuring forces acting on such a small object, an engineer proposes to conduct a scale-up test in a water channel, in which the cylinder diameter is 5 mm. Employ a procedure of your choice to obtain the appropriate non-dimensional parameters for this problem (show details of your work). What should the values of \(U_o\), \(\delta\) and \(f_o\) be for the test? (you may take the kinematic viscosity and density of water as ten and thousands of times that of air, respectively)
Formula Sheet

Bernoulli Equation:

Bernoulli Equation between two points \( \text{1} \) and \( \text{2} \) on a streamline for a steady, incompressible flow:

\[
\frac{P_1}{\rho} + \frac{V_1^2}{2} + \frac{\Delta z_1}{g} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \frac{\Delta z_2}{g}.
\]

Integral equations:

Mass: \( 0 = \frac{\partial}{\partial t} \int \rho dV + \int \rho \vec{V} \cdot d\vec{A} \)

Momentum: \( \vec{F} = \vec{F}_e + \vec{F}_n = \frac{\partial}{\partial t} \int \vec{V} \rho dV + \int \vec{V} \rho \vec{V} \cdot d\vec{A} \)

Angular momentum: \( \sum (\vec{r} \times \vec{F}) = \frac{\partial}{\partial t} \int (\vec{r} \times \vec{V}) \rho dV + \int (\vec{r} \times \vec{V}) \rho \vec{V} \cdot d\vec{A} \)

Differential Equations - Continuity

Rectangular Coordinates \((x, y, z)\):

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0
\]

Cylindrical Coordinates \((r, \theta, z)\):

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0
\]

Differential Equations – Momentums for Incompressible, Constant Viscosity (\(\mu\))

Rectangular Coordinates \((x, y, z)\):

\[
\rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = \mu \left[ \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] - \frac{\partial \rho}{\partial x} + \rho g_x
\]

\[
\rho \left( \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = \mu \left[ \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right] - \frac{\partial \rho}{\partial y} + \rho g_y
\]

\[
\rho \left( \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left[ \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] - \frac{\partial \rho}{\partial z} + \rho g_z
\]

Cylindrical Coordinates \((r, \theta, z)\):

\[
\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{\partial}{\partial r} (\rho v_r) + \frac{v_y}{r} \frac{\partial v_r}{\partial \theta} \right) = \mu \left[ \frac{\partial^2 v_r}{\partial r^2} + \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} \right] - \frac{\partial \rho}{\partial r} + \rho g_r
\]

\[
\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_r}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} \right) = \mu \left[ \frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} \right] - \frac{\partial \rho}{\partial \theta} + \rho g_\theta
\]

\[
\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + \frac{v_z}{r} \frac{\partial v_z}{\partial \theta} \right) = \mu \left[ \frac{\partial^2 v_z}{\partial r^2} + \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] - \frac{\partial \rho}{\partial z} + \rho g_z
\]
Integral mass conservation equation:
\[ 0 = \frac{\partial}{\partial t} \int_{\mathcal{C}_v} \rho \, d\mathcal{V} + \int_{\mathcal{C}_S} \rho \vec{V} \cdot d\vec{A} \]

Integral momentum equation:
\[ \vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t} \int_{\mathcal{C}_v} \vec{V} \rho \, d\mathcal{V} + \int_{\mathcal{C}_S} \vec{V} \rho \vec{V} \cdot d\vec{A} \quad \text{with} \quad \vec{F}_s \quad \text{due to pressure given by} \]
\[ \vec{F}_{s,p} = -\int_{\mathcal{C}_S} p \, d\vec{A} \]

Integral angular momentum equation:
\[ \sum (\vec{r} \times \vec{F}) = \frac{\partial}{\partial t} \int_{\mathcal{C}_v} (\vec{r} \times \vec{V}) \rho \, d\mathcal{V} + \int_{\mathcal{C}_S} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot d\vec{A} \]