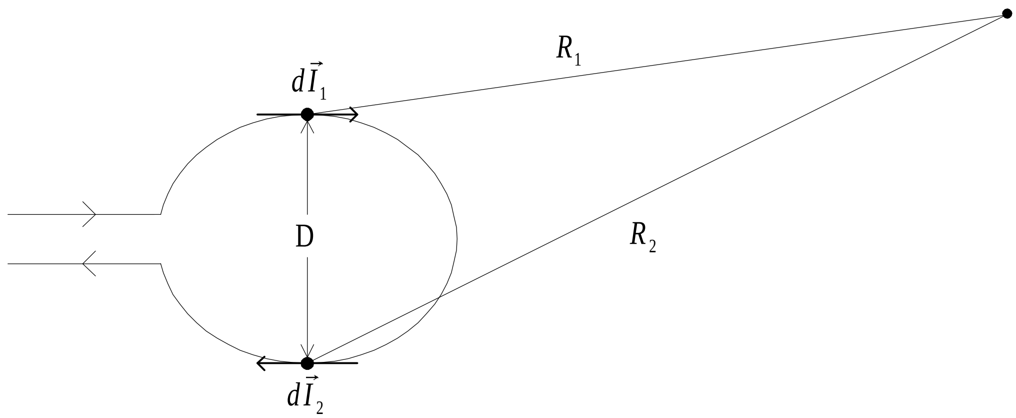
Module 6:
Antennas

6.0 Introduction

In chapter 2, the fundamental concepts associated with electromagnetic radiation were examined. In this chapter, basic antenna concepts will be reviewed, and several types of antennas will be examined. In particular, the antennas commonly used in making EMC-related measurements will be emphasized.

6.1 The Radiation Mechanism

Antennas produce fields which add in phase at certain points of space. Consider a loop of wire that carries a current.



Here two elements of current $d\vec{I}_1$ and $d\vec{I}_2$ are separated by a distance D. The current elements are located at distances R_1 and R_2, respectively from a distant observation point. If

$$R_2 - R_1 \lesssim 0.1\lambda$$

$$D \lesssim 0.1\lambda$$

Then the fields produced by the current elements add out of phase, and the amount of radiation is small. However, if

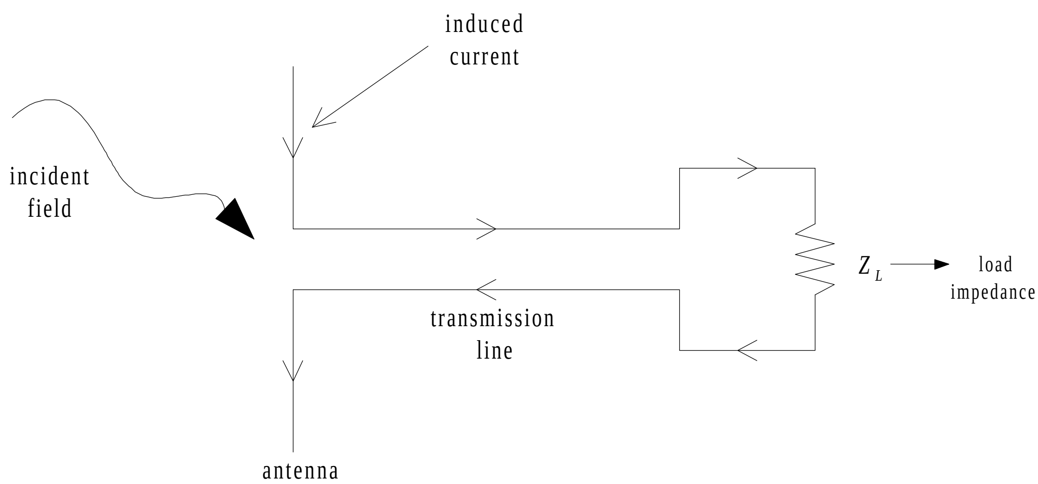
$$R_2 - R_1 \gtrsim 0.1\lambda$$

$$D \gtrsim 0.1\lambda$$

Then the fields produced by the current elements add in phase, and the amount of radiation is large.

-Reception Mechanism

Electromagnetic fields which are incident upon an antenna induce currents on the surface of the antenna which deliver power to the antenna load.



6.2 Radiated Power

The power radiated by a distribution of sources is that power which passes through a sphere of infinite radius. This, therefore, is the power which leaves the vicinity of the source system, and never returns.

In chapter 2 the time-average Poynting vector was presented

$$\left\langle \vec{P} \right\rangle = \frac{1}{2} \mathrm{Re}\left\{ \vec{E} \times \vec{H}^* \right\}$$

At points far from the antenna (the radiation zone)

$$\vec{E}(\vec{r}) \approx -j\omega \frac{\mu}{4\pi} \frac{e^{-jkr}}{r} \left[\hat{\theta} N_\theta + \hat{\phi} N_\phi \right]$$

$$\vec{H}(\vec{r}) \approx \frac{\hat{r} \times \vec{E}(\vec{r})}{\eta}$$

where

$$\vec{N}(\theta, \phi) = \int_v \vec{J}_S(\vec{r}') e^{jk(\hat{r}\cdot\vec{r}')} dv'$$

is known as the “radiation vector.” The radiation vector is related to the vector potential by

$$\vec{A}(r, \theta, \phi) = \frac{\mu}{4\pi} \frac{e^{-jkr}}{r} \vec{N}(\theta, \phi)$$

with

$$\vec{A}(\vec{r}) \approx \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\int_v \vec{J}_S(\vec{r}')e^{jk(\hat{r}\cdot\vec{r}')}dv'$$

Now in the radiation zone

$$\left\langle \vec{P} \right\rangle \approx \frac{1}{2}\mathrm{Re}\left\{\vec{E}\times\left(\frac{\hat{r}\times\vec{E}^*}{\eta}\right)\right\}$$

Using the vector identity $\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{A}\cdot\vec{B}\right)$

$$\left\langle \vec{P} \right\rangle \approx \frac{1}{2\eta}\mathrm{Re}\left\{\left(\vec{E}\cdot\vec{E}^*\right)\hat{r} - \left(\vec{E}\cdot\hat{r}\right)\vec{E}^*\right\}$$

$$\approx \hat{r}\frac{\vec{E}\cdot\vec{E}^*}{2\eta} \qquad \text{......because } \vec{E}\cdot\hat{r} = 0$$

$$\approx \hat{r}\frac{1}{2\eta}\left[\frac{\omega\mu}{4\pi r}\right]^2\left[N_\theta N_\theta^* + N_\phi N_\phi^*\right]$$

$$\text{......because } \left[\hat{\theta}N_\theta + \hat{\phi}N_\phi\right]\cdot\left[\hat{\theta}N_\theta^* + \hat{\phi}N_\phi^*\right] = \left[N_\theta N_\theta^* + N_\phi N_\phi^*\right]$$

Finally

$$\left\langle \vec{P} \right\rangle \approx \hat{r}\frac{1}{r^2}\frac{\eta}{8\lambda^2}\left[\left|N_\theta\right|^2 + \left|N_\phi\right|^2\right]$$

This represents the average power flow density and lies in the direction of wave propagation.

The power radiated through a sphere of infinite radius is given by

$$W = \lim_{r\to\infty}\oint_s \hat{n}\cdot\left\langle \vec{P} \right\rangle ds$$

Applying the expression for the time average Poynting vector leads to

$$W = \lim_{r\to\infty}\int_0^\pi\int_0^{2\pi}\hat{r}\cdot\left[\hat{r}\frac{1}{r^2}\frac{\eta}{8\lambda^2}\left(\left|N_\theta\right|^2 + \left|N_\phi\right|^2\right)\right]r^2\sin\theta d\theta d\phi$$

$$= \int_0^{\pi} \int_0^{2\pi} \frac{\eta}{8\lambda^2} \left(|N_\theta|^2 + |N_\phi|^2 \right) \sin\theta d\theta d\phi$$

Let $K = \dfrac{dW}{d\Omega} = \dfrac{\eta}{8\lambda^2} \left(|N_\theta|^2 + |N_\phi|^2 \right)$ ......"radiation intensity"

= power radiated per unit solid angle. ......where $d\Omega = \sin\theta d\theta d\phi$.

The total radiated power is then

$$W = \int_0^{\pi} \int_0^{2\pi} K(\theta,\phi) d\Omega.$$

6.3 Antenna Terminology

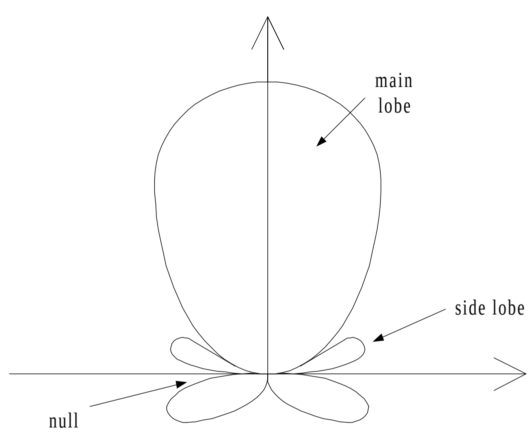
Antenna Patterns

Radiation pattern - A plot of the radiation characteristics of an antenna. There are two types of radiation patterns:

1. **Power pattern** - A plot of the radiated power at a constant radius.

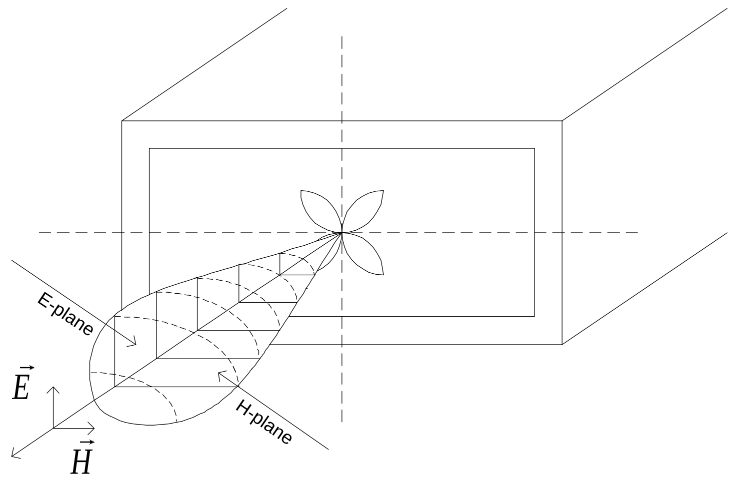
2. **Field pattern** - A plot of the electric or magnetic field magnitude at a constant radius.

An antenna pattern consists of a number of lobes. The largest lobe is usually called the main lobe, while the other smaller lobes are called side lobes. The minima between lobes are called **nulls**.



Radiation patterns are three-dimensional, but are usually measured and displayed as twodimensional patterns, which are sometimes called cuts. For most antennas, two cuts give a good representation of the three-dimensional pattern.

The radiation patterns of linearly polarized antennas are often specified in terms of **E-plane** and **H-plane** patterns. The E-plane contains the direction of maximum radiation and the electric field vector. The H-plane contains the direction of maximum radiation and the magnetic field vector.



No antenna has a truly isotropic pattern (one which is the same in all directions). Rather antennas (real ones anyway) tend to radiate more effectively in some directions rather than others.

Directive gain - The ratio of the radiation intensity K(θ,φ) to the uniform radiation intensity for an isotropic radiator with the same total radiation power W.

$$g_d(\theta,\phi) = \frac{K(\theta,\phi)}{\left(\frac{W}{4\pi}\right)} = \frac{4\pi}{W} K(\theta,\phi)$$

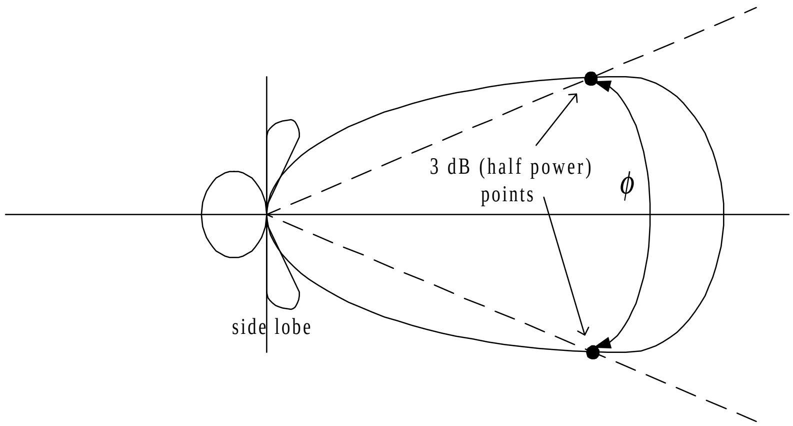
.....where $\left(\frac{4\pi}{W}\right)$ is the total power radiated by an isotropic radiator per unit solid angle.

Directivity - The maximum value of directive gain.

Gain - Directivity expressed in dB.

$$G = 10 \log_{10} (\text{directivity}) = \text{gain in dB}$$

Beamwidth - The beamwidth of a radiation pattern is the angle between the half-power points of the pattern.



Radiation efficiency - The radiation efficiency of an antenna is the ratio of the power radiated by the antenna to the total power supplied to the antenna. The total power supplied to the antenna consists of the power radiated and the power given up to resistive losses.

$$E = \frac{W}{W + W_L}$$

......where E = radiation efficiency
W = power radiated
W_L = power lost

Radiation resistance - The radiation resistance of an antenna is the equivalent resistance through which its input current must flow in order that the power dissipated in the resistance is equal to the total radiated power.

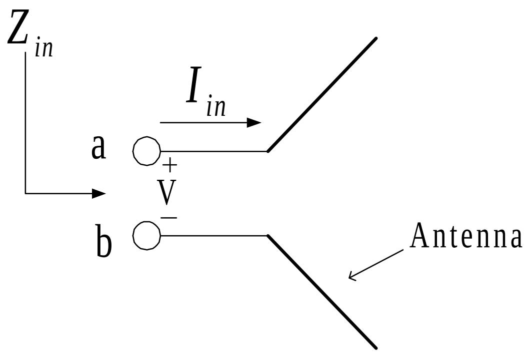
$$\frac{1}{2} I_0 I_0^* R_r = W$$

or $R_r = \dfrac{2W}{I_0 I_0^*}$ ......radiation resistance

......where I_0 is the input current to the antenna.

From the stand point of the source that drives an antenna, radiation resistance is indistinguishable from Ohmic resistance. In both cases, the source must continuously supply energy to the antenna in order to keep the current amplitude constant with time. In the case of Ohmic resistance, this resistance converts energy into propagating electromagnetic waves.

Input impedance - An arbitrary antenna with a pair of input terminals ‘a’ and ‘b’ is shown below.



When the antenna is not receiving power from waves generated by other sources the Thevenin equivalent circuit looking into the terminals of the antenna consists only of an impedance

$$Z_{in} = \frac{V}{I_{in}} = R_{in} + jX_{in}$$

where R_{in} is the input resistance and X_{in} is the input reactance. The input resistance is the sum of two components

$$R_{in} = R_{ri} + R_L$$

where R_{ri} is the input radiation resistance and R_L is the input loss resistance. R_L accounts for that portion of the input power that is dissipated as heat, while the input radiation resistance R_{ri} accounts for power that is radiated by the antenna. R_{ri} is related to R_r by

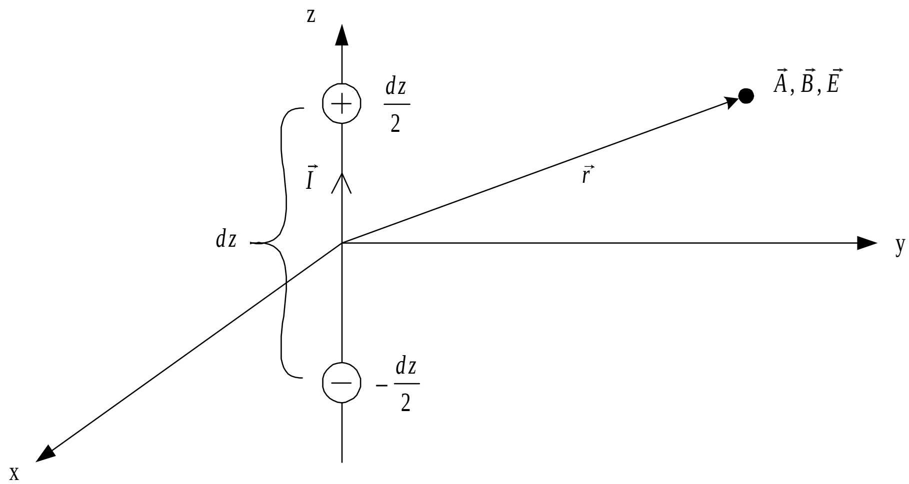
$$R_{ri} = \left[\frac{I_{max}}{I_0}\right]^2 R_r \ .$$

Radiation efficiency can be expressed

$$\eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{ri}}{R_{ri} + R_L} \ .$$

6.4 Hertzian Dipole

The simplest radiation source consists of a short segment of current



A Hertzian dipole consists of a uniform current I flowing in a short wire dz terminated by point charges.

Here

$$\vec{J}(\vec{r}) = \begin{cases} I\hat{z}\delta(x)\delta(y) \ldots\ldots \ for \ \frac{-dz}{2} \leq z \leq \frac{dz}{2} \\ 0 \qquad \ldots\ldots elsewhere \end{cases}$$

It is seen that

$$I = \int \vec{J} \cdot d\vec{s} = \int_x \int_y I\delta(x)\delta(y)dxdy = I$$

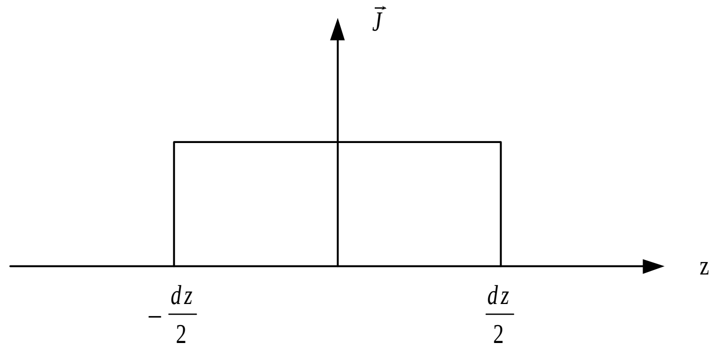
The charge associated with the current is found using the continuity equation

$$\nabla \cdot \vec{J} = -j\omega\rho \Rightarrow \rho = -\frac{1}{j\omega}\frac{dJ_z}{dz}$$

The current density may be expressed

$$\vec{J} = I\hat{z}\delta(x)\delta(y)\left[u\left(z + \frac{dz}{2}\right) - u\left(z - \frac{dz}{2}\right)\right]$$

where u(t) represents the unit step function.



$$\rho = \frac{1}{j\omega} I\delta(x)\delta(y)\frac{d}{dy}\left[u\left(z+\frac{dz}{2}\right)-u\left(z-\frac{dz}{2}\right)\right]$$

$$= \frac{I}{j\omega}\delta(x)\delta(y)\left[-\delta\left(z+\frac{dz}{2}\right)+\delta\left(z-\frac{dz}{2}\right)\right] \qquad \text{......because } \frac{du(t)}{dt}=\delta(t)$$

= (+) point charge at $z = \frac{dz}{2}$, (-) point charge at $z = -\frac{dz}{2}$

Vector potential

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_v \vec{J}(\vec{r}\,')\frac{e^{-jkR}}{R}dv'$$

where

$$R = |\vec{r}-\vec{r}\,'| \approx |\vec{r}| = r \quad \text{for } r >> dz$$

so,

$$\vec{A}(\vec{r}) \approx \frac{\mu_0}{4\pi}\frac{e^{-jkr}}{r}\int_v \vec{J}(\vec{r}\,')dv'$$

$$\approx \frac{\mu_0}{4\pi}\frac{e^{-jkr}}{r}\int_x\int_y \delta(x')\delta(y')dx'dy' \int_{z'=-\frac{dz}{2}}^{\frac{dz}{2}} I\hat{z}dz'$$

or

$$\vec{A}(\vec{r}) \approx \hat{z}\frac{\mu_0}{4\pi}Idz\frac{e^{-jkr}}{r}$$

now use $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$ to get

$$\vec{A}(\vec{r}) \approx \left(\hat{r}\cos\theta - \hat{\theta}\sin\theta\right)\frac{\mu_0}{4\pi} Idz \frac{e^{-jkr}}{r}$$

E-M fields

$$\vec{B} = \nabla \times \vec{A} = \frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial \theta}\right]$$

$$= \frac{\mu_0}{4\pi} Idz \frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}\left(-\sin\theta e^{-jkr}\right) - \frac{\partial}{\partial \theta}\left(\cos\theta \frac{e^{-jkr}}{r}\right)\right]$$

$$= \frac{\mu_0}{4\pi} Idz \frac{\hat{\phi}}{r}\left[jk\sin\theta + \frac{\sin\theta}{r}\right]e^{-jkr}$$

so

$$\vec{B} = \hat{\phi}\frac{\mu_0}{4\pi} Idz \frac{1+jkr}{r^2}\sin\theta e^{-jkr}$$

$$\vec{E} = -\frac{j}{\omega\mu_0\varepsilon_0}\nabla \times \vec{B} \quad \text{at all points where } \vec{J} = 0$$

$$= -\frac{j}{\omega\mu_0\varepsilon_0}\left[\frac{\hat{r}}{r\sin\theta}\frac{\partial}{\partial\theta}\left(B_\phi \sin\theta\right) - \frac{\hat{\theta}}{r}\frac{\partial}{\partial r}\left(rB_\phi\right)\right]$$

$$= -\frac{j}{\omega\varepsilon_0}\frac{Idz}{4\pi}\left[\frac{\hat{r}}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{1+jkr}{r^2}\sin^2\theta e^{-jkr}\right) - \frac{\hat{\theta}}{r}\frac{\partial}{\partial r}\left(\frac{1+jkr}{r}\sin\theta e^{-jkr}\right)\right]$$

$$\vec{E} = -\frac{j}{\omega\varepsilon_0}\frac{Idz}{4\pi}\left[\frac{\hat{r}}{r\sin\theta}\frac{1+jkr}{r^2}2\sin\theta\cos\theta e^{-jkr}\right.$$
$$\left. - \frac{\hat{\theta}}{r}\sin\theta\left(-j\frac{k}{r}\left[1+jkr\right] + \frac{jkr-1-jkr}{r^2}\right)e^{-jkr}\right]$$

$$= -\frac{j}{\omega\varepsilon_0}\frac{Idz}{4\pi}\left[\hat{r}\frac{1+jkr}{r^3}2\cos\theta + \hat{\theta}\sin\theta\left(\frac{1+jkr-k^2r^2}{r^3}\right)\right]e^{-jkr}$$

now use

$$\frac{1}{\omega\varepsilon_0} = \frac{1}{\omega\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{\eta_0}{k} \quad \text{.....where } \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Then

$$\vec{E} = \hat{r}\frac{Idz}{4\pi}\frac{e^{-jkr}}{r}\left[\frac{2\eta_0}{r} + \frac{2}{j\omega\varepsilon_0 r^2}\right]\cos\theta + \hat{\theta}\frac{Idz}{4\pi}\frac{e^{-jkr}}{r}\left[j\omega\mu_0 + \frac{\eta_0}{r} + \frac{1}{j\omega\varepsilon_0 r^2}\right]\sin\theta\,.$$

Thus the fields may be expressed:

$$\vec{B} = \hat{\phi}\frac{\mu_0 Idz}{4\pi}\sin\theta\left[\frac{jk}{r} + \frac{1}{r^2}\right]e^{-jkr}$$

$$E_r = \frac{Idz}{4\pi}2\eta_0\cos\theta\left[\frac{1}{r^2} - \frac{j}{kr^3}\right]e^{-jkr}$$

$$E_\theta = \frac{Idz}{4\pi}\eta_0\sin\theta\left[\frac{jk}{r} + \frac{1}{r^2} - \frac{j}{kr^3}\right]e^{-jkr}$$

It is seen that these expressions contain terms having three different rates of decay: 1/r, $1/r^2$, and $1/r^3$.

Near-zone fields (induction zone)

The near zone fields are those which are strongest near $r = 0$, or when $r << \frac{\lambda}{2\pi}$. Thus these are the terms of $\vec{B}$ which vary as $1/r^2$ and the terms of $\vec{E}$ which vary as $1/r^3$:

$$\vec{E} \approx -j\frac{Idz}{4\pi}\frac{e^{-jkr}}{r^3}\frac{\eta_0}{k}\left[2\hat{r}\cos\theta + \hat{\theta}\sin\theta\right]$$

$$\vec{B} \approx \hat{\phi}\frac{\mu_0}{4\pi}Idz\sin\theta\frac{e^{-jkr}}{r^2}$$

The near zone $\vec{E}$ -field looks like the field of an electrostatic dipole.

The near zone fields do not contribute to radiated power. Instead they result in reactive power (time changing energy stored in the fields near the antenna). Only the 1/r terms contribute to radiated power.

Far-zone fields (radiation zone fields)

The far-zone fields are those which are strongest as $r \to \infty$. Thus, these are the terms of $\vec{E}$ and $\vec{B}$ which vary as 1/r:

$$\vec{B} \approx \hat{\phi}\,\frac{\mu_0 I dz}{4\pi}\, jk \sin\theta\, \frac{e^{-jkr}}{r}$$

$$\vec{E} \approx \hat{\theta}\,\frac{jkIdz}{4\pi}\eta_0 \sin\theta\, \frac{e^{-jkr}}{r}$$

The radiation zone fields are those that contribute to radiated power. Note that fields form a transverse wave.

Radiated Power

$$W = \lim_{r\to\infty} \oint_s \hat{n}\cdot\left\langle \vec{P}\right\rangle ds$$

$$\left\langle \vec{P}\right\rangle = \frac{1}{2}\mathrm{Re}\left\{\vec{E}\times\vec{H}^*\right\} \quad \text{where } \hat{n} = \hat{r} \text{ and } ds = r^2 d\Omega$$

$\vec{B}$ has terms that go as 1/r and $1/r^2$ and $\vec{E}$ has terms that go as 1/r, $1/r^2$ and $1/r^3$.

Since $ds \approx r^2$ as $r \to \infty$ only the 1/r terms contribute to W.

$$W = \lim_{r\to\infty} \oint \frac{1}{2}\mathrm{Re}\left\{\vec{E}^r \times \vec{H}^{r*}\right\}\cdot \hat{r} r^2 d\Omega$$

where $\vec{E}^r$ and $\vec{H}^r$ are the 1/r terms from $\vec{E}$ and $\vec{H}$ called the "radiation zone" fields.

$$W = \int_0^{\pi}\int_0^{2\pi} \frac{1}{2} E_\theta^r H_\phi^{r*} r^2 d\Omega$$

$$= \frac{1}{2}\left[\frac{Idz}{4\pi}\right]^2 \int_0^{\pi}\int_0^{2\pi}\left[\frac{\omega\mu_0}{r}\sin\theta\right]\left[\frac{k}{r}\sin\theta\right] r^2 \sin\theta d\theta d\phi$$

$$= \frac{1}{2}\left[\frac{Idz}{4\pi}\right]^2 \omega\mu_0 k \underbrace{\int_0^{2\pi} d\phi}_{=2\pi} \underbrace{\int_0^{\pi} \sin^3\theta d\theta}_{=4/3}$$

$$= \frac{1}{2}\frac{I^2 dz^2}{16\pi^2}\omega\mu_0 k\left(\frac{8\pi}{3}\right) \quad \text{use: } \omega\mu_0 = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\frac{\mu_0}{\varepsilon_0}} = k\eta_0$$

$$= \frac{1}{12}\frac{I^2 dz^2}{\pi}k^2\eta_0 \quad \text{use: } k = \frac{2\pi}{\lambda}$$

$$= \eta_0 \frac{\pi}{3} I^2 \left(\frac{dz}{\lambda}\right)^2 \text{ Watts}$$

$$W = 40\pi^2 I^2 \left(\frac{dz^2}{\lambda^2}\right)$$

Radiation resistance

$$\frac{1}{2} II^* R_r = W = \eta_0 \frac{\pi}{3} I^2 \left(\frac{dz}{\lambda}\right)^2$$

$$\text{or } R_r = \eta_0 \frac{2\pi}{3}\left(\frac{dz}{\lambda}\right)^2 = 80\pi^2\left(\frac{dz}{\lambda}\right)^2$$

Example: Calculate the radiation resistance of a 1 cm length of uniform current if the frequency is 900 MHz and the host medium is air:

$$\lambda = \frac{c}{f} = \frac{3\times 10^8 \, m/s}{900\times 10^6 \, Hz} = 33.3cm$$

$$R_r = 80\pi^2\left[\frac{1\times 10^{-2} m}{33.3\times 10^{-2} m}\right]^2 = 0.711\Omega$$

It is seen that the radiation resistance of a short current segment is only on the order of a fraction of an Ohm, making it a relatively inefficient radiator.

Directive gain

$$g_d(\theta) = \frac{K(\theta)}{W/4\pi} \qquad\qquad W = \int_0^{\pi}\int_0^{2\pi} K(\theta)\,d\Omega$$

$$g_d(\theta) = \frac{\frac{1}{2}\left[\frac{Idz}{4\pi}\right]^2 \left[\omega\mu_0 \sin\theta\right]\left[k\sin\theta\right]}{\frac{1}{4\pi}\left\{\frac{1}{2}\left[\frac{Idz}{4\pi}\right]^2 \omega\mu_0 k(2\pi)\left(\frac{4}{3}\right)\right\}} = \frac{4\pi}{(2\pi)\left(\frac{4}{3}\right)}\sin^2\theta$$

$$g_d(\theta) = \frac{3}{2}\sin^2\theta$$

Directivity

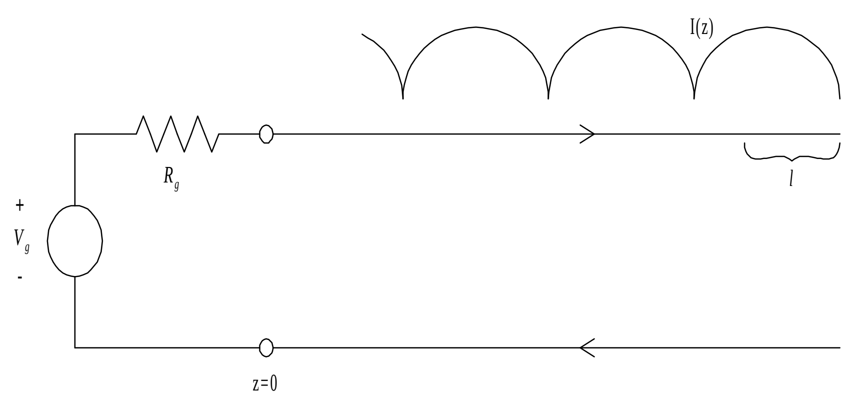
$$D = \max\left[g_d(\theta)\right] = g_d\left(\frac{\pi}{2}\right) = 1.5$$

$$G = gain = 10\log_{10}(1.5) = 1.76dB$$

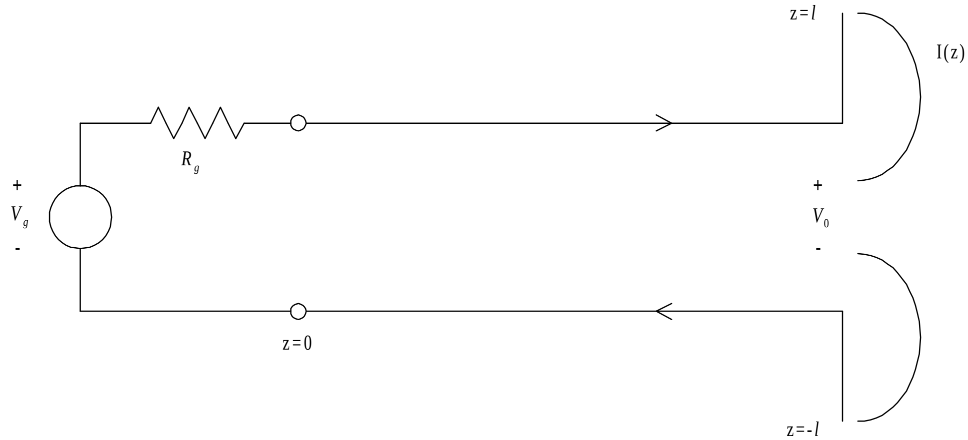
6.5 Radiation from a cylindrical dipole

The cylindrical dipole antenna is one of the most commonly used antenna in the VHF/UHF frequency range.

The cylindrical dipole may be viewed as an open-ended transmission line which has been flared out.

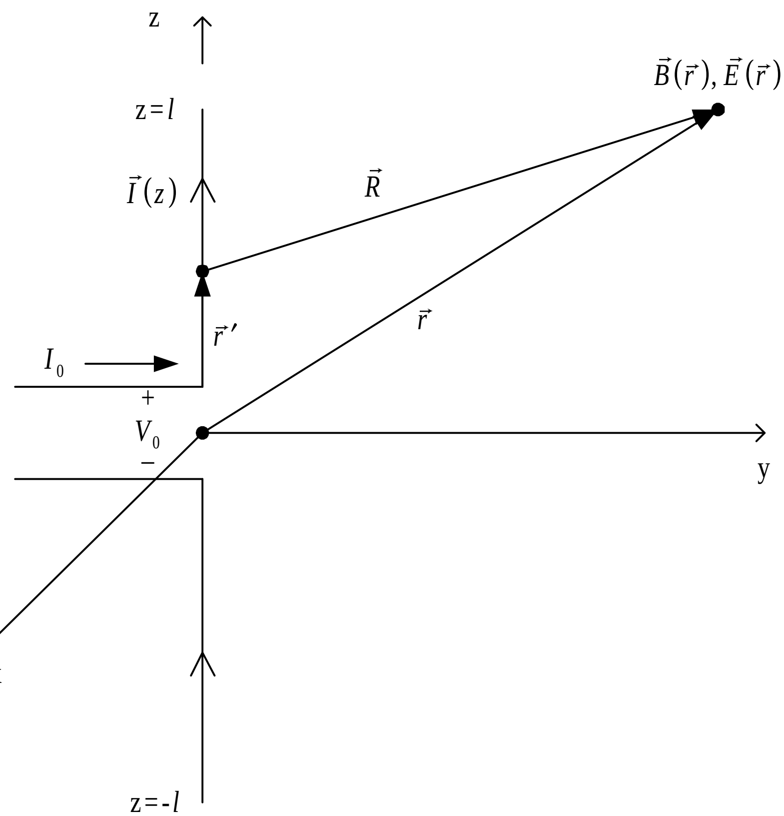


Open circuited transmission line



Dipole antenna

Far zone fields



It is noted that $I(z=\pm l)=0$ (current at the tips of the antenna is zero).

The current density present on the dipole is given by

$$\vec{J}(\vec{r}) = \begin{Bmatrix} \hat{z} I(z)\delta(x)\delta(y) \\ 0 \end{Bmatrix} \qquad \begin{matrix} \ldots for\ |z| \le l \\ \ldots elsewhere \end{matrix}$$

$$\text{....where } I(z) = I_0 \frac{\sin k(l - |z|)}{\sin kl}$$

The radiation vector is

$$\vec{N}(\theta,\phi) = \int_v \vec{J}(\vec{r}\,')e^{jk(\hat{r}\cdot\vec{r}\,')}dv'$$

where $\vec{r}\,' = z'\hat{z}$ $\qquad \hat{r}\cdot\vec{r}\,' = z'(\hat{z}\cdot\hat{r}) = z'\cos\theta$

$$\vec{N}(\theta,\phi) = \frac{I_0\hat{z}}{\sin kl}\int_x\int_y \delta(x')\delta(y')dx'dy' \int_{z'=-l}^{l} \sin k(l-|z'|)e^{jkz'\cos\theta}dz'$$

$$= \frac{I_0\hat{z}}{\sin kl}\int_{z'=-l}^{l} \sin k(l-|z'|)e^{jkz'\cos\theta}dz'$$

$$= \frac{I_0\hat{z}}{\sin kl}\int_0^{l} \sin k(l-z')e^{jkz'\cos\theta}dz' + \frac{I_0\hat{z}}{\sin kl}\int_{-l}^{0} \sin k(l+z')e^{jkz'\cos\theta}dz'$$

Using the relationship

$$\int e^{ax}\sin(bx+c)dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c) - b\cos(bx+c)\right]$$

gives

$$\vec{N}(\theta,\phi) = \hat{z}\frac{zI_0}{k\sin kl}\frac{\cos(kl\cos\theta) - \cos kl}{\sin^2\theta}$$

Let

$$F_0(\theta,kl) = \frac{\cos(kl\cos\theta) - \cos kl}{\sin\theta} \qquad \text{“radiation function”}$$

then

$$\vec{N}(\theta,\phi) = \frac{2I_0}{k\sin kl}\frac{F_0(\theta,kl)}{\sin\theta}\underbrace{\left(\hat{r}\cos\theta - \hat{\theta}\sin\theta\right)}_{\hat{z}}$$

$$\vec{E}(\vec{r}) = -j\omega \frac{\mu_0}{4\pi} \frac{e^{-jkr}}{r} \left[\hat{\theta} N_\theta + \hat{\phi} N_\phi \right]$$

$$= \frac{j\omega\mu_0 I}{2\pi k} \frac{e^{-jkr}}{r} F(\theta, kl)\hat{\theta} \quad \text{......far zone } \vec{E} \text{ -field}$$

$$\text{where } F(\theta, kl) = \frac{\cos(kl\cos\theta) - \cos(kl)}{\sin(kl)\sin\theta}$$

$$\vec{H}(\vec{r}) = \frac{\hat{r} \times \vec{E}(\vec{r})}{\eta} = \frac{j\omega\mu_0 I_0}{2\pi k\eta} \frac{e^{-jkr}}{r} F(\theta, kl)\hat{\phi}$$

Use $\omega\mu_0 = k\eta_0$ then

$$\vec{E}(\vec{r}) = \hat{\theta} \frac{j\eta_0 I_0}{2\pi} \frac{e^{-jkr}}{r} F(\theta, kl) \qquad \text{Radiation zone fields}$$

$$\vec{H}(\vec{r}) = \hat{\phi} \frac{jI_0}{2\pi} \frac{e^{-jkr}}{r} F(\theta, kl) \qquad \text{produced by a dipole}$$

Radiation pattern

$$K(\theta, \phi) = \frac{\eta_0}{8\lambda^2} \left(|N_\theta|^2 + |N_\phi|^2 \right)$$

so

$$K(\theta) = \frac{\eta_0}{8\lambda^2} |N_\theta|^2 = \frac{\eta_0}{8\lambda^2} \frac{4|I_0|^2}{k^2} F^2(\theta, kl)$$

$$= \eta_0 \frac{|I_0|^2}{8\pi^2} F^2(\theta, kl)$$

Often times $\sqrt{K(\theta, \phi)}$ is plotted as opposed to $K(\theta, \phi)$ since $\sqrt{K}$ describes the pattern of the far-zone field:

$$\sqrt{K(\theta)} \approx F(\theta, kl) = \frac{\cos(kl\cos\theta) - \cos kl}{\sin\theta}$$

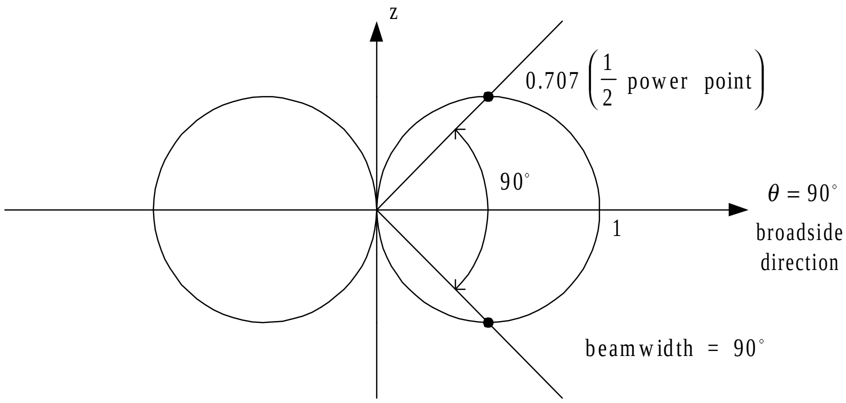
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

Special case: $kl << 1 \rightarrow l << \lambda$

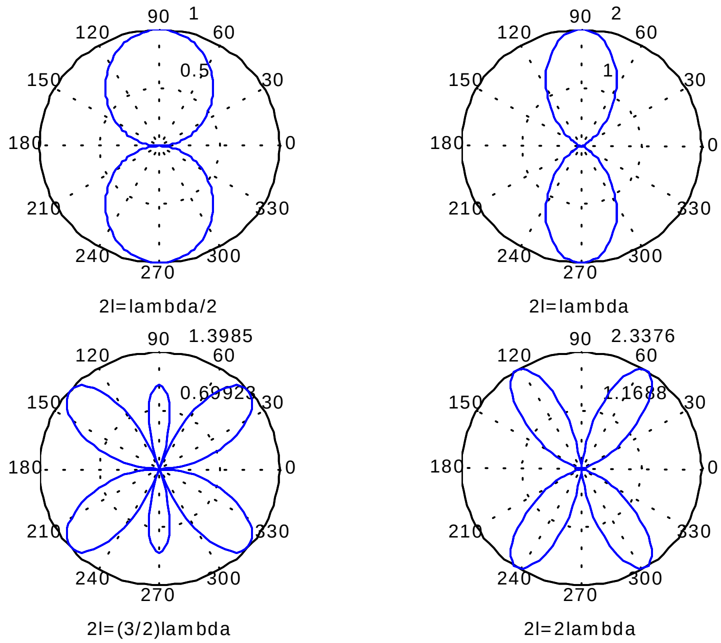
$$F(\theta, kl << 1) \approx \frac{1 - \frac{1}{2}(kl\cos\theta)^2 - 1 + \frac{1}{2}(kl)^2}{\sin\theta}$$

$$\approx \frac{\frac{1}{2}(kl)^2\left[1 - \cos^2\theta\right]}{\sin\theta}$$

$$\approx \frac{1}{2}(kl)^2 \sin\theta \rightarrow$$ Same pattern as Hertzian dipole.



Other cases:



Most often the length $l=\lambda/4$ (half-wave dipole) is used, since if it is a nearly resonant structure with its current maximum at the driving point (z=0). For a dipole with a non-zero wire radius, the length must be slightly shorter than $\lambda/4$ to produce resonance.

Note if we define the input impedance Z_{in} as

$$Z_{in} = \frac{V_0}{I_0} \qquad \frac{input\ voltage}{input\ current}$$

then resonance occurs when Z_0 is purely real (just as in circuit theory).

Radiated power

$$W = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} K(\theta,\phi) d\Omega = \eta_0 \frac{|I_0|^2}{8\pi^2} \int_0^{2\pi} d\phi \int_0^{\pi} F^2(\theta,kl) \sin\theta d\theta$$

$$= \eta_0 \frac{|I_0|^2}{4\pi} \int_0^{\pi} F^2(\theta,kl) \sin\theta d\theta$$

This expression must be integrated numerically.

Special case #1: $kl<<1$ (Short dipole)

$$F^2(\theta,kl) \approx \left[\frac{1}{2}(kl)\sin\theta\right]^2$$

$$W \approx \eta_0 \frac{|I_0|^2}{4\pi}\frac{1}{4}(kl)^2 \underbrace{\int_0^{\pi} \sin^3 \theta d\theta}_{4/3} \qquad \eta_0 \approx 120\pi\Omega$$

$$W \approx (120\pi)\frac{|I_0|^2}{4\pi}\frac{1}{4}\left(\frac{2l\pi}{\lambda}\right)^2 \frac{4}{3}$$

$$\approx 40\pi^2 |I_0|^2 \left(\frac{l}{\lambda}\right)^2 \text{ Watts}$$

Special case #2: $\left(kl = \frac{\pi}{2}\right)$ (Half-wave dipole)

$$F\left(\theta, \frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right) - \cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)\sin\theta} = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$W = \eta_0 \frac{|I_0|^2}{4\pi} \underbrace{\int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta}_{1.22 \text{ by numerical integration}}$$

thus

$$W = 36.6|I_0|^2$$

Directivity

$$D = \frac{\max\{K(\theta)\}}{W/4\pi}$$

Special case #1: *kl*<<1 (Short dipole)

$$K(\theta) = \eta_0 \frac{|I_0|^2}{8\pi^2} F^2(\theta, kl) \approx \eta_0 \frac{|I_0|^2}{8\pi^2}\left[\frac{1}{2}(kl)\sin\theta\right]^2$$

$$\Rightarrow K(\theta)_{max} = K\left(\theta = \frac{\pi}{2}\right) = \eta_0 \frac{|I_0|^2}{8\pi^2} \frac{1}{4}(kl)^2$$

so

$$D = \frac{\eta_0 \frac{|I_0|^2}{8\pi^2} \frac{1}{4}(kl)^2}{\frac{1}{4\pi}\left(\eta_0 \frac{|I_0|^2}{4\pi} \frac{1}{4}(kl)^2 \frac{4}{3}\right)} = \frac{3}{2}$$

$$G = 10\log_{10}(D) = 10\log_{10}\left(3/2\right) = 1.76dB$$

Special case #2: $kl = \frac{\pi}{2}$ (half-wave dipole)

$$K(\theta)_{max} = K\left(\theta = \frac{\pi}{2}\right) = \eta_0 \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos\left(\frac{\pi}{2}\cos\left(\frac{\pi}{2}\right)\right)}{\sin\left(\frac{\pi}{2}\right)}\right]^2$$

$$= \eta_0 \frac{|I_0|^2}{8\pi^2}$$

$$D = \frac{\eta_0 \frac{|I_0|^2}{8\pi^2}}{\frac{1}{4\pi}\left(\eta_0 \frac{|I_0|^2}{4\pi}(1.22)\right)} = \frac{2}{1.22} = 1.64$$ more directive than short dipole

$$G = 10\log_{10}(1.64) = 2.15dB$$

Radiation resistance

$$R_r = \frac{2W}{I_0 I_0^*} = \frac{2W}{|I_0|^2}$$

$$= 60\int_0^{\pi} F^2(\theta, kl)\sin\theta d\theta$$

Special case #1: $kl \ll 1$ (short dipole)

$$R_r = 60\int_0^{\pi}\left[\frac{1}{2}(kl)\sin\theta\right]^2 \sin\theta d\theta$$

$$= 80\pi^2\left(\frac{l}{\lambda}\right)^2 \Omega$$

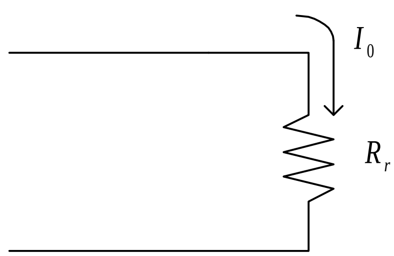
Special case #2: $kl = \pi/2$ (half-wave dipole)

$$R_r = \frac{2\left(36.6|I_0|^2\right)}{|I_0|^2} = 73.2\Omega$$

Note that a resonant half-wave dipole ($Z_{in} = R_{in} + jX_{in} = 73.2 + j0$) is matched quite well by a 75Ω coaxial cable transmission line.

Example

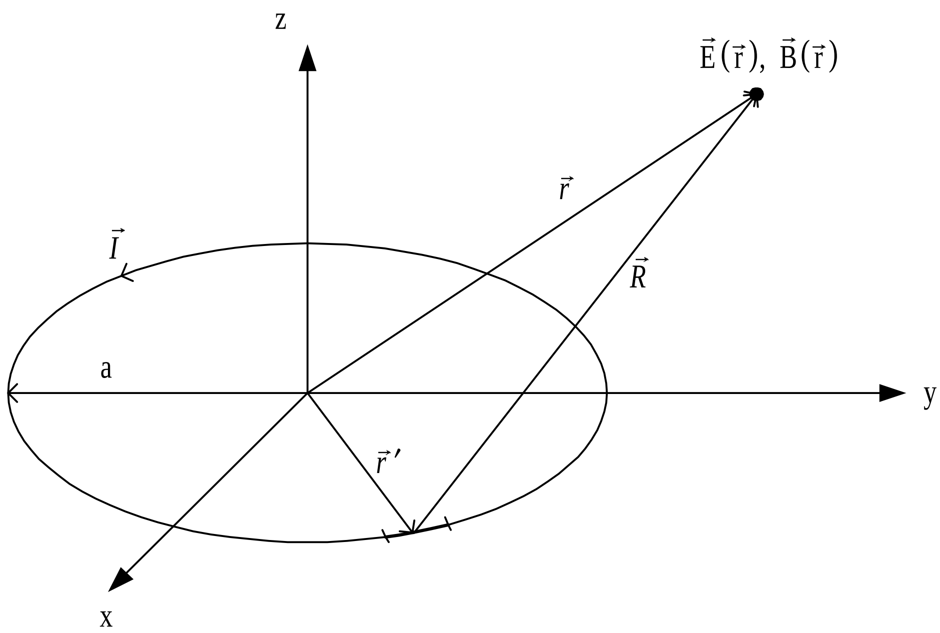
What input currents are needed to a short dipole of length $\left(\frac{l}{\lambda}\right) = \frac{1}{10}$ and to a half-wave dipole to radiate a power of 1kW?



short dipole: $R_r = 80\pi^2\left(\frac{1}{10}\right)^2 = 7.9\Omega \rightarrow I_0 = 15.9\,A$

half-wave dipole: $R_r = 73.2\Omega \rightarrow I_0 = 5.23\,A$

6.6 Radiation from a small loop antenna



The fields associated with a small loop antenna can be shown to be

$$\vec{E}(\vec{r}) = \hat{\phi}\,\frac{\eta_0 I (ka)^2}{4}\,\frac{e^{-jkr}}{r}\sin\theta$$

$$\vec{H}(\vec{r}) = \frac{\hat{r}\times\vec{E}(\vec{r})}{\eta} = -\hat{\theta}\,\frac{I(ka)^2}{4}\,\frac{e^{-jkr}}{r}\sin\theta$$

Radiated Power

$$K(\theta,\phi) = K(\theta) = \frac{\eta_0}{8\lambda^2}\left[\left|N_\theta\right|^2 + \left|N_\phi\right|^2\right] = \frac{\eta_0}{8\lambda^2}\left|N_\phi\right|^2$$

$$= \frac{\eta_0}{8\lambda^2} I^2 k^2 \left(\pi a^2\right)^2 \sin^2\theta$$

using $\eta_0 = 120\pi \quad \lambda = \dfrac{2\pi}{k}$

$$K(\theta,\phi) = \frac{15\pi}{4} I^2 (ka)^4 \sin^2\theta$$

Note that the radiation pattern of the small loop is identical to the radiation pattern of the Hertzian dipole. The small loop is the magnetic dipole analog of the electric (Hertzian) dipole

$$W = \int\int K(\theta) d\Omega = \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \frac{15\pi}{4} I^2 (ka)^4 \sin^3 \theta d\theta$$

$$= \frac{15\pi^2}{2} I^2 (ka)^4 \underbrace{\int_{\theta=0}^{\pi} \sin^3 \theta d\theta}_{4/3}$$

$$= 10\pi^2 I^2 (ka)^4 Watts$$

Radiation resistance

$$R_r = \frac{2W}{I^2} = 20\pi^2 (ka)^4 \Omega$$

example: $a/\lambda = 0.5 \rightarrow R_r = 20\pi^2 [2\pi(0.05)]^4 = 1.92\Omega$

Directivity

Since the radiation pattern of the small loop is the same as that of the Hertzian dipole, the directive gain, the directivity, and the gain are the same:

D = 3/2 G = 1.76dB

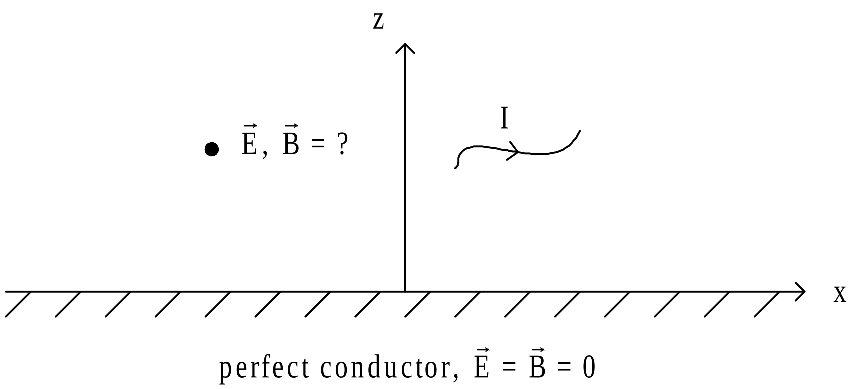
6.7 Currents above a perfectly conducting ground

Antennas are often placed above conducting surfaces, for purposes of measurements in the lab, or in practical situations as when an antenna is placed on a car roof. Often the earth itself is modeled as a perfect conductor (although this not always a good approximation because the conductivity of the earth is fairly low).

Fields produced by currents by antennas above a ground plane can be calculated using the method of images.

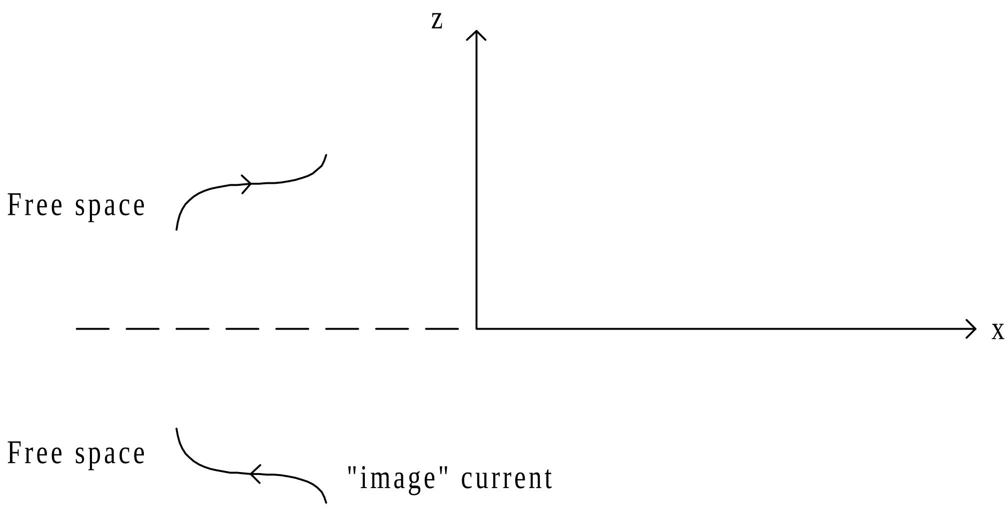
Method of images

Consider a current carrying element above a perfectly conducting plane:



The current element will produce an $\vec{E}$ -field which will induce currents to flow on the surface of the conductor. These currents will produce an additional "scattered field". The total field must obey the boundary condition $E_{tangential} = 0$ on the conductor surface at z = 0.

We may replace the problem by an equivalent problem. The ground plane is removed and replaced by an "image" current.



Here the fields produced by both the current and its image will be identical to the fields produced by the current above the ground plane as long as the boundary condition on the total field $E_{tan} = 0$ at z = 0 is obeyed.

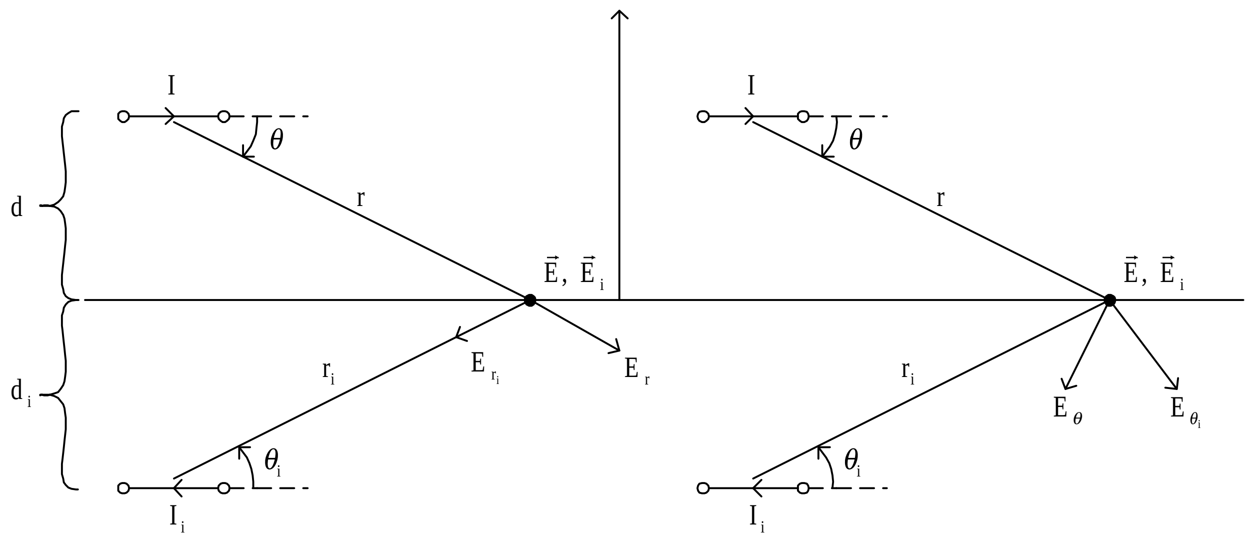
Question: What image current will result in the boundary condition being satisfied?

Since any current distribution can be viewed as a superposition of Hertzian dipoles, we only need to identify the image of a Hertzian dipole.

The field due to a Hertzian dipole on the z-axis is given by

$$\vec{E} = \hat{r}\frac{Idz}{4\pi}\frac{e^{-jkr}}{r}\left[\frac{2\eta_0}{r} + \frac{2}{j\omega\varepsilon_0 r^2}\right]\cos\theta + \hat{\theta}\frac{Idz}{4\pi}\frac{e^{-jkr}}{r}\left[j\omega\mu_0 + \frac{\eta_0}{r} + \frac{1}{j\omega\varepsilon_0 r^2}\right]\sin\theta$$

Case I: Horizontal dipole



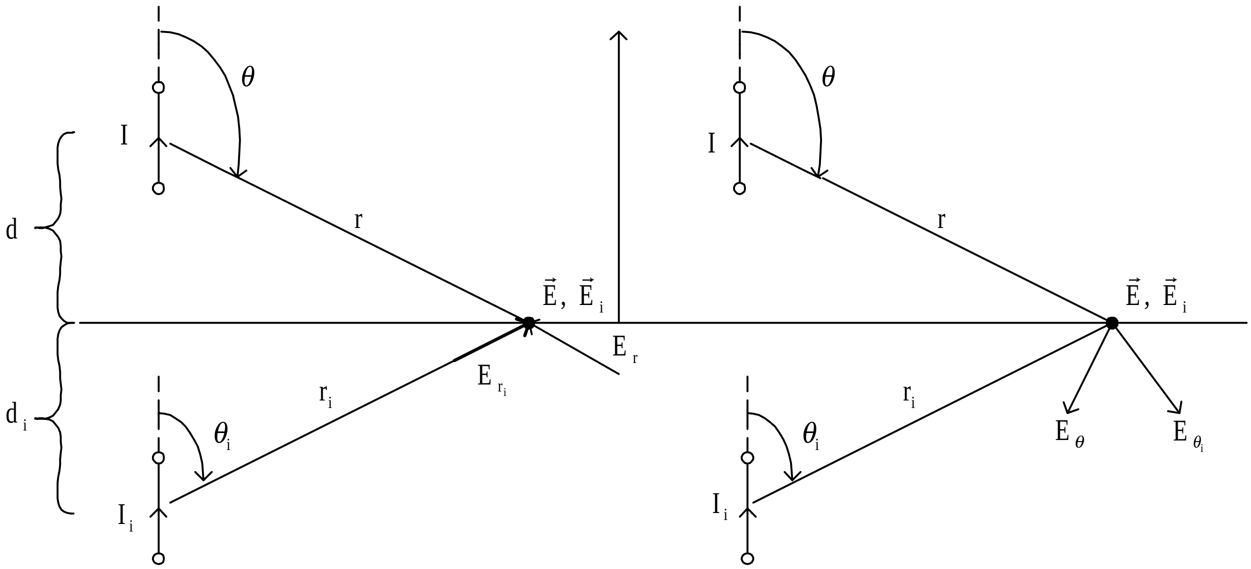
choose: $d_i = d$, $|I_i| = |I|$, $\vec{I}_i = -\vec{I}$

thus: $r_i = r$, $\theta_i = \theta$

and: $(E_{ri})_{tan} = -(E_r)_{tan}$, $(E_{\theta i})_{tan} = -(E_\theta)_{tan}$

So, $(\vec{E} + \vec{E}_i)_{tan} = 0 \rightarrow$ then boundary condition is satisfied.

Case II: Vertical dipole



choose: $d_i = d$, $|I_i| = |I|$, $\vec{I}_i = \vec{I}$

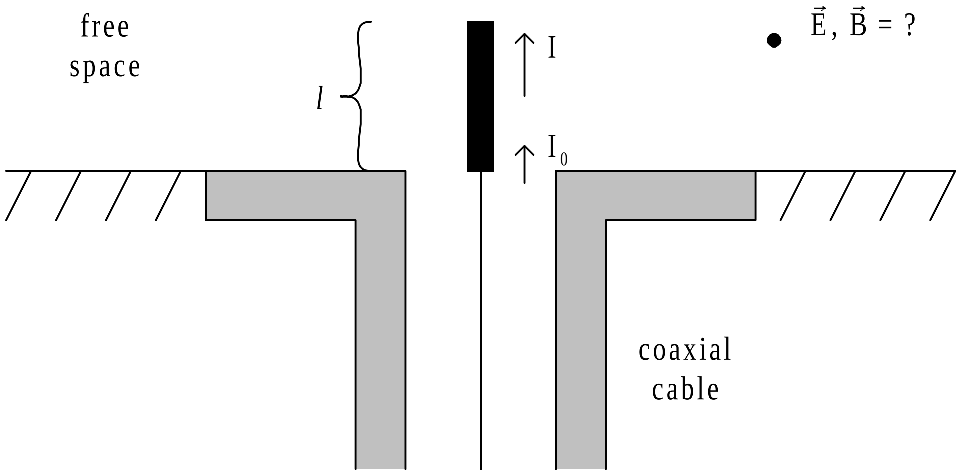
thus: $r_i = r$, $\theta_i = \pi - \theta \rightarrow \cos\theta_i = -\cos\theta$

and: $(E_{ri})_{\tan} = -(E_r)_{\tan}$, $(E_{\theta i})_{\tan} = -(E_\theta)_{\tan}$

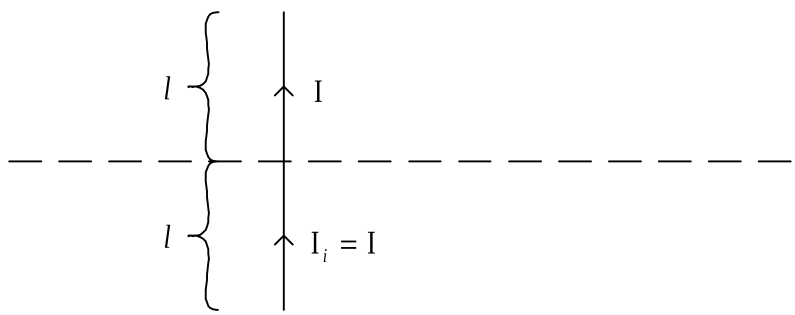
So, $(\vec{E} + \vec{E}_i)_{\tan} = 0 \rightarrow$ then boundary condition is satisfied.

Summary:
1. Horizontal currents image in opposite directions.
2. Vertical currents image in the same direction.

6.8 Monopole above a ground plane



The equivalent problem is a dipole.



Thus the fields radiated by a monopole above a ground plane are identical to those of a dipole in free space. However, the radiated power, and thus the radiation resistance are half that of the dipole, since the monopole only radiates into half the space that the dipole does.

$$W_{monopole} = \frac{1}{2} W_{dipole} = \eta_0 \frac{\left|I_0\right|^2}{4\pi} \int_0^{\pi/2} F^2(\theta, kl) \sin\theta d\theta$$

$$R_{r_{monoploe}} = \frac{1}{2} R_{r_{dipole}} = 60 \int_0^{\pi/2} F^2(\theta, kl) \sin\theta d\theta$$

6.9 Broadband antennas

Although dipole antennas possess many attractive characteristics for measurement of radiated emissions, they are not ideal for gathering data over a wide range of frequencies. The radiated emissions range typically extends from 30 MHz to 16 GHz, and the length of a dipole must be physically adjusted to provide a length of ½ λ at each measurement frequency.

A more practical technique is to employ broadband measurement antennas. A broadband antenna has the following characteristics:

1. The input/output impedance is fairly constant over the frequency band.
2. The antenna pattern is fairly constant over the frequency band.

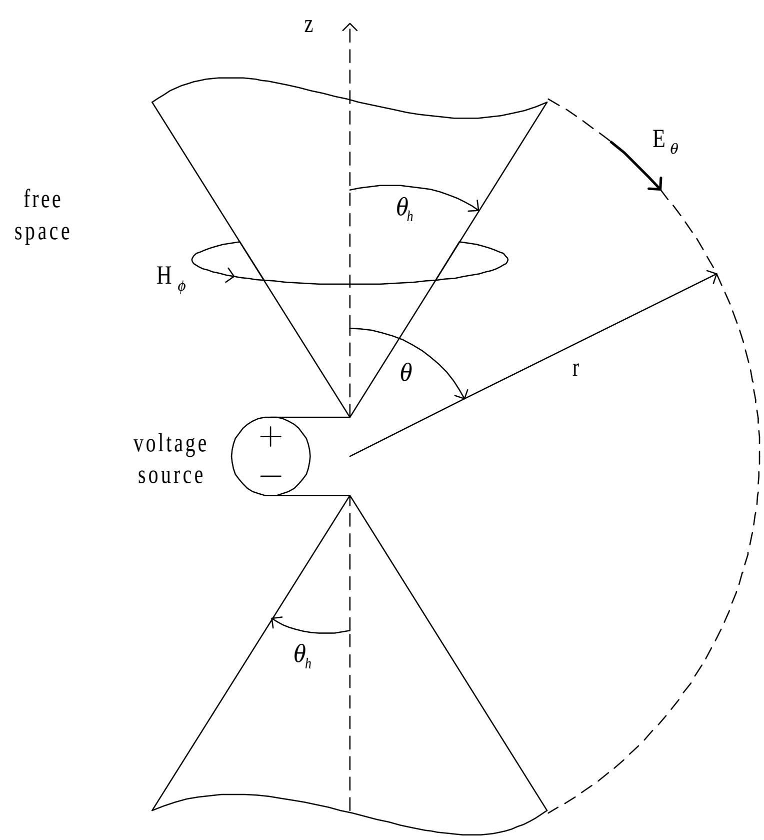
Two types of broadband antennas will be examined: The biconical antenna, and the log-periodic antenna.

The biconical antenna is typically used in the frequency range of 30 MHz to 200 MHz.

The log-periodic antenna is typically used in the band from 200 MHz to 1 GHz.

Biconical Antennas

An infinite biconical antenna consists of two cones of half angle θ_h with a small gap at the feed point.



In the space surrounding the cones $\vec{J} = 0$. Symmetry suggests that the fields are $\vec{H} = \hat{\phi} H_\phi$ and $\vec{E} = \hat{\theta} E_\theta$. Maxwell's equations can be solved to give the form of the field as

$$H_\phi = \frac{H_0}{\sin\theta} \frac{e^{-jk_0 r}}{r}$$

and

$$E_\theta = \frac{k_0}{\omega\varepsilon_0} \frac{H_0}{\sin\theta} \frac{e^{-jk_0 r}}{r} = \eta_0 H_\phi$$

Where H_0 is constant.

It is noted that these fields form transverse electromagnetic (TEM) waves (the electric and magnetic fields are orthogonal and transverse to the direction of propagation). Therefore, a unique voltage between two points on the cones may be defined.

The voltage produced between two points on opposite cones that are both a distance 'r' from the feed point is

$$V(r) = -\int_{\theta=\pi-\theta_h}^{\theta_h} \vec{E} \cdot d\vec{l}$$

$$= 2\eta_0 H e^{-jk_0 r} \ln\left(\cot\frac{1}{2}\theta_h\right)$$

The current on the surface of the cones is given by

$$I(r) = \int_{\phi=0}^{2\pi} H_\phi r \sin\theta d\phi$$

$$= 2\pi H_0 e^{-jk_0 r}$$

and the input impedance is then

$$Z_{in} = \left.\frac{V(r)}{I(r)}\right|_{r=0} = \frac{\eta_0}{\pi} \ln\left(\cot\frac{1}{2}\theta_h\right)$$

$$= 120 \ln\left(\cot\frac{1}{2}\theta_h\right)$$

which is purely resistive.

Usually, the cone half-angle is chosen to provide a match to the feed line characteristic impedance.

The total time average radiated power is given by

$$W = \oint_s \langle \overline{P} \rangle \cdot d\overline{s}$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=\theta_h}^{\pi-\theta_h} \frac{|E_\theta|^2}{2\eta_0} r^2 \sin\theta d\theta d\phi$$

$$= \pi\eta_0 H_0{}^2 \int_{\theta=0}^{\theta_h} \frac{d\theta}{\sin\theta}$$

$$= 2\pi\eta_0 |H_0|^2 \ln\left(\cot\frac{1}{2}\theta_n\right)$$

Radiation resistance is given by

$$R_r = \frac{2W}{|I(0)|^2} = \frac{4\pi\eta_0 |H_0|^2 \ln\left(\cot\frac{1}{2}\theta_h\right)}{4\pi^2 |H_0|^2}$$

or

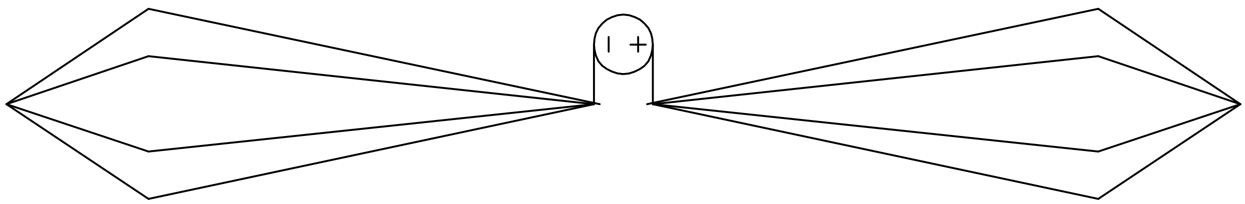
$$R_r = 120 \ln\left(\cot\frac{1}{2}\theta_h\right)$$

which is the same as the input impedance Z_{in}.

It is noted that the radiated fields are spherical waves with $\vec{E}$ in the θ direction and $\vec{H}$ in the ϕ direction. For linearly polarized waves incident on the antenna from the broadside direction ($\theta = 90^o$), the antenna responds to the field component that is parallel to its axis. Also the input impedance and pattern are theoretically constant over an infinite range of frequencies.

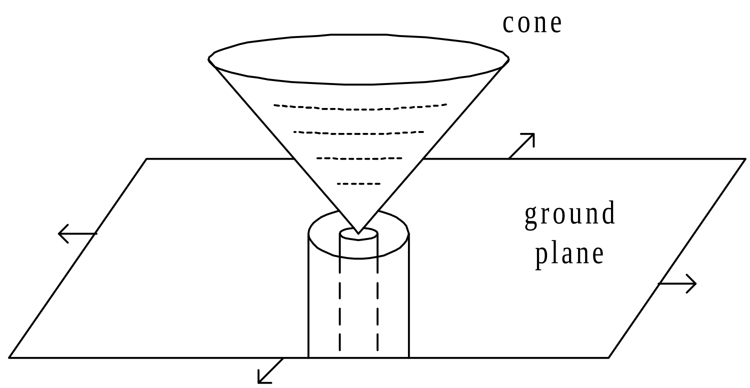
Infinite length cones are obviously impossible to construct, therefore real biconical antennas consist of truncated cones. The finite length of the cones causes reflections as the waves travel outward along the cones. This produces standing waves that result in the input impedance having an imaginary (reactive) component, rather than being purely real.

Often wires are used to approximate the cone surfaces:

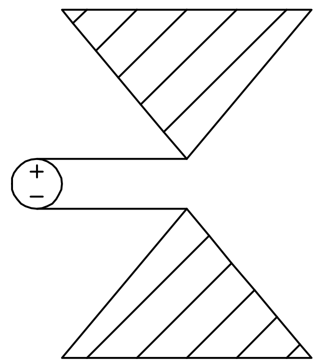


truncated biconical antenna
composed of wire elements

Other variations:



Discone



Bowtie

The fields of the discone antenna above the ground plane are the same as those of the biconical antenna by the method of images. The radiation resistance of the discone is ½ that of the biconical antenna.

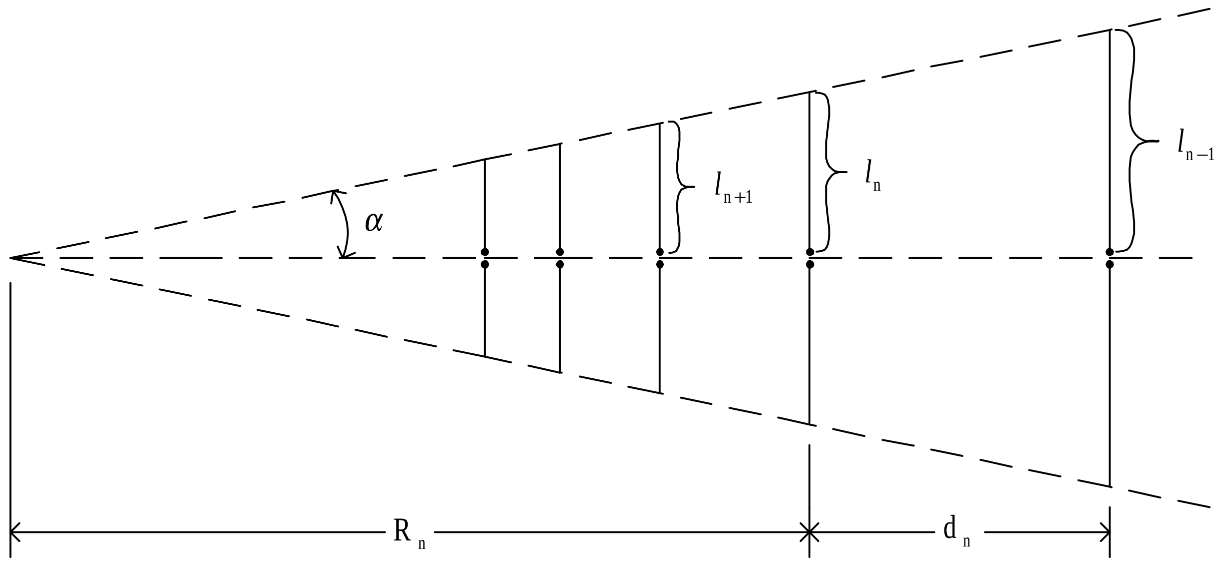
The bow-tie antenna consists of flat triangular plates or a wire which outlines the same area as the plates. The bow-tie antenna is frequently used for reception of the UHF television signals. Using wires instead of solid metal triangles tends to reduce the bandwidth of the bow-tie antenna.

Log-periodic antennas

The log-periodic antenna achieves a large operational bandwidth through repetitive dimensioning of structures. The structural dimensions increase in proportion to the distance from the origin of the structure. As a result, the input impedance and radiation properties repeat periodically and are functions of the logarithm of frequency.

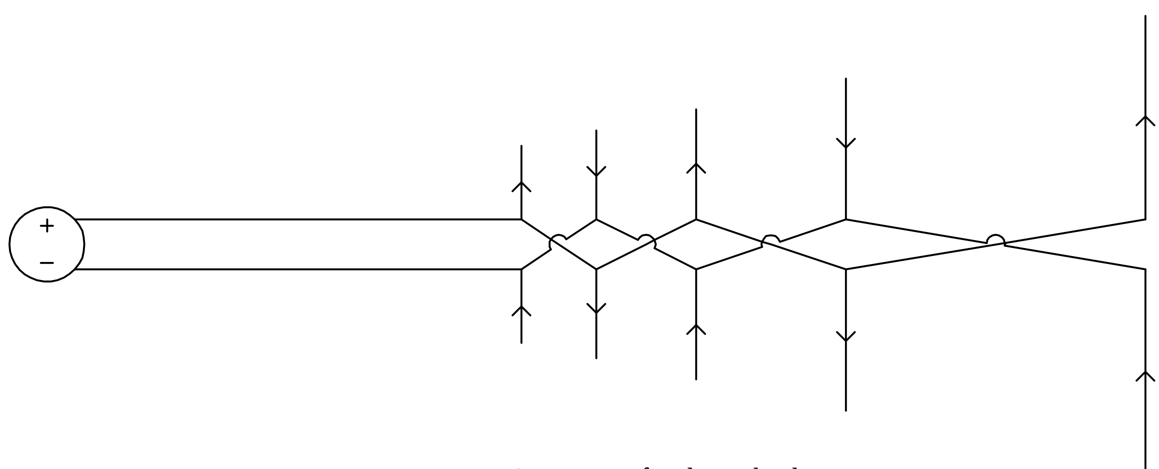
The log-periodic dipole array is a common log-periodic measurement antenna. This antenna shares the properties of all log-periodic antennas in that element distances, lengths, and separations are related by a constant such that

$$\tau = \frac{l_n}{l_{n-1}} = \frac{d_n}{d_{n-1}} = \frac{R_n}{R_{n-1}}$$



log-periodic array

The most efficient way of operating a log-periodic array is such that the currents on adjacent elements are reversed in phase.

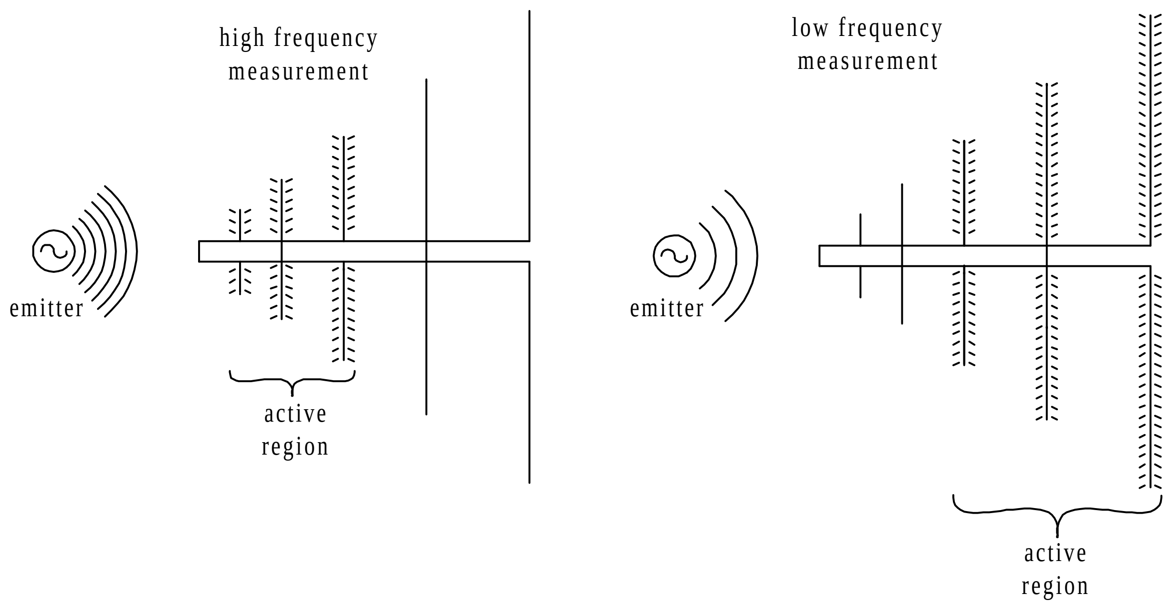


criss-cross feed method

In this way the shorter elements will not interfere with elements to the right.

The bandwidth of the log-periodic antenna is approximated by determining the frequency at which the shortest element is, and the frequency at which the longest element is ½ λ.

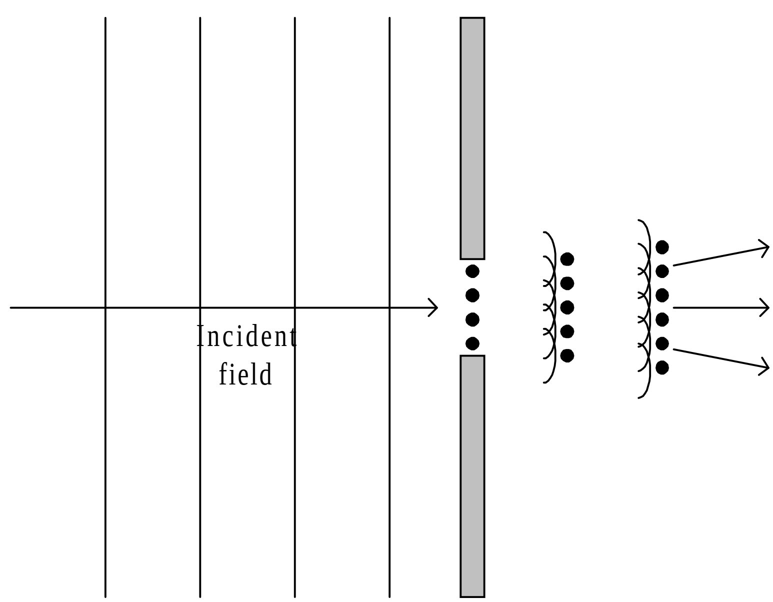
At a particular frequency only the elements which are at or near a resonance are active. Thus, the active region of the antenna adjusts depending on frequency.



6.10 Aperture antennas

Aperture antennas are characterized by an aperture or opening from which radiated fields are emitted. These include horns, slots and microstrip patch antennas. The operation of such antennas is best explained by Huygen's principle:

Each point in an advancing wavefront acts as a source of spherical, secondary wavelets, that propagate outward.



The secondary wavelets cause the wave to spread as it travels away from the aperture.

This is known as **diffraction**. The antenna pattern of an aperture antenna is actually a diffraction pattern.

In the far zone of a simple aperture the magnitude of the electric field is given by

$$E(\vec{r}) \approx \frac{jk}{2\pi} \int_S E_a(\vec{r}\,') \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} ds'$$

where S is the aperture surface, $E_a(\vec{r}\,')$ is the magnitude of the electric field in the aperture and $\vec{r}\,'$ is a position vector that sweeps over points in the aperture.

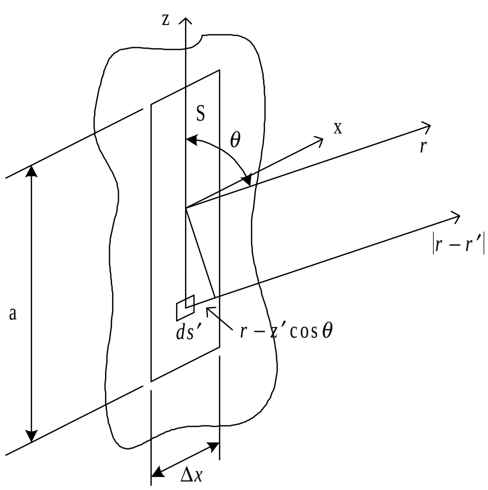
At points far from the aperture

$$|\vec{r}-\vec{r}\,'| \approx r - z'\cos\theta$$

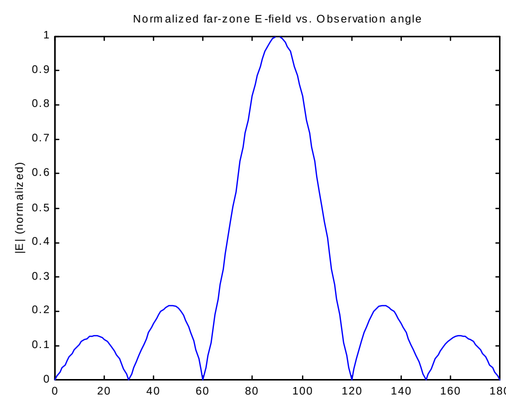
Assuming that the field in the aperture is uniform and that the aperture width is small

$$E(r,\theta,\phi) \approx \frac{jk\Delta x e^{-jkr}}{2\pi r} \int_{-a/2}^{a/2} E_a e^{jkz'\cos\theta} dz' = jE_a \Delta \times \frac{e^{-jkr}}{r} \frac{\sin\left(\pi \frac{a}{\lambda}\cos\theta\right)}{\pi\cos\theta}$$

The size of the main lobe in the resulting pattern is inversely proportional to the aperture width ‘a’. Thus in order for the main lobes to be narrow, the aperture dimensions must be on the order of a wavelength or greater.



Radiation from a narrow aperture

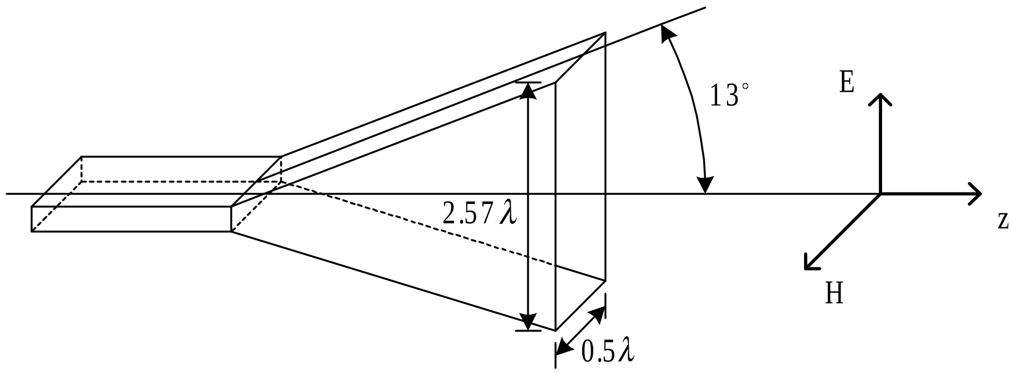


Horn antennas

Horn antennas are flared waveguides. There are three basic types of horn antennas.

E-plane horn - flare is in the plane that contains the $\vec{E}$ -field vector.
H-plane horn - flare is in the plane that contains the $\vec{H}$ -field vector.
Pyramidal horn - flare is in both planes.

The aperture distribution of a horn is typically the same as the mode of the feeding waveguide, with a phase taper across the aperture.



Geometry of a typical horn antenna

The antenna as a receiving element

Any transmitting antenna can also be used for the purpose of "receiving" – intercepting a portion of the power radiated by some source. Instead of being driven by transmission line, a receiving antenna delivers power to a load connected at its terminals.

Consider a spherical wave radiated by some distant source, and incident on a receiving antenna. Over the local region of the antenna, the spherical wave can be approximated as a plane wave. The plane wave induces a current in the antenna, which in turn produces an additional scattered field. But the induced current also causes a voltage to appear across the load impedance. This voltage then acts like a driving voltage causing additional currents to flow, just as in a transmitting antenna, which produce still another scattered field.

Thus, by superposition, the total current flowing on the antenna may be viewed as that due to a scatterer interacting with a plane wave *plus* that of a transmitting antenna.

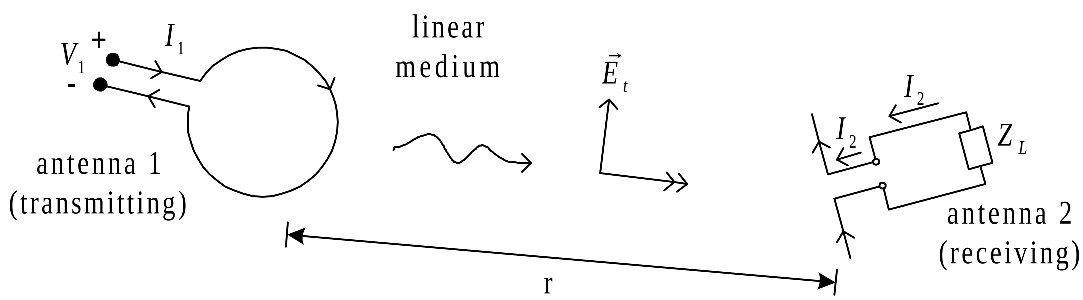
Transmitting/receiving equivalent circuits

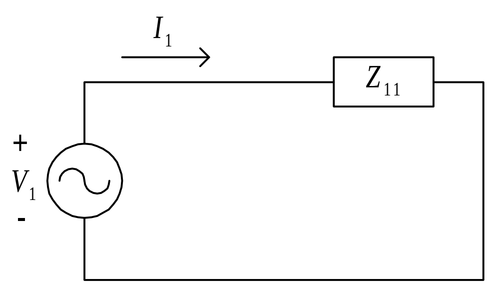
$$\left.\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}\right\}$$ network description of transmitting/receiving system

Assume: $Z_{12} << Z_{11}$ for $r \rightarrow$ large (little coupling from receiving to transmitting antenna)

$$V_1 = Z_{11} I_1 + \overbrace{Z_{12} I_2}^{\approx 0} \approx Z_{11} I_1$$

$$\left(Z_1\right)_{in} = \frac{V_1}{I_1} \approx Z_{11}$$

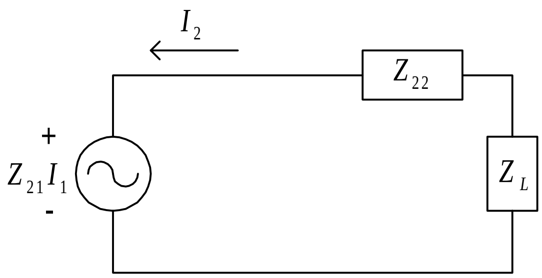




Equivalent circuit for transmitting antenna

$$V_2 = -I_2 Z_L = Z_{21} I_1 + Z_{22} I_2$$

$$I_2 = \frac{-Z_{21} I_1}{Z_{22} + Z_L}$$



Equivalent circuit for receiving antenna

Note: When antenna 2 is in receiving, we can not assume $Z_{21} \approx 0$ since this term describes the coupling effect between transmitter and receiver.

Note: When antenna 2 is transmitting, then $Z_{21} \approx 0$ so that

$$V_2 = \overbrace{Z_{21}I_1}^{\approx 0} + Z_{22}I_2 \approx Z_{22}I_2$$

$$\left(Z_2\right)_{in} = \frac{V_2}{I_2} \approx Z_{22}$$

$$Z_{22} \approx \left(Z_2\right)_{in}$$

Receiving/transmitting reciprocity

There are three basic reciprocity relations between receiving and transmitting antennas:

1) The antenna pattern for reception is identical to that for transmission.
2) The equivalent impedance in the receiving antenna equivalent circuit is identical to the input impedance of the antenna when it is transmitting.
3) The effective receiving cross-section area of an antenna is proportianal to its diective gain as a transmitting element.

We have already considered (2) above. The others require the use of the Lorentz reciprocity theorem. From this theorem we can show that $Z_{12} = Z_{21}$.

Relationship between gain and effective receiving cross-sectional area

Definition: $A_{er} = \dfrac{W_r}{P_{av}}$ effective receiving cross-section area (m^2).

where: W_r = received power (power delivered to the load)

P_{av} = average Poynting vector (power density) maintained by transmitting antenna (Watts/m^2)

A_{er} is a function of: 1) Load impedance
2) Aspect of antenna to oncoming wave
3) Polarization of oncoming wave

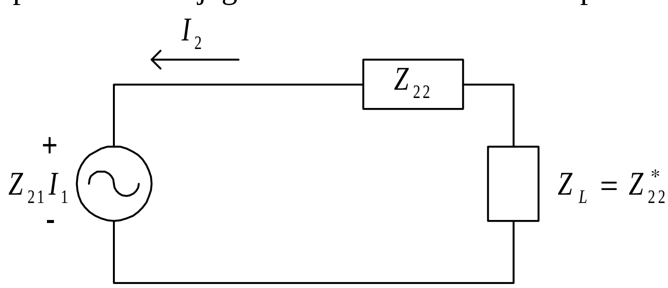
Note: $W_r = A_{er} P_{av}$ = total power intercepted by receiving antenna.

Recall: $g_{dt}(\theta, \phi) = \dfrac{\dfrac{dW_t}{d\Omega}}{\left(\dfrac{W_t}{4\pi}\right)}$ directive gain of transmitting antenna, W_t = transmitted power.

$$= \frac{4\pi r^2 P_{av}}{W_t} \Rightarrow P_{av} = \frac{W_t g_{dt}}{4\pi r^2}$$

So: $W_r = W_t \dfrac{A_{er} g_{dt}}{4\pi r^2}$ received power in terms of transmitted power.

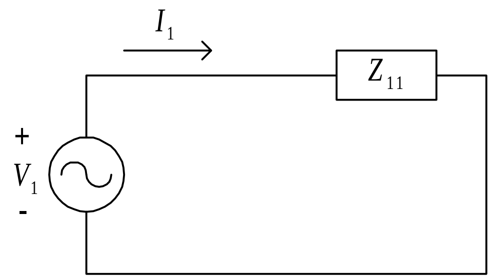
Maximum power transfer relation:
Assume load impedance is conjugate matched $\Rightarrow$ maximum power transferred to load



receiving antenna

$$Z_L = Z_{22}^* = R_2 - jX_2$$

$$W_r = \frac{1}{2}|I_2|^2 R_2 = \frac{1}{2}\left|\frac{-Z_{21} I_1}{2R_2}\right|^2 R_2 = \frac{|Z_{21} I_1|^2}{8R_2}$$



transmitting antenna

$$Z_{11} = R_1 + jX_1$$

$$W_t = \frac{1}{2}\left|I_1\right|^2 R_1$$

$$\frac{W_r}{W_t} = \frac{\dfrac{\left|Z_{21} I_1\right|^2}{8R_2}}{\dfrac{1}{2}\left|I_1\right|^2 R_1} = \frac{\left|Z_{21}\right|^2}{4R_1R_2}$$

but:

$$\frac{W_r}{W_t} = \frac{A_{er}\, g_{dt}}{4\pi r^2} = \frac{\left|Z_{21}\right|^2}{4R_1R_2}$$

so:

$$\left|Z_{21}\right|^2 = \frac{R_1R_2A_{er_2}\, g_{dt_1}}{\pi r^2}$$ antenna 1 transmitting, antenna 2 receiving

$$\left|Z_{12}\right|^2 = \frac{R_2R_1A_{er_1}\, g_{dt_2}}{\pi r^2}$$ antenna 1 receiving, antenna 2 transmitting

now use $Z_{12} = Z_{21}$ (Reciprocity of network)

$$g_{dt_1} A_{er_2} = g_{dt_2} A_{er_1}$$

or

$$\frac{A_{er_1}}{g_{dt_1}} = \frac{A_{er_2}}{g_{dt_2}}$$

Now since antennas 1 and 2 were arbitrary,

$$\frac{A_{er}}{g_{dt}} = \text{constant}$$

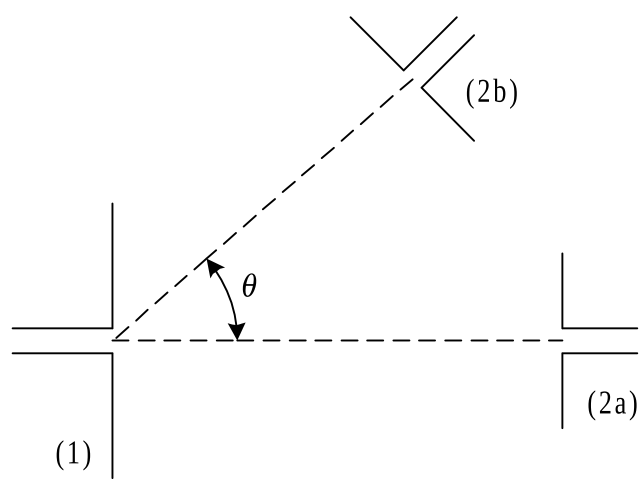
For polarization matched conditions (receiving antenna oriented to intercept maximum amount of power), the constant can be shown to be $\frac{4\pi}{\lambda^2}$ (Derivation is kind of messy).

Thus, $\frac{A_e}{g_d} = \frac{\lambda^2}{4\pi}$ Universal relationship between gain of an antenna acting as a transmitter and effective area of same antenna acting as a receiver.

So, $W_r = W_t \left(\frac{\lambda}{4\pi r} \right)^2 D_r D_t$ "Friis" equation

Reciprocity between transmission and reception patterns

Case 1) Antenna 1 transmitting, antenna 2 receiving

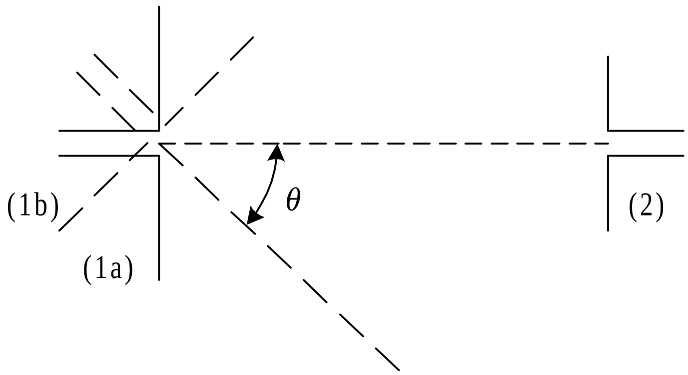


Measure transmitting pattern of antenna 1 by varying θ.

$$\frac{W_{r_2}}{W_{t_1}} = \left| Z_{21} \right|^2 \left(\frac{1}{4 R_1 R_2} \right)$$

$$\frac{W_{r_{2b}}}{W_{r_{2a}}} = \frac{\left|Z_{21}\right|_b^2}{\left|Z_{21}\right|_a^2}$$

Case 2) Antenna 2 transmitting, antenna 1 receiving



Measure receiving pattern of antenna 1 by varying θ.

$$\frac{W_{r_1}}{W_{t2}} = \left|Z_{12}\right|^2 \left(\frac{1}{4R_1R_2}\right)$$

$$\frac{W_{r_{1b}}}{W_{r_{1a}}} = \frac{\left|Z_{12}\right|_b^2}{\left|Z_{12}\right|_a^2}$$

By reciprocity: $Z_{12} = Z_{21} \Rightarrow \frac{W_{r_{2b}}}{W_{r_{2a}}} = \frac{W_{r_{1b}}}{W_{r_{1a}}}$ power patterns are equal.

Thus, the reception pattern of an antenna is identical to its transmission pattern.

References

1. Demarast, K. Engineering Electromagnetics, Prentice Hall Inc., 1998
2. Paul, C. Introduction to Electromagnetic Compatibility, John Wiley & Sons, 1992