Module 9:
Radiated Emissions

Overview

The term radiated emissions refers to the unintentional release of electromagnetic energy from an electronic device. The electronic device generates the electromagnetic fields that unintentionally propagate away from the device's structure. In general, radiated emissions are usually associated with non-intentional radiators, but intentional radiators can also have unwanted emissions at frequencies outside their intended transmission frequency band.

As was discussed in the EMC Regulations module, the allowable radiated emissions from electronic modules are regulated by various organizations and agencies. Electronic devices that have significant amounts of radiated emissions may interfere with their normal operation or the normal operation of other devices in close proximity. For these reasons, it is important to understand the concepts behind the origins of radiated emissions so that fundamental design techniques can be used to minimize the emissions.

This module will investigate the origins of radiated emissions, and discuss methods to predict, measure, and minimize the radiated emissions from an electronic device. Differential and common mode currents will be introduced, and their roles in radiated emissions will be investigated. Using current probes to measure the differential and common mode currents on current carrying wires will be discussed. Finally, a discussion will be given on the role of circuit geometry and device structure on the radiated emissions from an electronic device.

9.1 Common and Differential Mode Currents

Conductors carrying current at frequencies with wavelengths appreciable to the size of the conductors may efficiently radiate electromagnetic energy. Therefore a grasp of the types of current that can exist on simple conductor arrangements is crucial to an understanding of radiated emissions. In this section, two types of current modes that can exist on parallel conductors will be investigated.

Currents on parallel conductors can be split into two types: differential mode currents and common mode currents. A diagram of these current modes is given in **Figure 1**. Differential mode currents are equal in magnitude but oppositely directed currents on parallel conductors. This type of current mode is usually assumed in circuit theory. Common mode currents are equal in magnitude and have the same direction on parallel conductors. This type of current is not predicted by circuit theory and presents the largest problems for EMC issues (Paul 402).

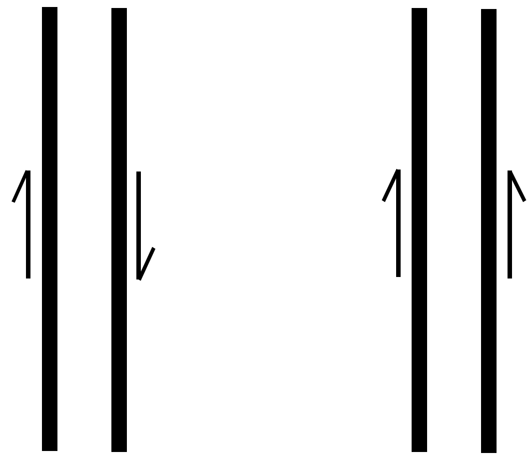

Figure 1: Differential and Common Mode Currents

An arbitrary current on a parallel conductor system is in general comprised of differential and common mode currents. A general current, therefore, can be decomposed into a differential mode component, and a common mode component. As shown in **Figure 2**, the current on the first conductor, I_1, can be written as the sum of the common mode current, I_C, and the differential mode current, I_D:

$$I_1 = I_C + I_D.$$

The current on the second conductor, I_2, can be written as the difference of the common mode and the differential mode currents:

$$I_2 = I_C - I_D.$$

Thus the differential and common mode currents are found to be:

$$I_D = \frac{I_1 - I_2}{2},$$

and

$$I_C = \frac{I_1 + I_2}{2}.$$

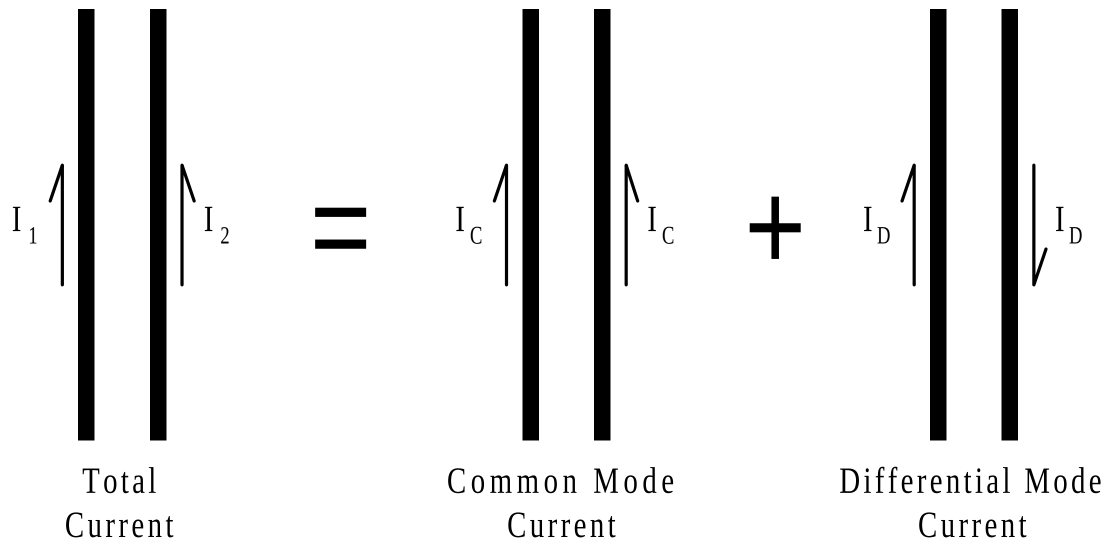

Figure 2: Decomposition of total current into common mode and differential mode currents

The importance of distinguishing between common mode and differential mode currents becomes evident when the radiated emissions from each type of current is examined. The radiated electric field components due to differential mode currents subtract, producing a small net radiated electric field. The radiated electric field components due to common mode currents add, producing a much larger net electric field (Paul 404). This effect is demonstrated in **Figure 3**. The electric field components due to the currents on conductor 1, $\vec{E}_1$, are identical for common mode and differential mode currents. The electric field is observed at a point in the same plane as the parallel conductors. Since conductor 2 is closer to the observation point (by a small distance *s*), the magnitude of the electric field due to currents on conductor 2 ($\vec{E}_2$) is slightly larger than that due to currents on conductor 1 for both current modes. However, $\vec{E}_2$ due to common mode currents is in the same direction as $\vec{E}_1$, and $\vec{E}_2$ due to differential mode currents is in the opposite direction as $\vec{E}_1$. Therefore, at distances *d* much greater than the conductor separation *s*, the total electric field due to the differential mode currents is almost negligible, whereas the total electric field due to the common mode currents is quite appreciable.

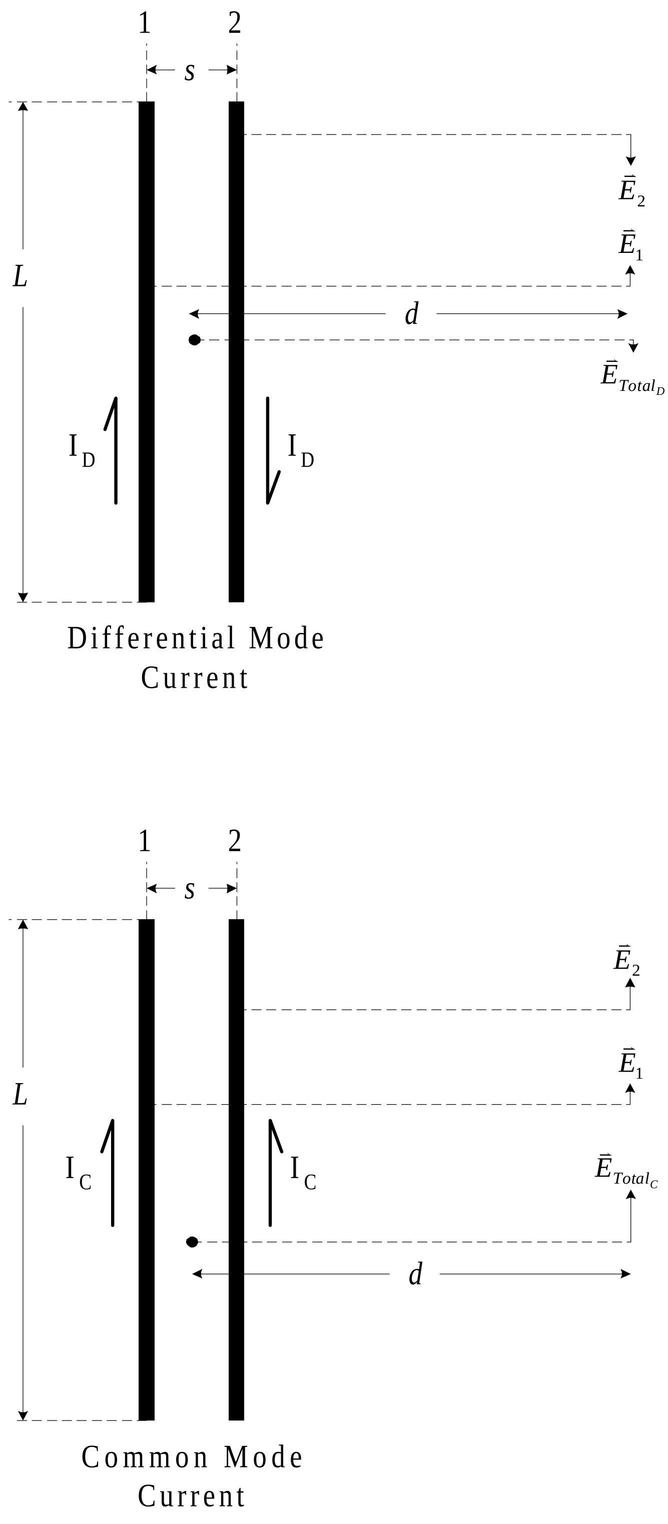

Figure 3: Demonstration of the difference in the total electric field due to differential mode and common mode currents.

9.2 Approximations for the Radiated Electric Fields due to Common Mode and Differential Mode Currents

In general, it is difficult to model complicated electronic devices using Maxwell's equations to find the electromagnetic fields radiated from the structures. However, simple geometries can be modeled to provide approximations to the radiated electric fields from common and differential mode currents. This section will present these approximate models.

We wish to approximate the radiated (far-zone) electric field from two parallel conductors of length L for both the common mode current and differential mode current cases as shown in **Figure 4**. This is done by assuming that the general currents on the conductors, $I_1(z)$ and $I_2(z)$, could have both common mode and differential mode current components. The approximation will be performed by treating the system as a two-element antenna array. Therefore, a simple introduction to antenna array theory will be given here.

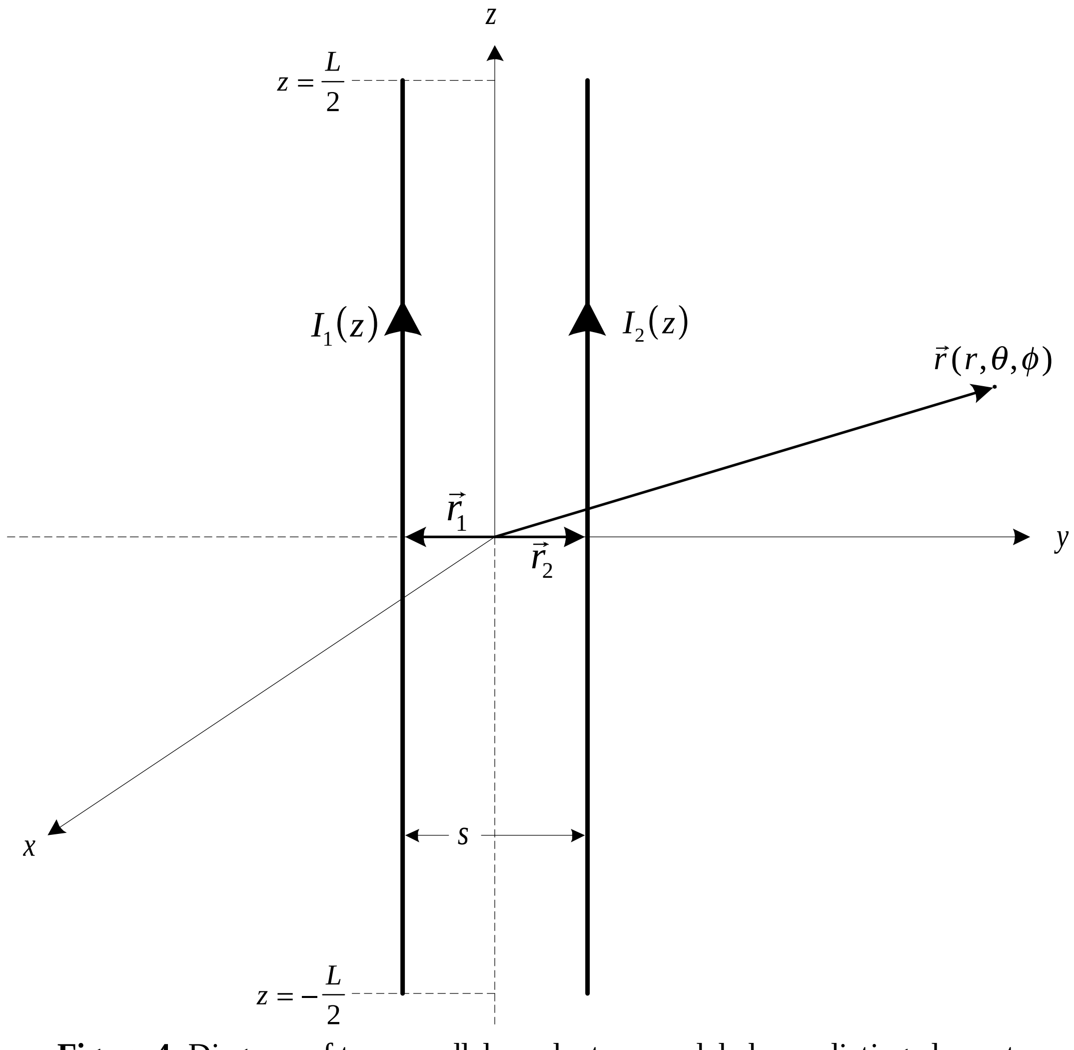

Figure 4: Diagram of two parallel conductors modeled as radiating elements.

9.2.1 Introduction to Antenna Array Theory

As shown in the Antennas module, the expression for the radiated electric field from an arbitrary electric current source at an observation point $\vec{r}$ in the far-zone of the antenna is

$$\vec{E}(\vec{r}) = -j\omega[\hat{\theta}A_\theta(z) + \hat{\varphi}A_\varphi(z)]$$

where $\vec{A}(\vec{r})$ is the magnetic vector potential. $\vec{A}(\vec{r})$ can be approximated in the far-zone as

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\vec{N}(\theta,\varphi)$$

where $\vec{N}(\theta,\varphi)$ is called the *directional weighting function*:

$$\vec{N}(\theta,\varphi) = \int_V \vec{J}(\vec{r}^{\,/})e^{jk\hat{r}\cdot\vec{r}^{\,/}}dv^{/}.$$

A general $N + 1$ element array configuration is shown in **Figure 5**. The observation point, $\vec{r}$, is assumed to be in the far-zone of any of the antenna elements. The vector $\vec{r}_i$ $(i = 0, 1, 2, ... N)$ points to a local origin on the ith antenna element. The vector $\vec{r}_i^{\,/}$ points from the ith element's local coordinate origin to a source point on the same element. Thus the vector $\vec{r}^{\,/} = \vec{r}_i + \vec{r}_i^{\,/}$ points from the global coordinate origin to a source point in the ith element. Then

$$\vec{N}(\theta,\varphi) = \int_V \vec{J}(\vec{r}_i + \vec{r}_i^{\,/})e^{jk\hat{r}\cdot(\vec{r}_i + \vec{r}_i^{\,/})}dv^{/}$$

is the directional weighting function at an observation point $\vec{r}$. By linear superposition,

$$\vec{N}(\theta,\varphi) = \sum_{i=0}^{N} e^{jk\hat{r}\cdot\vec{r}_i}\int_{V_i}\vec{J}_i(\vec{r}_i^{\,/})\,e^{jk\hat{r}\cdot\vec{r}_i^{\,/}}dv_i^{/}$$

or

$$\vec{N}(\theta,\varphi) = \sum_{i=0}^{N} e^{jk\hat{r}\cdot\vec{r}_i}\,\vec{N}_i(\theta,\varphi)$$

where

$$\vec{N}_i\,(\theta,\varphi) \;=\; \int_{V_i} \vec{J}_i(\vec{r}_i^{\,/})\,e^{jk\hat{r}\cdot\vec{r}_i^{\,/}}\,dv_i^{/}.$$

Thus the

$$\vec{N}_i\,(\theta,\varphi)$$

represent the directional weighting functions of each element with respect to their local coordinate origins. These functions are then weighted by an additional phase shift of

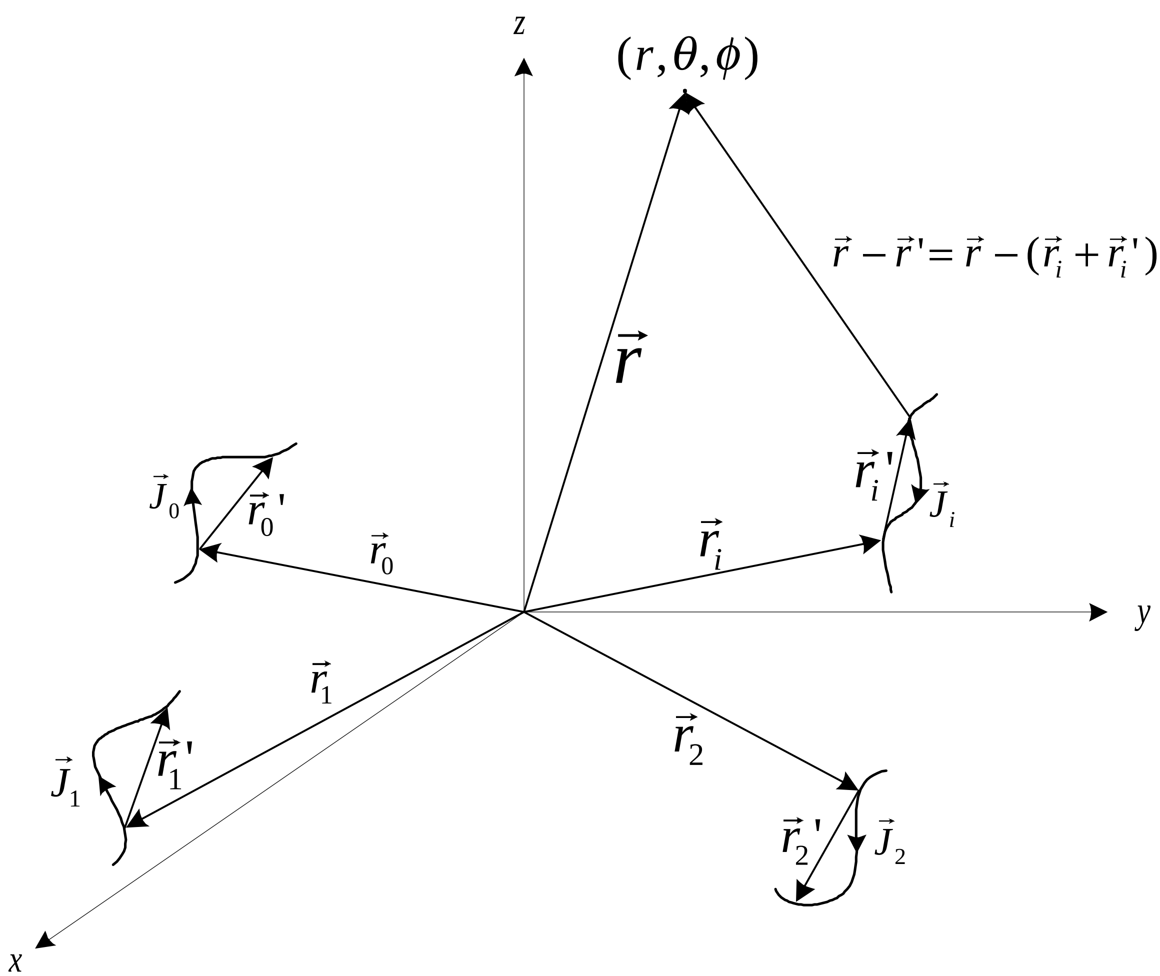

Figure 5: Diagram of general array configuration

$$e^{jk\hat{r}\cdot\vec{r}_i}$$

accounting for the locations of the ith local coordinate origin with respect to the global coordinate system. The total directional weighting function is then the superposition of the local weighting functions with adjusted phases.

The general array results will now be specialized to the case where all of the antenna elements are identical. We will assume that the current distribution will have the same spatial form on each element and the current densities will differ only by a scalar:

$$\vec{J}_i(\vec{r}_i^{\,/}) = \frac{I_i}{I_0}\vec{J}_0(\vec{r}_0^{\,/})$$

where $\vec{J}_0(\vec{r}_0^{\,/})$ is the current density on array element 0 excited by an input current I_0, and I_i is the input current to the ith element. Element 0 is then regarded as the reference element. Then we can write

$$\vec{N}(\theta,\varphi) = \vec{N}_e(\theta,\varphi)\ N_a(\theta,\varphi)$$

where

$$N_a(\theta,\varphi) = \sum_{i=0}^{N}\frac{I_i}{I_0}e^{jk\hat{r}\cdot\vec{r}_i}$$

is called the *array factor* and

$$\vec{N}_e(\theta,\varphi) = \int_{V_0}\vec{J}_0(\vec{r}_0^{\,/})\,e^{jk\hat{r}\cdot\vec{r}_0^{\,/}}dv_0^{/}$$

is the element pattern weighting function.

Now that a simple array theory has been developed, the theory will be applied to the parallel wire system for both a Hertzian dipole and a half-wave dipole current distribution. The array factor

$$N_a(\theta,\varphi)$$

for the parallel wire structure will be the same for both current distributions on the conductors. Again referring to **Figure 4**, the array factor can be found by

$$N_a(\theta,\varphi) = \sum_{i=1}^{2}\frac{I_i}{I_0}e^{jk\hat{r}\cdot\vec{r}_i}.$$

Using

$$\hat{r}\cdot\vec{r}_1 = -\frac{s}{2}\sin(\theta)\sin(\varphi) \text{ and } \hat{r}\cdot\vec{r}_2 = \frac{s}{2}\sin(\theta)\sin(\varphi),$$

$$N_a(\theta, \varphi) = \frac{I_1}{I_0} e^{-jk\frac{s}{2}\sin\theta\sin\varphi} + \frac{I_2}{I_0} e^{jk\frac{s}{2}\sin\theta\sin\varphi}$$

for both a Hertzian dipole and a half-wave dipole current distribution.

9.2.2 Specialization of Array Results Using a Hertzian Dipole Current Distribution

As a simple model for a radiating structure, a Hertzian dipole current distribution can be used to approximate the far-zone radiated electric field from the parallel wire system shown in **Figure 4**.

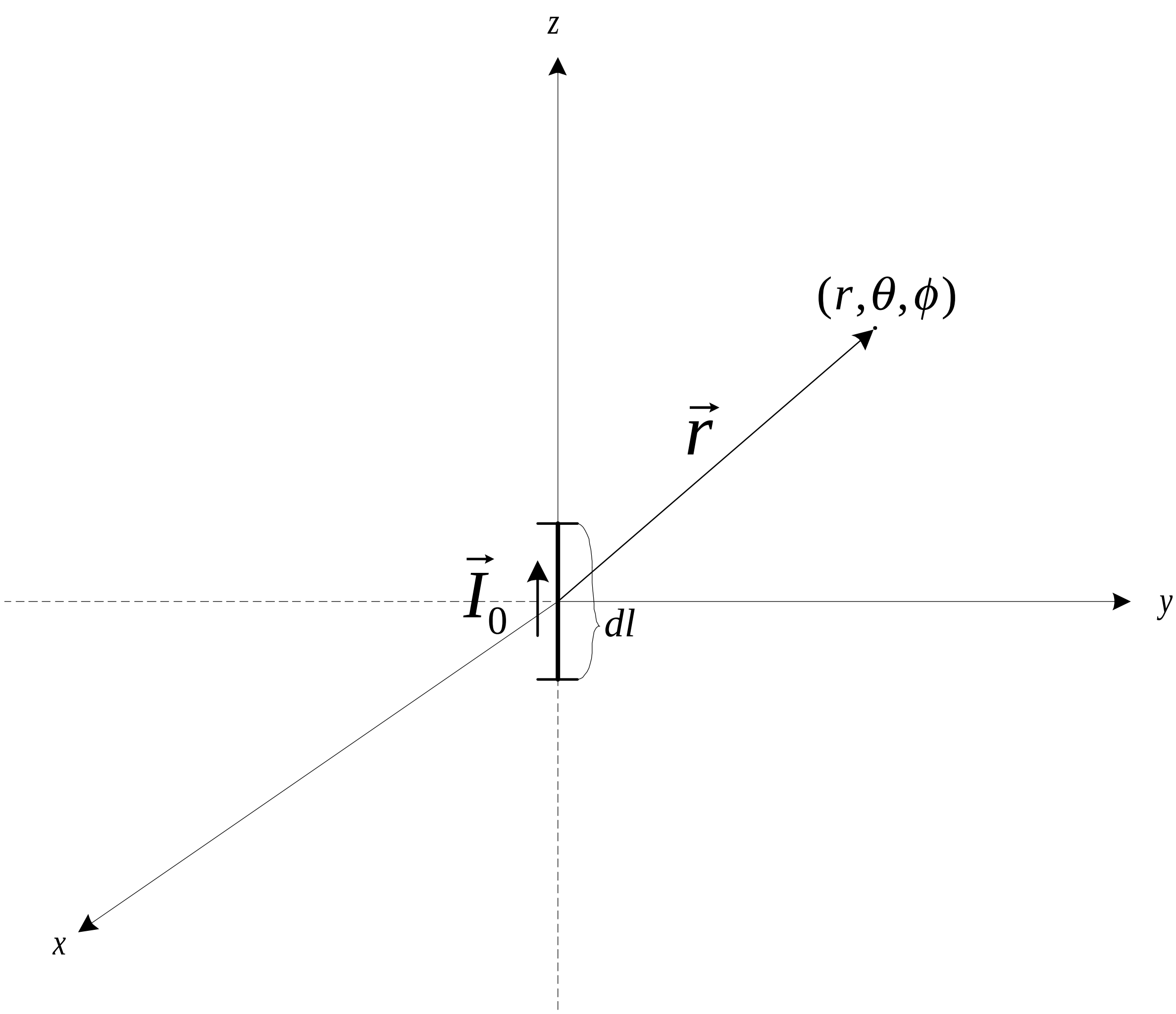

Figure 6: Diagram of general Hertzian dipole arrangement.

To find the element pattern weighting function for a Hertzian dipole, consider the system shown in **Figure 6**.

$$\vec{N}_e(\theta,\varphi) = \int_{V_0} \vec{J}_0(\vec{r}_0^{\,\prime})\, e^{jk\hat{r}\cdot\vec{r}_0^{\,\prime}} dv_0^{\prime}$$

where

$$\vec{J}_0(\vec{r}) = \begin{cases} \hat{z}\, I_0\, \delta(x)\, \delta(y) & \ldots \quad -\frac{dl}{2} < z < \frac{dl}{2} \\ 0 & \ldots \quad otherwise \end{cases}.$$

Then

$$\vec{N}_e\,(\theta,\varphi) = \int_{-dl/2}^{dl/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{z}\; I_0\; \delta(x')\delta(y')\, e^{jk(x'\cos\varphi\sin\theta + y'\sin\varphi\sin\theta + z'\cos\theta)} dx'dy'dz'$$

$$\vec{N}_e\,(\theta,\varphi) = \hat{z}\, I_0 \int_{-dl/2}^{dl/2} e^{jkz'\cos\theta} dz'.$$

Now assume that dl is sufficiently small compared to wavelength such that $kz'\cos\theta \approx 0$ for all z' from $-dl/2$ to $dl/2$ so that

$$e^{jkz'\cos\theta} \approx 1.$$

Then

$$\vec{N}_e\,(\theta,\varphi) = \hat{z}\, I_0 \int_{-dl/2}^{dl/2} dz' = \hat{z}\, I_0\, dl.$$

Assuming that the length of the parallel conductors,L, is sufficiently small compared to wavelength, we can approximate $L = dl$. Letting $I_1 = I_0$, the magnetic vector potential becomes

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\vec{N}_e\,(\theta,\varphi)\;\; N_a(\theta,\varphi)$$

$$\vec{A}(\vec{r}) = \hat{z}\,\frac{\mu}{4\pi}\frac{e^{-jkr}}{r} L\left[I_1\, e^{-jk\frac{s}{2}\sin\theta\,\sin\varphi} + I_2\, e^{jk\frac{s}{2}\sin\theta\,\sin\varphi} \right].$$

Now using $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$, the radiated far-zone electric field from the parallel wire system using a Hertzian dipole current distribution is

$$\vec{E}(\vec{r}) = -j\omega[\hat{\theta}A_\theta(\vec{r}) + \hat{\varphi}A_\varphi(\vec{r})]$$

$$\vec{E}(\vec{r}) = \hat{\theta}\,\frac{j\omega\mu}{4\pi}\,\frac{e^{-jkr}}{r}\,L\sin\theta\left[I_1\,e^{-jk\frac{s}{2}\sin\theta\sin\varphi} + I_2\,e^{jk\frac{s}{2}\sin\theta\sin\varphi}\right]$$

or

$$\vec{E}(\vec{r}) = \hat{\theta}\,\frac{j\eta k}{4\pi}\,\frac{e^{-jkr}}{r}\,L\sin\theta\left[I_1\,e^{-jk\frac{s}{2}\sin\theta\sin\varphi} + I_2\,e^{jk\frac{s}{2}\sin\theta\sin\varphi}\right].$$

This is the same result as found in Paul.

9.2.3 Specialization of Array Results Using a Half-Wave Dipole Current Distribution

As L approaches a half-wavelength, a Hertzian dipole current distribution may not be a very accurate model for approximating the far-zone radiated electric field. The far-zone field will now be approximated by using a half-wave dipole current distribution on each of the parallel conductors.

From the Antennas module, the element pattern weighting function of a half-wave dipole current distribution is:

$$\vec{N}_e(\theta,\varphi) = -\frac{2I_0}{K}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\hat{\theta} = -\frac{2I_0}{K}F(\theta)\,\hat{\theta}$$

where

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}.$$

Recall that for the parallel wire system,

$$N_a(\theta,\varphi) = \frac{I_1}{I_0}e^{-jk\frac{s}{2}\sin\theta\sin\varphi} + \frac{I_2}{I_0}e^{jk\frac{s}{2}\sin\theta\sin\varphi}.$$

Therefore, the far-zone radiated electric field from the parallel wire system assuming a half-wave dipole current distribution is:

$$\vec{E}(\vec{r}) = -j\omega\vec{A}_T = -j\omega\frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\vec{N}_e(\theta,\varphi)\,N_a(\theta,\varphi)$$

$$\vec{E}(\vec{r}) = j\frac{\eta_0}{2\pi}\frac{e^{-jkr}}{r}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\left[I_1 e^{-jk\frac{s}{2}\sin\theta\sin\varphi} + I_2 e^{jk\frac{s}{2}\sin\theta\sin\varphi}\right].$$

This result is also shown in Paul.

9.3.1 Specialization of Emissions Models for Differential Mode Currents

Now that models have been developed for the far-zone radiated electric field due to general input currents, these models will now be specialized assuming a differential mode current on the parallel-wire system. Thus, the input currents to the parallel wires will be equal and opposite:

$$I_1 = I_D$$

and

$$I_2 = -I_D$$

where I_D is the differential mode current on the parallel conductors. In order to appreciate the difference between radiated emissions due to differential mode and common mode currents, a worst-case scenario application of the radiated-electric field expressions will now be performed for differential mode currents. The case using common mode currents will be performed in the next subsection.

To develop a simple approximation for the fields due to differential mode currents, we will assume a Hertzian dipole current distribution. Thus this development is only valid when the length of the parallel wire system is short enough for the current distribution to be constant over the length of the wires. Recall that for a Hertzian dipole current distribution the far-zone radiated electric field of the parallel conductor system is

$$\vec{E}(\vec{r}) = \hat{\theta}\,\frac{j\eta k}{4\pi}\,\frac{e^{-jkr}}{r}\,L\sin\theta\left[I_1\,e^{-jk\frac{s}{2}\sin\theta\sin\varphi} + I_2\,e^{jk\frac{s}{2}\sin\theta\sin\varphi}\right].$$

Using $I_1 = I_D$ and $I_2 = -I_D$, this becomes

$$\vec{E}(\vec{r}) = \hat{\theta}\,\frac{j\eta k}{4\pi}\,\frac{e^{-jkr}}{r}\,L\sin\theta\left[I_D\,e^{-jk\frac{s}{2}\sin\theta\sin\varphi} - I_D\,e^{jk\frac{s}{2}\sin\theta\sin\varphi}\right].$$

If the term $k\frac{s}{2}\sin\theta\sin\varphi = 0$, then

$$\vec{E}(\vec{r}) = \hat{\theta}\,\frac{j\eta k}{4\pi}\,\frac{e^{-jkr}}{r}\,L\sin\theta\left[\,I_D\,e^{0} - I_D\,e^{0}\,\right] = 0\,\hat{\theta}.$$

Therefore, in order to achieve maximum radiation, let $\theta = \pi/2$ and $\varphi = \pi/2$. Then

$$\vec{E}_{\max}(r) = \hat{\theta}\,\frac{\eta k}{2\pi}\,\frac{e^{-jkr}}{r}\,L\,I_D\,\sin\!\left(k\frac{s}{2}\right).$$

Now an important assumption needs to be made about the distance s between the conductors. We will assume that this distance is small compared to the wavelength:

$$k\frac{s}{2} = \frac{2\pi}{\lambda}\,\frac{s}{2} = \pi\,\frac{s}{\lambda} \ll 1$$

With this assumption, $\sin(k\frac{s}{2}) \approx k\frac{s}{2}$, and then the radiated electric field becomes

$$\vec{E}_{\max}(r) = \hat{\theta}\,\frac{\eta_0 k}{2\pi}\,\frac{e^{-jkr}}{r}\,L\,I_D\,k\frac{s}{2} = \hat{\theta}\,\frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\,(2\pi f\sqrt{\mu_0\varepsilon_0})^2\,\frac{e^{-jkr}}{r}\,L\,I_D\,\frac{s}{2}$$

or

$$\vec{E}_{\max}(r) = \hat{\theta}\,1.317\times10^{-14}\,f^{2}\,\frac{e^{-jkr}}{r}\,L\,I_D\,s.$$

The magnitude of this result matches the result found in Paul:

$$|\vec{E}_{\max}(r)| = 1.317\times10^{-14}\,\frac{f^{2}\,L\,s\,|I_D|}{r}.$$

Examining this expression for a fixed measurement distance r, it is evident that the three contributors to differential mode current radiation are the current magnitude ($|I_D|$), the frequency of the current (f), and the loop area of the parallel wire system ($A = L s$). Therefore, to reduce the radiated emissions from differential mode currents, these quantities must be reduced. Note that the current magnitude $|I_D|$ is a function of frequency.

Reducing the magnitude of the current may in general be difficult because the functional performance of the system may be compromised. However, techniques such as lengthening the rise time of digital pulses and lowering the fundamental clock frequencies of digital signals will have the effect of lowering the value of the current magnitude at higher frequencies. Reducing loop area of the parallel conductor system, $A = L s$, is another method used to reduce the differential mode current radiation. Shortening the length of the wires, L, or decreasing the distance between the wires, s, would help in reducing the emissions (Paul, 409).

As an example problem with differential mode current radiation, consider a loop circuit of area A that carries a differential mode current I_D. Let the current I_D be a trapezoidal waveform such as the digital clock waveform studied in the Signals and Spectra module. The envelope of the magnitude spectrum of I_D is shown in **Figure 7**. From the formula for $|\vec{E}_{\max}(r)|$ given above, notice that for a given distance from the circuit the term $|\vec{E}_{\max}(r)/I_D|$ is proportional to f^2. This term increases at 40 dB per decade of frequency, as also shown in **Figure 7** ($\vec{E}_D = \vec{E}_{\max}(r)$). By multiplying the spectrum for $|\vec{E}_D/I_D|$ with the spectrum of $|I_D|$, we get the envelope of the magnitude spectrum for the radiated emissions $|\vec{E}_D|$ due to a differential mode current from a trapezoidal waveform. The spectrum of $|\vec{E}_D|$ is also shown in **Figure 7**. Notice that the spectrum of $|\vec{E}_D|$ increases at 40 dB per decade until the frequency $1/\pi\tau$, and then at 20 dB per decade until the frequency $1/\pi\tau_r$. After this frequency, the spectrum is flat. Thus for differential mode currents, radiated emissions problems for a trapezoidal current waveform are typically high in frequency, usually past the frequency $1/\pi\tau_r$.

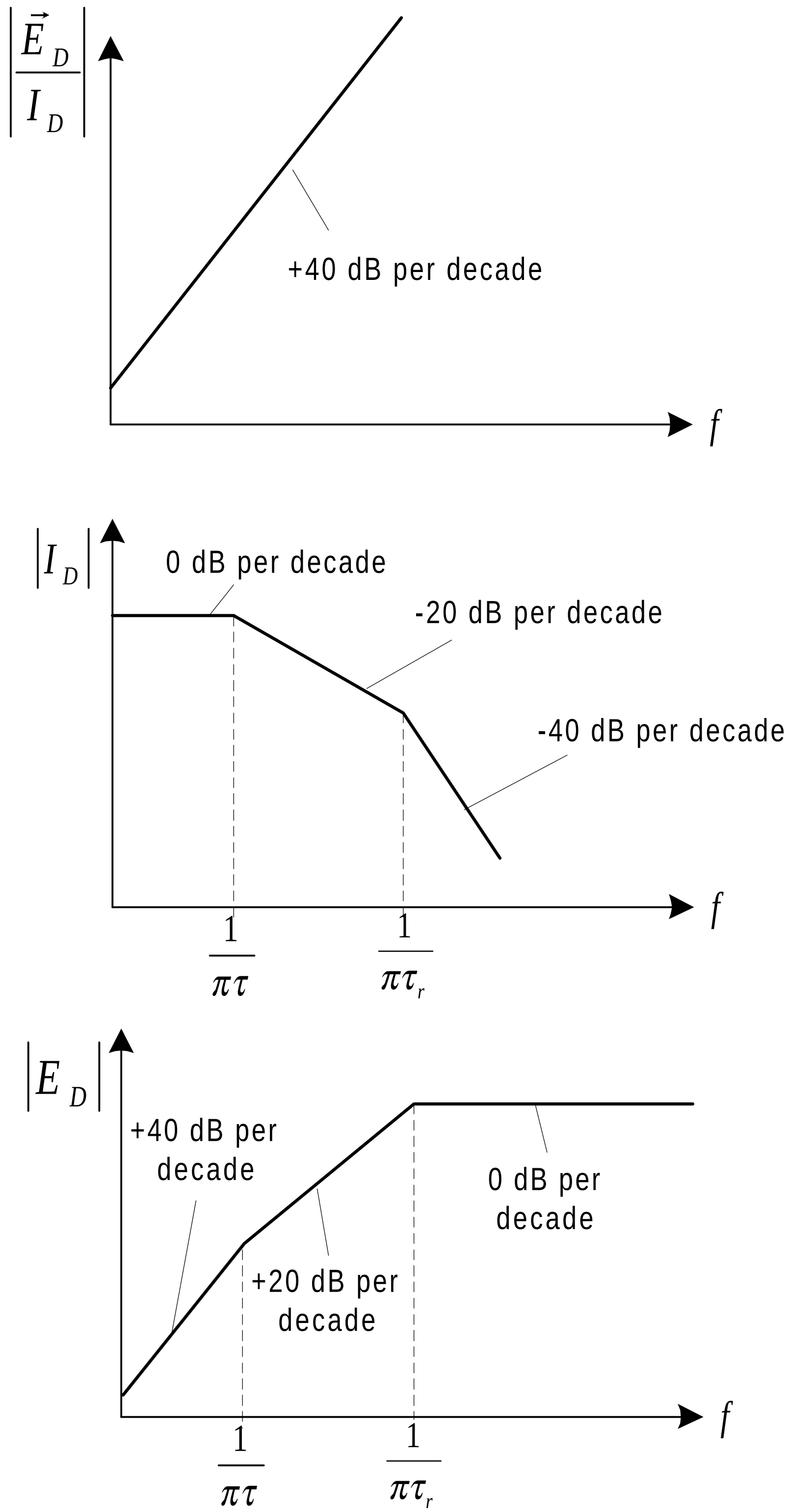

Figure 7: Qualitative plot of the radiated emissions due to a differential mode current from a trapezoidal waveform.

9.3.2 Specialization of Emissions Models for Common Mode Currents

Now that the emissions model for the far-zone electric field has been specialized for differential mode currents, the model will now be specialized for common mode currents. All of the same assumptions made for the differential mode case will also be assumed here (Hertzian dipole current distribution, separation between wires small compared to wavelength) except the signs of the input currents. For common mode current on the parallel wire structure,

$$I_1 = I_C$$

and

$$I_2 = I_C.$$

Recalling the results from the general parallel wire development using a Hertzian dipole current distribution,

$$\vec{E}(\vec{r}) = \hat{\theta}\,\frac{j\eta k}{4\pi}\,\frac{e^{-jkr}}{r}\,L\sin\theta\left[I_1\,e^{-jk\frac{s}{2}\sin\theta\,\sin\varphi} + I_2\,e^{jk\frac{s}{2}\sin\theta\,\sin\varphi}\right]$$

Assuming common mode currents on the conductors, this becomes

$$\vec{E}(\vec{r}) = \hat{\theta}\,\frac{j\eta k}{4\pi}\,\frac{e^{-jkr}}{r}\,L\sin\theta\,I_C\left[e^{-jk\frac{s}{2}\sin\theta\,\sin\varphi} + e^{jk\frac{s}{2}\sin\theta\,\sin\varphi}\right].$$

As in the development for the differential mode case, we will look for the maximum radiated electric field from the structure. This occurs when $\theta = \pi/2$ and $\varphi = \pi/2$. Then

$$\vec{E}_{\max}(r) = \hat{\theta}\,\frac{j\eta k}{4\pi}\,\frac{e^{-jkr}}{r}\,L\,I_C\left[e^{-jk\frac{s}{2}} + e^{jk\frac{s}{2}}\right] = \hat{\theta}\,\frac{j\eta k}{2\pi}\,\frac{e^{-jkr}}{r}\,L\,I_C\,\cos\left(k\frac{s}{2}\right).$$

Now assuming that $k\frac{s}{2} \ll 1$, $\cos\left(k\frac{s}{2}\right) \approx 1$. The far-zone radiated electric field now becomes

$$\vec{E}_{\max}(r) = \hat{\theta}\,\frac{j\eta k}{2\pi}\,\frac{e^{-jkr}}{r}\,L\,I_C.$$

The magnitude of the electric field is then

$$|\vec{E}_{\max}(r)| = 1.257 \times 10^{-6}\,\frac{f\,L\,|I_C|}{r}.$$

This expression indicates that the separation between conductors, *s*, does not significantly affect the radiated electric field due to a common mode current. The parameters impacting the radiated emissions for a fixed distance *r* are the magnitude of the current $|I_C|$ (a function of frequency), the frequency *f*, and the length of the conductors *L*.

As discussed in the previous subsection, using longer rise times and lower clock frequencies will decrease the frequency *f* and the magnitude of the current at these frequencies. However, unlike differential mode currents, common mode currents are usually never needed for the functional performance of the system. Therefore, methods of restricting $|I_C|$ may greatly reduce the common mode radiated emissions. Using common mode chokes or ferrite beads are methods of reducing the common mode currents on a parallel wire structure. Reducing the length of the conductors, *L*, also reduces the common mode radiated emissions (Paul, 415).

As an example problem with common mode current radiation, consider a loop circuit of length L with a total common mode current of $2\,I_C$ (the width of the circuit is much narrower than its length). Let the current I_C be a trapezoidal waveform such as the digital clock waveform studied in the Signals and Spectra module. The envelope of the magnitude spectrum of I_C is shown in **Figure 8**. From the above formula for the maximum radiated electric field due to a common mode current, notice that for a given distance from the circuit the term $|\vec{E}_C/I_C|$ ($\vec{E}_C = \vec{E}_{\max}(r)$) is proportional to f (recall that for differential mode currents, the term $|\vec{E}_D/I_D|$ is proportional to f^2). This term increases at 20 dB per decade as shown in **Figure 8**. By multiplying the spectrum for $|\vec{E}_C/I_C|$ with the spectrum of $|I_C|$, we get the envelope of the magnitude spectrum for the radiated emissions $|\vec{E}_C|$ due to a common mode current with a trapezoidal shape. The spectrum of $|\vec{E}_C|$ is also shown in **Figure 8**. Notice that $|\vec{E}_C|$ is a maximum between the frequencies $1/\pi\tau$ and $1/\pi\tau_r$, where the spectrum is flat. After the frequency $1/\pi\tau_r$, the spectrum rolls off. Thus for common mode currents, radiated emissions problems for trapezoidal current waveforms usually occur lower in frequency than they do for differential mode currents. The plateau where the envelope of the radiated emissions spectrum is flat is usually much higher for common mode currents than for differential mode currents.

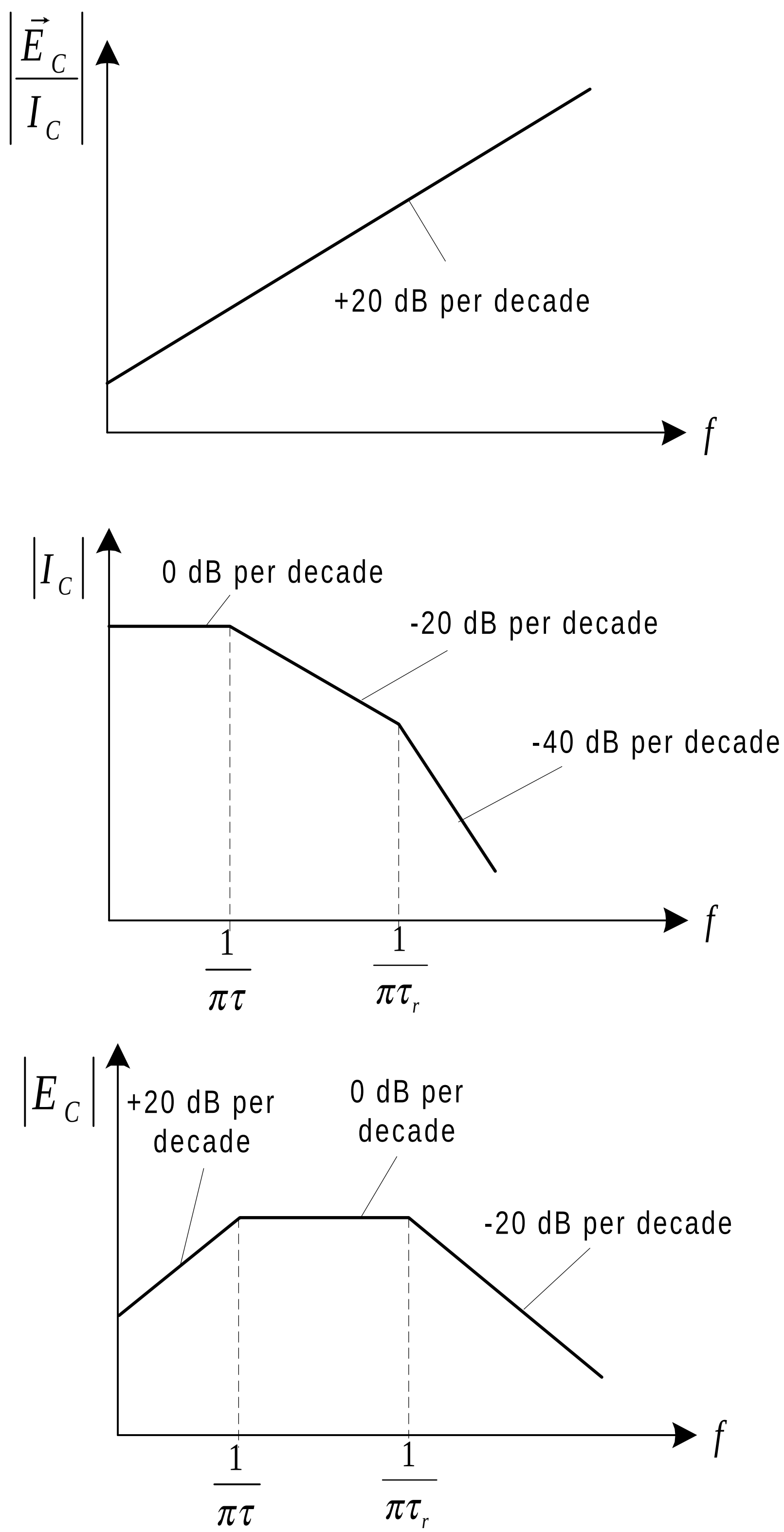

Figure 8: Qualitative plot of the radiated emissions due to a common mode current from a trapezoidal waveform.

9.4 Current Probes

This module has emphasized that the radiated emissions from an electronic system are greatly dependent on the common and differential mode currents in the system. Expressions for the approximate radiated electric field at a distance *r* from current-carrying wires were developed for both common mode and differential mode currents. However, to estimate the actual differential mode or common mode currents on wires within an electrical system may be difficult to achieve analytically. This is especially true for common mode currents, which are largely dependent on the geometry of the system an are very difficult to predict. In most cases, the common mode currents were not even part of the intended operation of the system. Therefore, an effective method to measure the approximate current on a conductor would be beneficial to an EMC engineer trying to predict the radiated emissions of an electronic system. This section describes a device called a current probe, which can be used to measure current.

A diagram of a current probe is shown in **Figure 9**. A ferrite core is wound with thin wire. Often the core will be separated into two halves and connected by a hinge so that the probe can be clamped around a wire without it needing to be disconnected from a circuit. The ferrite core is chosen such that it has a high relative permeability over the frequency band that the probe will be used. From Ampere's law, a conduction or displacement current passing through the surface of the hole's core will produce a magnetic field $\vec{H}$ in the ferrite core:

$$\oint_C \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s} + \frac{d}{dt}\varepsilon\int_S \vec{E}\cdot d\vec{s} .$$

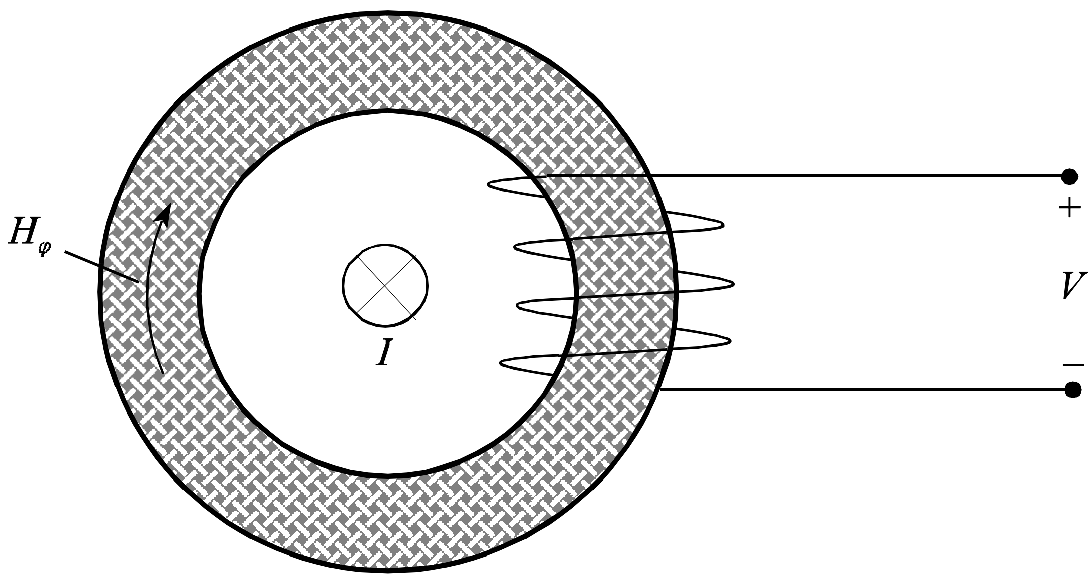

Figure 9: Current Probe

A diagram showing the application of ampere's law is shown in **Figure 10**. The surface S is taken to be the hole in the ferrite core, and the contour C is an imaginary contour within the ferrite core. At frequencies where the conduction current, $\vec{J}$, is much larger than the displacement current, $d/dt\,(\varepsilon\,\vec{E})$, Ampere's law can be approximated as:

$$\oint_C \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s} = I \;.$$

Thus the magnetic field is directly proportional to the total current I passing through the hole in the ferrite core.

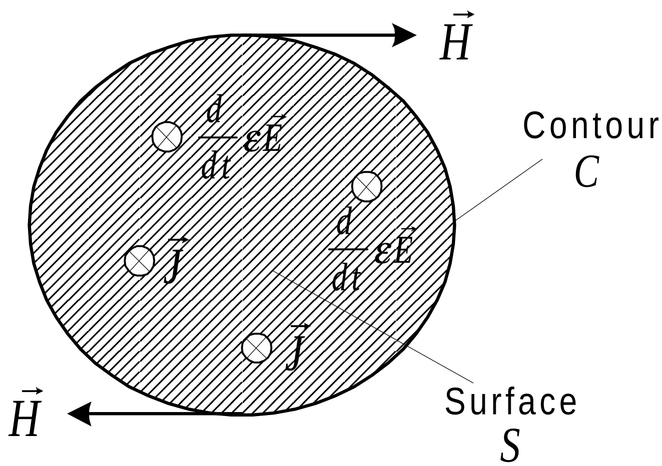

Figure 10:Application of Ampere's Law

A new surface $S^{/}$ bounded by a new contour $C^{/}$ can be defined as a cross section of the ferrite core, as shown in **Figure 11**. By Faraday's law, a time changing magnetic field through the surface $S^{/}$ produces a net circulation of an electric field around the contour $C^{/}$:

$$\oint_{C^{/}} \vec{E}\cdot d\vec{l}^{/} = -\frac{d}{dt}\int_{S^{/}} \vec{B}\cdot d\vec{S}^{/} \;.$$

Assuming that the ferrite core is a linear material ($\vec{B} = \mu_0\mu_r\vec{H} = \mu\vec{H}$),

$$\oint_{C^{/}} \vec{E}\cdot d\vec{l}^{/} = -\frac{d}{dt}\int_{S^{/}} \mu_0\mu_r\vec{H}\cdot d\vec{S}^{/} \;.$$

Recall that the ferrite core is wound with thin wire, and that the contour $C^{/}$ follows the path of each winding. The total emf for N turns of wire is then

$$emf_{total} = N \oint_{C^{/}} \vec{E} \cdot d\vec{l}^{\,/} = -N \frac{d}{dt} \int_{S^{/}} \mu_0 \mu_r \vec{H} \cdot d\vec{S}^{\,/} .$$

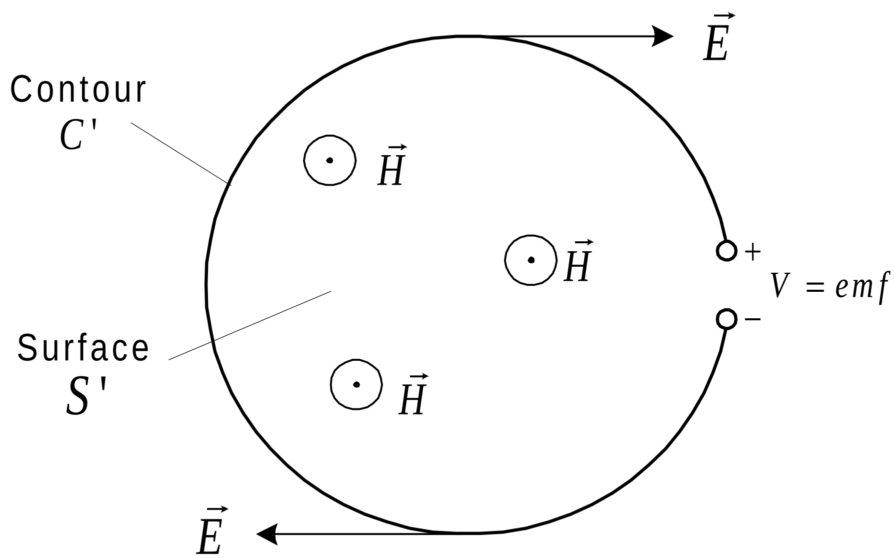

Figure 11: Application of Faraday’s Law

Thus the total frequency domain voltage read at the probe’s terminals (as shown in **Figure 9**) is

$$V(\omega) = emf(\omega)_{total} = -N j \omega \int_{S^{/}} \mu_0 \mu_r(\omega) \vec{H}(\omega) \cdot d\vec{S}^{\,/} .$$

Since the magnetic field $\vec{H}(\omega)$ is directly proportional to the current $\vec{J}(\omega)$ from Ampere’s law, the probe’s terminal voltage $V(\omega)$ is directly proportional to the total current $I(\omega)$ passing through the hole in the ferrite core. A transfer impedance can then be defined as

$$Z_T(\omega) = \frac{V(\omega)}{I(\omega)} .$$

By applying a known current at each frequency of interest and measuring a current probe’s terminal voltage, the transfer impedance of a particular current probe can be determined. When the current probe is then used to measure an unknown current, the amplitude of the measured voltage is divided by the transfer impedance to determine the current:

$$I(\omega) = \frac{V(\omega)}{Z_T(\omega)}$$

If the current is desired in dB, this expression becomes

$$|I(\omega)|_{dB\mu A} = |V(\omega)|_{dB\mu V} - |Z_T(\omega)|_{dB\Omega}$$

or

$$20\log\left(\frac{|I(\omega)|}{1\mu A}\right) = 20\log\left(\frac{|V(\omega)|}{1\mu V}\right) - 20\log\left(\frac{|Z_T(\omega)|}{1\Omega}\right).$$

Once the transfer impedance of a current probe is measured, the above equations are only good for measuring currents when the measurement device has the same input impedance as the device that measured the transfer impedance. Most measuring devices, such as most spectrum analyzers, have a 50Ω input impedance. If a measurement device with a different input impedance is to be used to measure unknown currents, the transfer impedance of the current probe also must be re-measured.

The transfer impedance of a current probe from a manufacturer is usually given by a calibration curve. An example of a calibration curve for a current probe can be found in Paul (pg. 418).

Current probes are excellent diagnostic tools that enable one to estimate the maximum radiated emissions from a wire within an electronic system without having to make a difficult calibrated absolute field strength measurements in a semianechoic chamber. Since common mode current radiation usually dominates differential mode current radiation, using a current probe to measure the common mode current on a group of wires provides a good estimate of the total radiation from the wires by using the formula for radiation due to a common mode current. When trying to meet a regulatory limit at a certain frequency, the formula for common mode current radiation,

$$|\vec{E}_{C,\,\max}(r)| \;=\; 6.285 \times 10^{-7}\;\frac{f\,L\;|I_{meas}|}{r},$$

can be rearranged as

$$|I_{meas,\,\max}| \;=\; 1.591 \times 10^{6}\;\frac{r}{f\,L}\;|\vec{E}_{C,\,\max}(r)|$$

so that the maximum allowable common mode current can be found. Thus when measuring a voltage on a spectrum analyzer at a certain frequency using a current probe, this approximate method can be used to determine whether the regulatory emission limits have been exceeded. In the above equation for the maximum allowable field strength, $|\vec{E}_{C,\,\max}(r)|$, the measured current has been divided by two because the common mode current is half of the total measured current:

$$I_{meas} \;=\; 2\,I_C\;.$$

It should be emphasized that this is only an approximate method of measuring the radiated emissions from a device, and to meet most regulations, the device must be measured in a semianechoic chamber under specific measurement guidelines. However, this approximate method is a simple and effective way to estimate the emissions before an actual regulatory test is appropriate.