Module 13:
Network Analysis and Directional Couplers

13.0 Introduction

Directional couplers represent an important class of devices that are frequently used in EMC-related testing. In this section, network theory will be briefly reviewed, and simple directional couplers will be discussed.

13.1 Impedance

- The term 'impedance' first used by Oliver Heaviside in the nineteenth century to describe the complex ratio V/I in AC circuits
- Later this concept applied to transmission lines (lumped element equivalent circuits, distributed series impedance, shunt admittance)
- In the 1930's, impedance concept extended to EM fields by Schelkunoff. Noted that impedance is a characteristic of the type of field, as well as the medium of transmission.

Several types of impedance exist:

- intrinsic impedance of a medium $\eta = \sqrt{\frac{\mu}{\epsilon}}$: dependent only on the material parameters of the medium, equal to wave impedance for plane waves.

- wave impedance $Z_w = \frac{E_t}{H_t} = \frac{1}{Y_w}$: characteristic of a particular type of wave; TEM, TM, and TE waves each have different wave impedances (Z_{TEM}, Z_{TM}, Z_{TE}) which may depend on the type of line or guide, the material of transmission, and operating frequency

- characteristic impedance $Z_0 = \frac{1}{Y_0} = \sqrt{\frac{L}{C}}$: ratio of voltage to current for a traveling wave (since voltage and current are uniquely defined for TEM waves, the characteristic impedance associated with a TEM wave is unique. TE and TM waves do not have a uniquely defined voltage and current, thus the associated impedances are not unique.)

13.2 Network theory

- **two port networks, S-parameters, Z-parameters, Y-parameters**

The study of two port networks is important in the field of electrical engineering because most electric circuits and electronic modules have at least two ports, namely input and output terminal pairs. Two-port parameters describe a system in terms of the voltage and current that may be measured at each port. A typical generalized two-port network is indicated in the figure below.

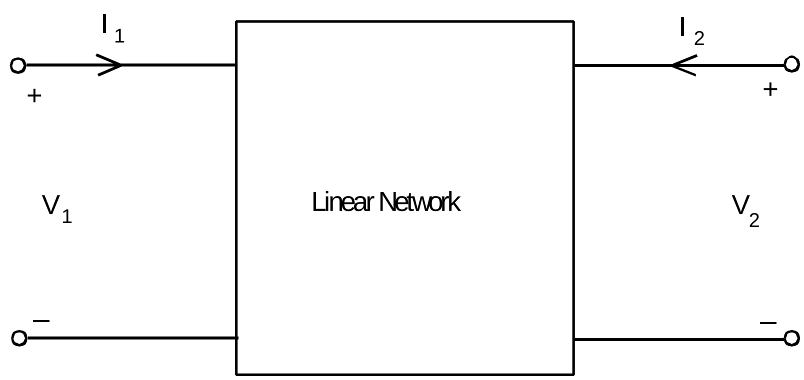


Figure 1. General linear network.

Here I_1 is the current entering port 1, I_2 is the current entering port 2, and V_1 and V_2 are voltages that exist at ports 1 and 2, respectively.

- Y-parameters

If the network that exists between ports 1 and 2 is *linear*, and contains no independent sources, then the principle of superposition may be applied to determine the currents I_1 and I_2 in terms of voltages V_1 and V_2

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 .$$

These may be written in matrix form as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

Here the terms Y_{11}, Y_{12}, Y_{21}, and Y_{22} are known as *admittance*, or *Y-parameters*. It is apparent form the matrix equation above that the parameter Y_{11} may be determined by measuring I_1 and V_1 when V_2 is equal to zero (or more accurately when port 2 is short-circuited)

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}.$$

The remaining Y-parameters may be determined in a similar manner

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}.$$

Because the parameter Y_{11} is an admittance which is measured at the input of a two-port network when port 2 is short circuited, it is known as the *short-circuit input admittance*. Likewise, the parameters Y_{12} and Y_{21} are known as *short-circuit transfer admittances*, and Y_{22} is the *short-circuit* output admittance. It is evident that, using these relationships, the properties of an unknown, linear two-port network can be completely specified by values measured experimentally at ports 1 and 2.

- **Z-parameters**

A similar set of relationships may be established in which the voltages at the input and output of a linear two-port network are expressed in terms of the input and output currents

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

which may be described in matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

It is apparent from this expression that the parameter Z_{11} may be determined by open-circuiting port 2 (letting I_2 equal zero)

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0}$$

and the remaining *impedance* or *Z-parameters* are determined similarly as

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1 = 0}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0}$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1 = 0}.$$

The term Z_{11} is known as the *open-circuit input impedance*, Z_{12}, and Z_{21} are known as *open-circuit transfer impedances*, and Z_{22} is the *open-circuit output impedance*.

- **S-parameters**

The representations above are useful if voltage and current can easily be measured at the input and output of the two-port network. While it is usually possible to directly measure both voltage and current in low frequency electric circuits, it is not always possible to do this with high-frequency circuits or particularly with waveguides. In such cases it is often necessary to determine impedances from measured standing wave ratios or reflection coefficients. It is thus convenient to describe unknown high-frequency networks in terms of outgoing and incoming wave amplitudes, instead of voltages and currents. The figure below represents a linear two-port network

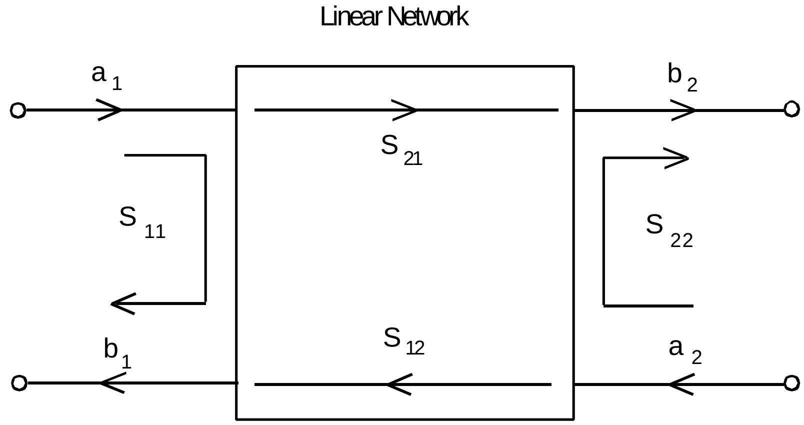


Figure 2. General scattering network.

where a_1, and a_2 are incoming wave amplitudes, and b_1, and b_2 are outgoing wave amplitudes. These parameters represent the normalized incident and reflected voltage wave amplitudes. For an n-port network they are

$$a_n = \frac{V_n^+}{\sqrt{Z_{on}}}, \qquad b_n = \frac{V_n^-}{\sqrt{Z_{on}}}$$

where Z_{on} is the equivalent characteristic impedance at the n^{th} terminal port. The voltage and current at some reference plane within the network can thus be represented by

$$V_n = V_n^+ + V_n^- = \sqrt{Z_{on}}(a_n + b_n)$$

and

$$I_n = \frac{1}{Z_{on}}(V_n^+ - V_n^-) = \frac{1}{\sqrt{Z_{on}}}(a_n - b_n) .$$

Solving for a_n and b_n yields

$$a_n = \frac{1}{2}\left(\frac{V_n}{\sqrt{Z_{on}}} + \sqrt{Z_{on}}\, I_n \right)$$

$$b_n = \frac{1}{2}\left(\frac{V_n}{\sqrt{Z_{on}}} - \sqrt{Z_{on}}\, I_n \right) .$$

The average power flowing into the nth terminal is given by

$$(P_n)_{ave} = \frac{1}{2} Re(V_n I_n^*) = \frac{1}{2} Re\left[(a_n a_n^* - b_n b_n^*) + (b_n a_n^* - b_n^* a_n)\right].$$

The term $(a_n a_n^* - b_n b_n^*)$ is purely real, and the term $(b_n a_n^* - b_n^* a_n)$ is purely imaginary, therefore

$$(P_n)_{ave} = \frac{1}{2}(a_n a_n^* - b_n b_n^*)$$

where $(P_n)_{ave}^+ = 1/2\,(a_n a_n^*)$ represents power flow into the terminal, and $(P_n)_{ave}^+ = 1/2\,(a_n a_n^*)$ represents the power reflected out of the terminal.

Consider again the case of a network with two terminal ports. The principle of superposition may be applied to represent the outgoing waves in terms of a linear combination of the incoming waves

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

which may be written in matrix form as

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

The individual *scattering*, or *S-parameters* are determined in a way similar to those above. The parameter S_{11} is the ratio of the outgoing wave amplitude at port 1 to the incoming wave amplitude at port 1 when the incoming wave amplitude at port 2 is zero (or more correctly when port 2 is match terminated)

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2 = 0} .$$

It is noted that the ratio of the outgoing wave amplitude at port 1 to the incoming wave amplitude at port 1 is simply the reflection coefficient at port 1. Therefore S_{11} is the reflection amplitude at port 1 is simply the reflection coefficient at port 1. Therefore S_{11} is the reflection coefficient at port 1 when port 2 is match terminated. The remaining S-parameters are determined as follows

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}.$$

– Transmission parameters

Z, Y, and S-parameters are used to characterize microwave networks having arbitrary numbers of ports. In practice many microwave networks consist of multiple, cascaded, two-port networks. It is therefore convenient to define a 2x2 transmission (ABCD) matrix for each two-port network. The ABCD matrix of the total network can be found by multiplying the ABCD matrices of the individual two-ports.

The ABCD matrix is defined for a two-port network in terms of total voltages and currents

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

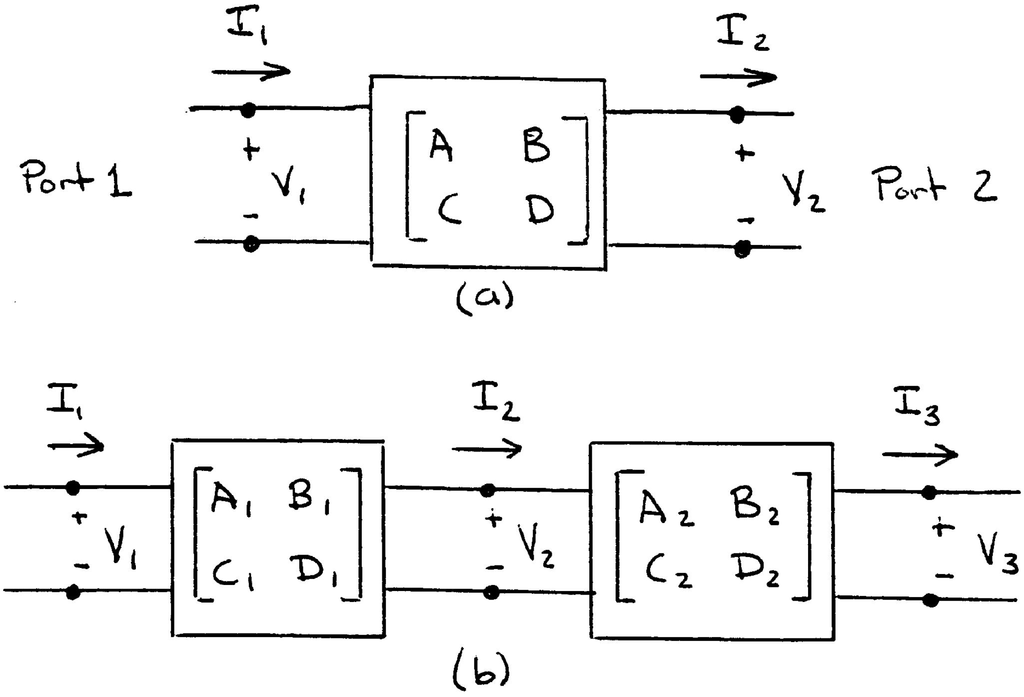


Figure 3. (a) two port network; (b) cascaded connection of two-port networks.

This may be expressed in matrix form as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

It should be noted that in this convention I_2 flows <u>out</u> of port 2 (in the previous convention I_2 flowed into I_2).

The left-hand side of the matrix equation above represents the voltage and current at port 1 of the network, and the right hand side represents the voltage and current at port 2.

It is clear that the parameter A is found by measuring the open circuit voltage V_2 at port 2, when a voltage V_1 is applied at port 1

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} .$$

The remaining parameters are given by

$$B = \left. \frac{V_1}{I_2} \right|_{V_2=0}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$$

$$D = \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

A cascaded connection of two two-port networks can be described by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

and

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} .$$

The total cascaded network is then represented by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} .$$

The ABCD parameters of a network can be determined from the Z-parameters. The defining relations for Z-parameters of a two-port network give

$$V_1 = I_1 Z_{11} - I_2 Z_{12}$$
$$V_2 = I_1 Z_{21} - I_2 Z_{22}$$

where I_2 is consistent with the sign convention used with ABCD parameters. The relationships between impedance parameters and ABCD parameters are thus

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{I_1 Z_{11}}{I_1 Z_{21}} = \frac{Z_{11}}{Z_{21}}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \left.\frac{I_1 Z_{11} - I_2 Z_{12}}{I_2}\right|_{V_2=0} = Z_{11}\left.\frac{I_1}{I_2}\right|_{V_2=0} - Z_{12}$$

$$= Z_{11}\frac{I_1 Z_{22}}{I_1 Z_{21}} - Z_{12} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{I_1 Z_{21}} = \frac{1}{Z_{21}}$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_2 Z_{22}/Z_{21}}{I_2} = \frac{Z_{22}}{Z_{21}}$$

13.4 Directional Couplers

A directional coupler is a four port device that is designed to couple in a separable fashion to the positively and negatively traveling waves in a guide.

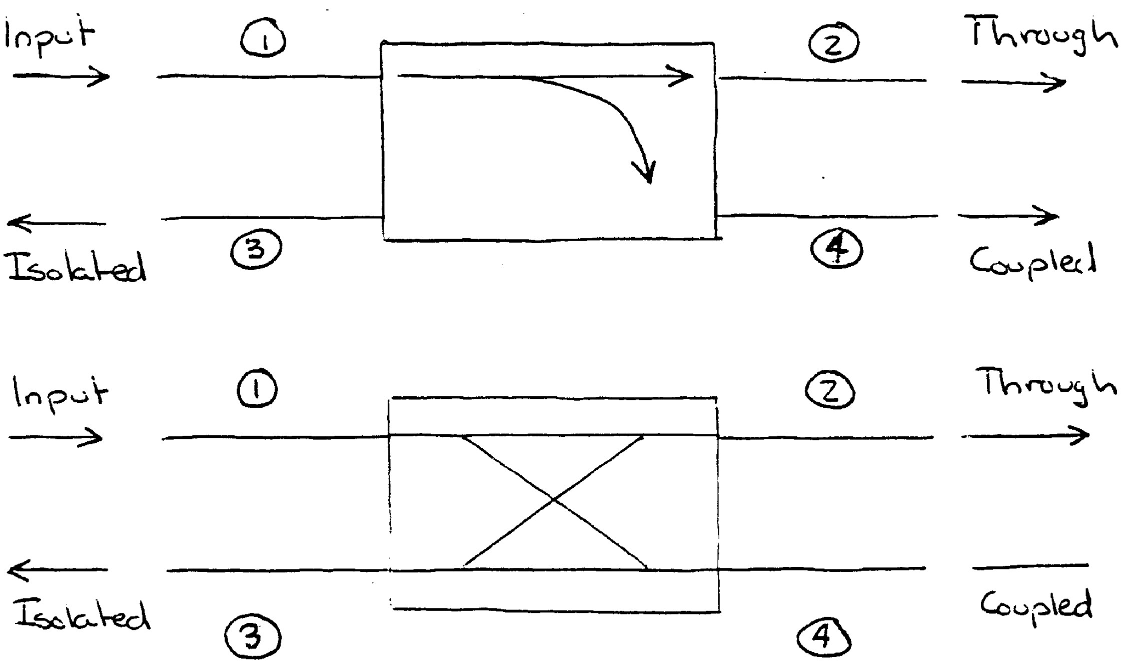


Figure 4. Two common symbols for directional couplers.

Two commonly used symbols representing directional couplers are shown in Figure 4.

- Power supplied to port 1 is coupled to port 4 (the coupled port).

- The remainder of the input power is delivered to port 2 (the through port).

- For an ideal directional coupler, no power is delivered to port 3 (the isolated port).

Directional couplers are characterized using three quantities

$$\text{Coupling} = C = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{directivity} = D = 10 \log_{10} \frac{P_4}{P_3}$$

$$\text{Isolation} = I = 10 \log_{10} \frac{P_1}{P_3}$$

- The coupling factor indicates the fraction of input power coupled to the output port.

- Directivity measures the ability of the coupler to isolate forward and backward traveling waves.

These three quantities are related by

$$I = D + C \quad dB.$$

An ideal directional coupler infinite directivity and isolation.

13.5 Wave guide directional couplers

One of the simplest types of directional coupler consists of two parallel waveguides which share a common wall. Two small holes placed $\lambda/4$ apart couple the two waveguides.

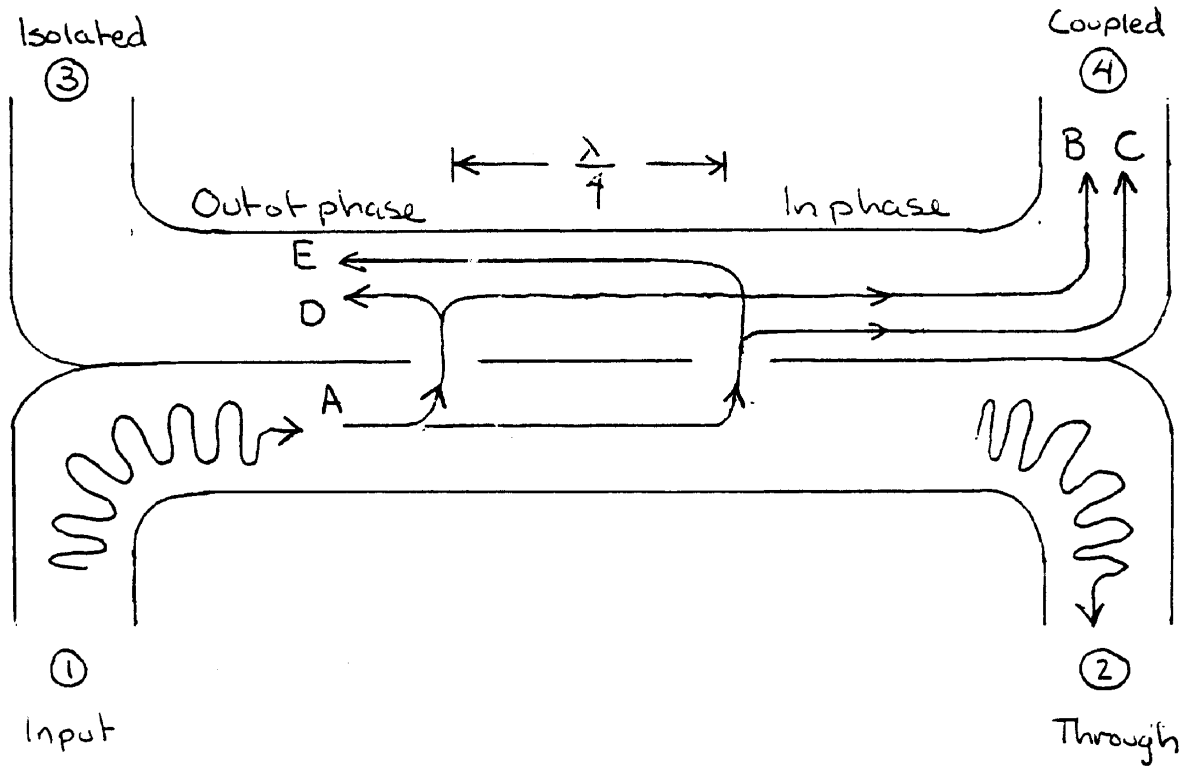


Figure 5. Two-hole directional coupler

A two-hole waveguide directional coupler is shown in Figure 5.

- An incident wave at point 'A' progresses to the right
- The incident wave couples to waves which emanate from the holes in the waveguide wall.
- The coupled waves add in phase at port 4 after following paths B and C, which are of equal lengths.
- The coupled waves add out of phase at port 3 because paths E and D differ in length by a half-wavelength.

It is clearly that this type of coupler is extremely frequency dependent, and also narrow band. The bandwidth of the coupler may be extended by employing multiple coupling holes.

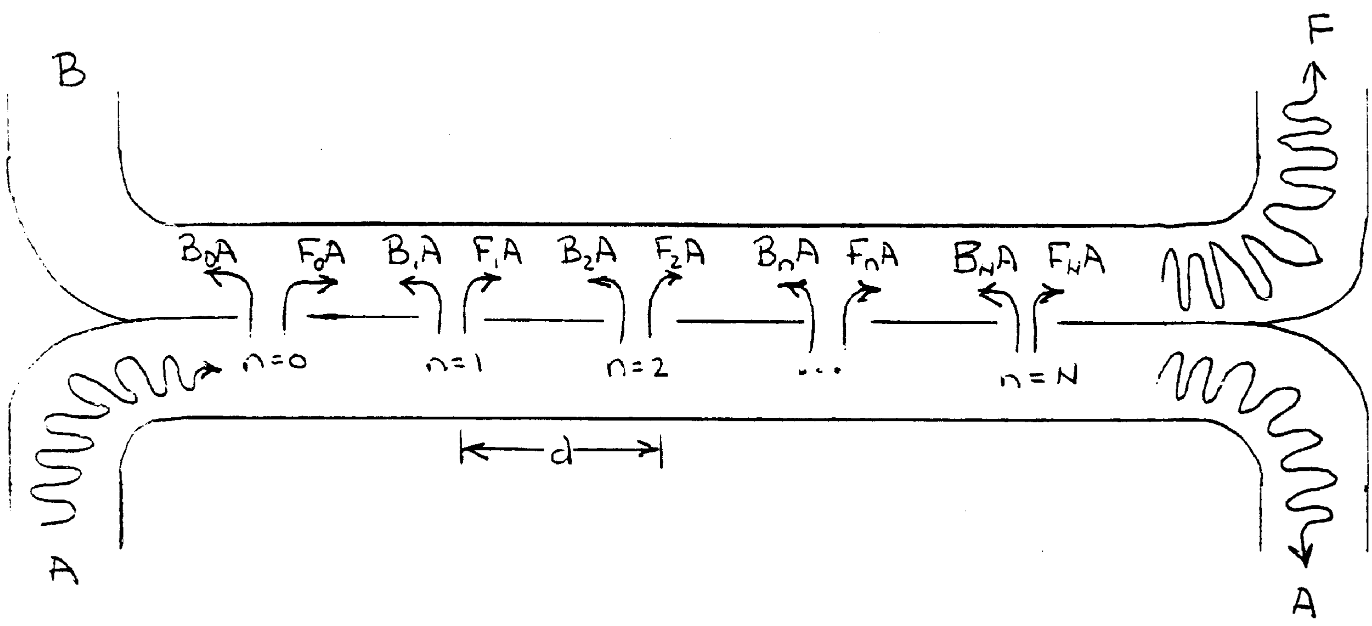


Figure 6. N+1 hole waveguide directional coupler.

The amplitude of the incident wave is 'A'. For small coupling, the amplitude of the through wave is nearly the same as that of the incident wave.

Now let

F_n = the coupling coefficient of the n^{th} aperture in the forward direction

B_n = the coupling coefficient of the n^{th} aperture in the backward direction

The amplitude of the forward coupled wave is then

$$F = Ae^{-j\beta Nd} \sum_{n=0}^{N} F_n$$

where β is a propagation constant. The amplitude of the backward traveling wave is

$$B = A \sum_{n=0}^{N} B_n e^{-2j\beta nd}$$

Since the path length for the n^{th} component is $2\beta nd$, where d is the spacing between the aperatures (In each case the phase reference is taken at the $n=0$ aperature).

If the apenatures are round holes with identical positions relative to the center of the wave guide then

$$F_n = K_f\, r_n^3$$

$$B_n = K_b\, r_n^3$$

where K_f and K_b are constants for the forward and backward coupling coefficients, and r_n is the radius of the n^{th} aperture. For this case

$$C = -20 \log \left| \frac{F}{A} \right| = -20 \log \left| \sum_{n=0}^{N} F_n \right|$$

$$= -20 \log |K_f| - 20 \log \sum_{n=0}^{N} r_n^3 \quad dB$$

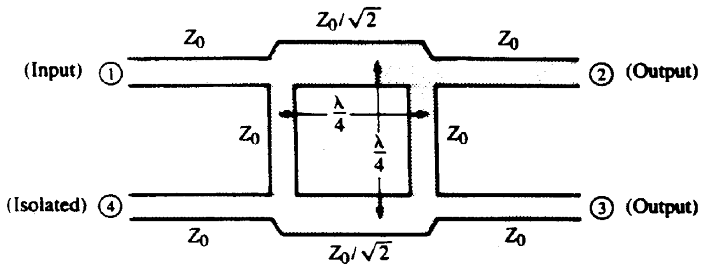
$$D = -20 \log \left| \frac{B}{F} \right| = -20 \log \left| \frac{\sum_{n=0}^{N} B_n e^{-2j\beta n d}}{\sum_{n=0}^{N} F_n} \right|$$

$$= C - 20 \log \left| \sum_{n=0}^{N} B_n e^{-2j\beta n d} \right|$$

$$= -C - 20 \log |K_b| - 20 \log \left| \sum_{n=0}^{N} r_n^3 e^{-2j\beta n d} \right| \quad dB$$

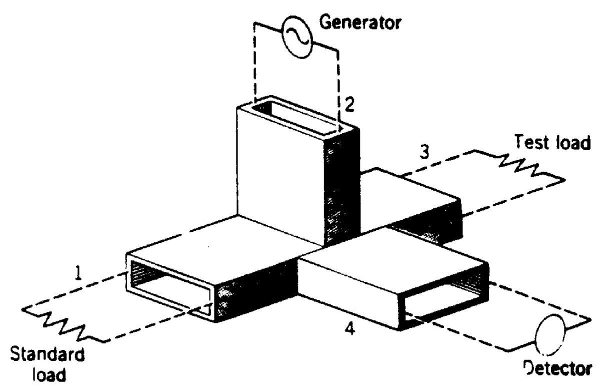
Other common couplers:

- microstrip branch line coupler



Known as a quadrature hybrid directional coupler. With all ports matched, power entering port 1 is evenly divided between ports 2 and 3 with a 90° phase shift between these ports. No power is coupled to port 4 (ideally).

- magic T coupler



Used for rectangular waveguides for the TE_{10} mode. A wave introduced into arm 2 divides equally between arms 1 and 3, but will not couple to arm 4. A wave introduced into arm 4 will divide between arms 1 and 3, but will not couple to arm 2.

References

1. Ramo, Whinnery, and Van Duzer, "Fields and Waves in Communication Electronics," John Wiley & Sons, second edition, 1984

2. Pozar, D., "Microwave Engineering," Addison-Wesley Publishing, 1990