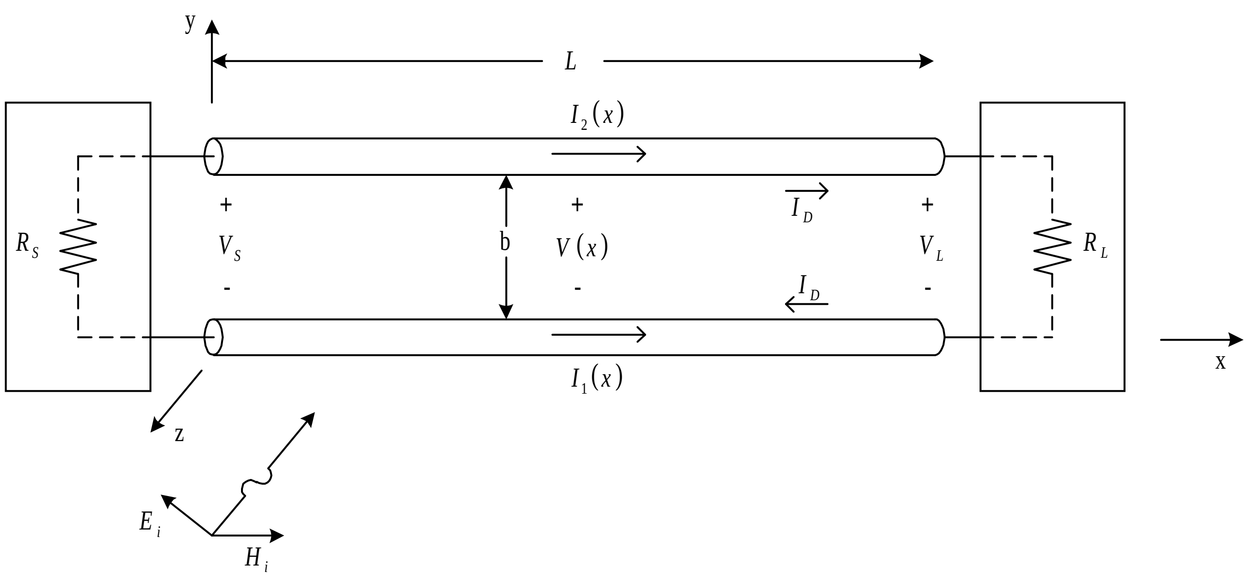
# Module 10:
# Radiated Immunity

## Introduction

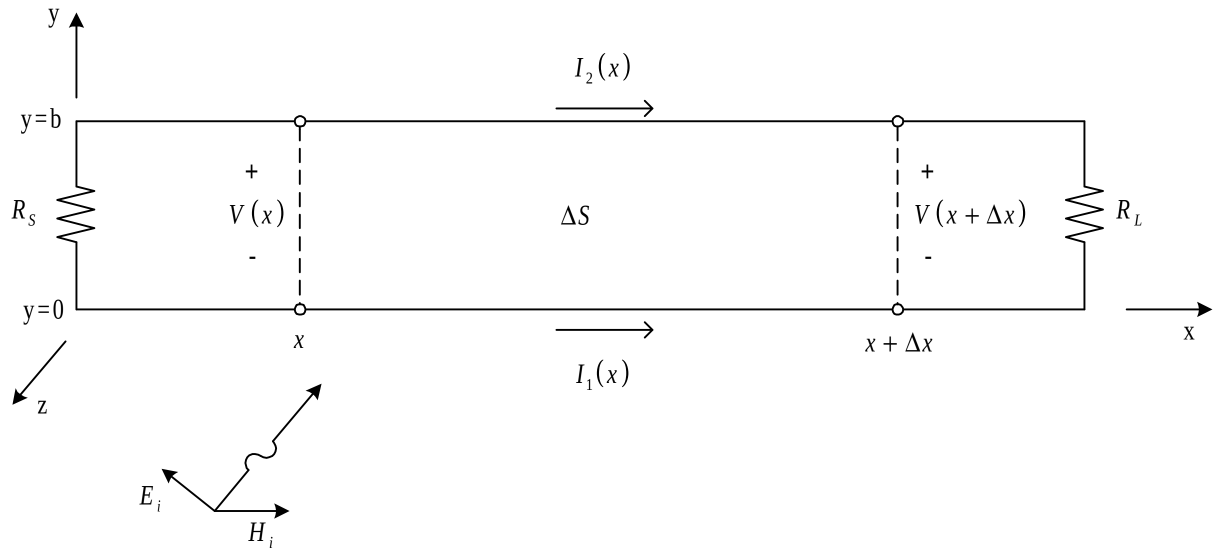
Although the law requires that all digital devices comply with regulatory limits on radiated and conducted emissions, no laws exist regulating a product's ability to withstand interference from such emissions. Nevertheless, it is important that digital devices are immune to these radiated and conducted emissions. Many products would be unsafe or unmarketable if they were susceptible to external disturbances such as fields from radio transmitters or radar. With this in mind, this module will discuss the effects that radiated fields have on digital devices. In particular, we will develop models for two-wire transmission lines and shielded coaxial cables and discuss the effects that radiated fields have on them.

## The two-wire transmission line

Consider an arbitrary two-conductor line illuminated by an impressed electromagnetic field. This impressed field will induce currents on the transmission line, and those currents will in turn create a scattered field that will eminate from the transmission line. Thus, the total electromagnetic field is composed of both the impressed and scattered field. In this development we will use the fact that the total fields can be decomposed into impressed and scattered fields to derive a pair of coupled transmission line equations for the two-wire transmission line.




The field incident on this transmission line system will induce voltages across the terminals. We will be studying the effects of these induced voltages in this chapter. In order to do this, however, we must develop a system of equations to describe this transmission line. These equations will allow us to construct an equivalent circuit that will simulate a two-conductor transmission line illuminated with an incident electromagnetic field.




In order to obtain equations for the line currents and voltage for this system the large scale form of Faraday's law is applied in the form

$$\oint E \cdot dl = -j\omega \int_{\Delta S} \vec{B} \cdot \hat{n} dS$$

Applying this to the geometry described in the figure, for the total electromagnetic field on the transmission line leads to:

$$\int_0^b \left[E_y(x+\Delta x, y) - E_y(x,y)\right]dy - \int_x^{x+\Delta x} \left[E_x(x,b) - E_x(x,0)\right]dx = -j\omega \int_x^{x+\Delta x} \int_0^b B_z(x,y)dydx$$

We need to simplify this equation into a more useful form. We will begin by assuming that the top and bottom wires are identical and share a common per unit length impedance $Z^i$.

Therefore the total electric field in the x-direction is

$$\left.\begin{matrix} E_x(x,0) = Z^i I_1(x) \\ E_x(x,b) = Z^i I_2(x) \end{matrix}\right\} \quad \text{.....where } I_1(x) \text{ and } I_2(x) \text{ are the axial currents on the wires}$$

From this we obtain

$$\lim_{\Delta x \to 0} \frac{1}{\Delta x} \int_{x}^{x+\Delta x} \left[ E_x(x,b) - E_x(x,0) \right] dx = Z^i \left[ I_2(x) - I_1(x) \right]$$

Breaking $I_1(x)$ and $I_2(x)$ into common and differential mode currents

$$I_2(x) = I_C + I_D$$
$$I_1(x) = I_C - I_D$$

leads to

$$\lim_{\Delta x \to 0} \frac{1}{\Delta x} \int_{x}^{x+\Delta x} \left[ E_x(x,b) - E_x(x,0) \right] dx = 2 Z^i I_D$$

It is observed that this equation is of the same form as the second term in the equation for the total electromagnetic field on the transmission line, and represents the total electromagnetic field in the x-direction. We will now simplify the first term of that equation and substitute both simplified forms back into the original equation. The first term describes the total electromagnetic field in the y-direction and can be written:

$$-\frac{\partial V}{\partial x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \int_{0}^{b} \left[ E_y(x + \Delta x, y) - E_y(x, y) \right] dy$$

Thus, by substituting these equations for total electromagnetic field in the x- and y- directions into the equation describing the electromagnetic fields about the entire transmission line, we get

$$\frac{\partial V}{\partial x} + 2 Z^i I_D(x) = j\omega \int_{0}^{b} B_z(x, y) dy$$

The total magnetic field in the z-direction is a composition of the impressed and scattered magnetic fields in the z-direction. Thus

$$B_z(x, y) = B_z^i(x, y) + B_z^s(x, y)$$

The scattered field is commonly expressed as

$$\int_0^b B_z^s(x,y)dy \cong -l^e I_D(x) \quad .....\text{where } l^e = \tfrac{\mu_0}{\pi}\ln\left(\frac{b}{a}\right) \text{ and } a \text{ is the conductor radius}$$

Thus

$$\frac{\partial V(x)}{\partial x} + ZI(x) = j\omega\int_0^b B_z^i(x,y)dy \quad .....\text{where } Z = 2Z^i + j\omega l^e \text{ and } I = I_D$$

This is our first transmission line equation, where Z is the distributed series impedance of the transmission line. In order to derive the second transmission line equation we will begin with the expression for line voltage

$$V(x) = -\int_0^b E_y(x,y)dy$$

From Faraday's Law: $\nabla \times \vec{E} = -j\omega\vec{B}$ , it can be shown that

$$E_y(x,y) = \frac{j\omega}{k^2}\frac{\partial B_z}{\partial x}$$

Thus

$$V(x) = -j\frac{\omega}{k^2}\frac{\partial}{\partial x}\int_0^b B_z(x,y)dy$$

If we again use

$$\int_0^b B_z^s(x,y)dy \cong -l^e I(x) \quad .....\text{where } l^e = \tfrac{\mu_0}{\pi}\ln\left(\frac{b}{a}\right) \text{ and } a \text{ is the conductor radius}$$

then

$$V(x) = -j\frac{\omega}{k^2}\frac{\partial}{\partial x}\int_0^b B_z^i(x,y)dy + \frac{j\omega l^e}{k^2}\frac{\partial}{\partial x}I(x)$$

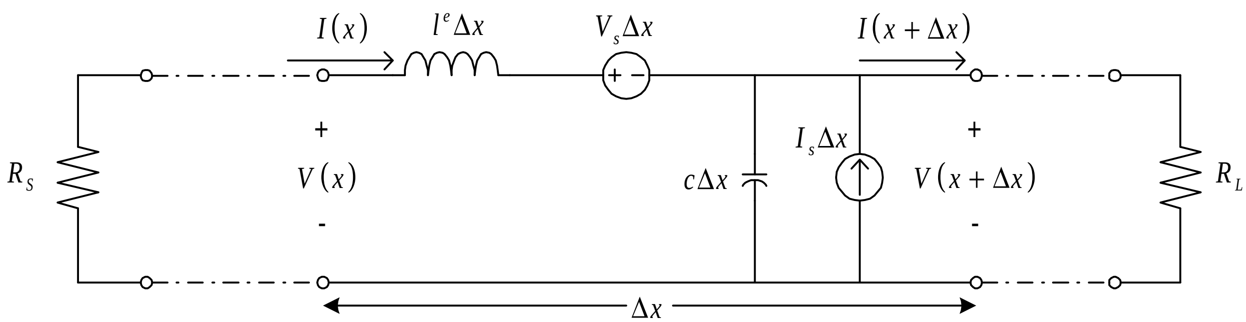
bringing the current term to the left side and converting the magnetic field term back to an electric field term, this becomes

$$\frac{jk^2}{\omega l^e}V(x)+\frac{\partial I(x)}{\partial x}=-\frac{jk^2}{\omega l^e}\int_0^b E_y^i(x,y)dy$$

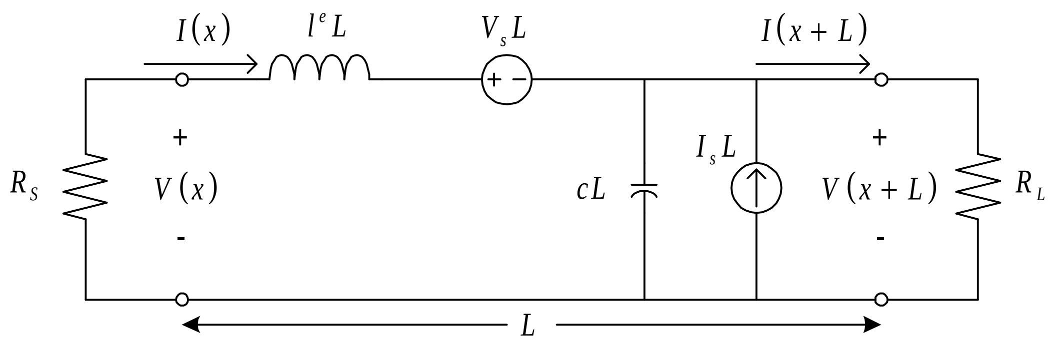
Finally, if we let $Y=\frac{jk^2}{\omega l^e}$ which is the distributed shunt admittance of the line conductors, then our second transmission line equation is

$$\frac{\partial I(x)}{\partial x}+YV(x)=-Y\int_0^b E_y^i(x,y)dy$$

Thus, we have derived a pair of coupled transmission line equations which simulate a two-conductor transmission line illuminated by an incident electromagnetic field. The equivalent per-unit-length circuit described by these coupled transmission line equations is shown in the following figure, where the entire transmission line consists of a series of such segments as shown..

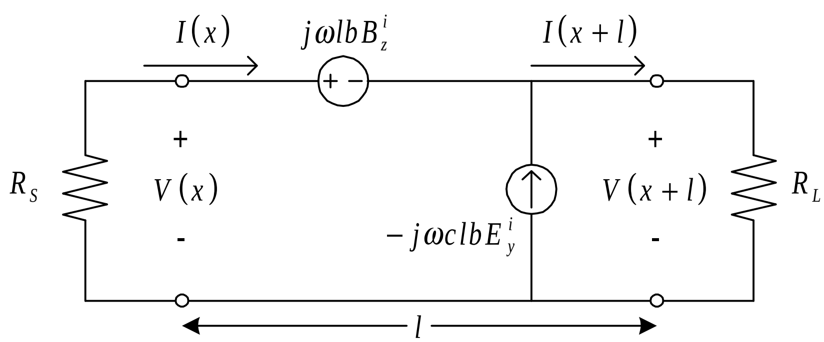



These coupled transmission line equations can be solved to find exact terminal voltages, but for estimation purposes, such complex solutions are not necessary. Instead, by further simplifying the model, we can easily obtain approximate solutions. For many practical applications, the line length is electrically short at the frequency of interest; that is $L << \lambda_0$. When this is the case, we can lump the distributed parameters into a single section that represents the entire line. In this case, the entire line is represented by a single segment shown in the above figure and $\Delta x$ is replaced with the line length $L$. For calculation purposes, this means that the per-unit-length elements and sources will be multiplied by the total line length $L$.




The terminal voltages can be calculated from this model for electrically short lines, but we can further simplify this model that will still be valid for a large variety of practical situations. As long as the terminal impedances are not extreme values such as open or short circuits, the per-unit-length parameters for inductance and capacitance can be ignored. Furthermore, since it is assumed that the line length *l* is small when compared to a wavelength, the wire separation *b*, which is much smaller than the line length, is also electrically small. Thus the field vectors do not vary appreciably across the wire cross section. Therefore the integrals in the sources with respect to y can be replaced with the wire separation *b*. Applying these simplifications to the coupled transmission line equations for electrically short transmission lines yields:

$$V_S L \cong j\omega B_z^i A$$
$$I_S L \cong -j\omega c E_y^i A$$
$$\text{....where } A = bL$$



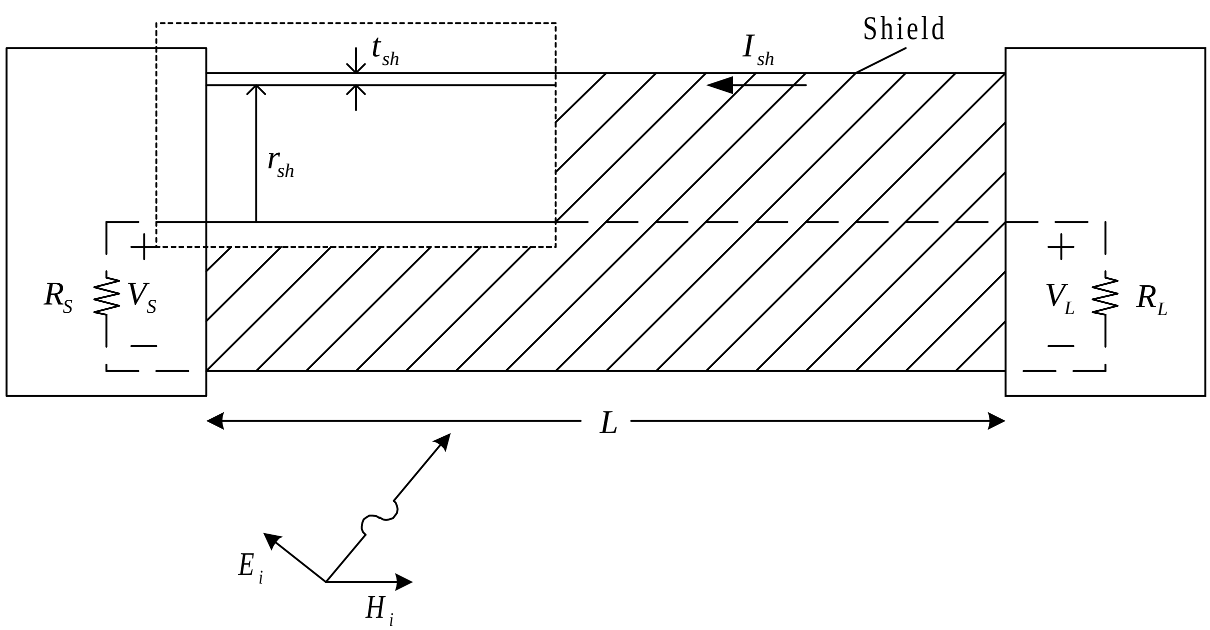

From this simplified model we can determine the terminal voltages by applying superposition:

$$V_S = \frac{R_S}{R_S + R_L} j\omega L b B_z^i - \frac{R_S R_L}{R_S + R_L} j\omega L b E_y^i$$
$$V_L = \frac{-R_L}{R_S + R_L} j\omega L b B_z^i - \frac{R_S R_L}{R_S + R_L} j\omega L b E_y^i$$

Thus, we have determined a method for approximating the voltages that will occur at the terminations of a two-wire transmission line that is illuminated by impressed electromagnetic fields. For a device to operate properly, it is desirable to minimize these line-end voltages, or to develop circuitry that will prevent them from interfering with the device. Using these models, engineers can determine which parameters of a device using two-wire transmission lines are desirable to minimize interference caused by electromagnetic fields.

**Shielded cables**

Coaxial cables consist of a center conducting wire surrounded by a cylindrical conducting shield. The shield is intended to entirely surround a circuit in order to prevent coupling to the line terminations from incident electromagnetic fields. Although a solid conducting shield would accomplish this task, the shield would need to be free of breaks or discontinuities to entirely prevent incident fields from penetrating to the center conducting wire and inducing currents. Although actual shields used on coaxial cables are not made with perfectly constructed shield, the imperfections are small enough that they can be ignored in common applications. Furthermore, breaks in the shield, called pigtails, often occur at the line ends. For the full effectiveness of the coaxial cable shielding to be realized, cables with pigtails should be avoided, and cables with connectors peripherally bonded at the cable ends should be used. For the purposes of this discussion we will assume that imperfections in the cable shield are negligible and breaks in the shield, such as pigtails, are not present. A model for the shielded coaxial cable is shown here:




External fields penetrate non ideal shields via diffusion of the current that is induced on the surface of the shield by the exterior field. In other words, the shield has an impedance, known as the surface transfer impedance of the shield. As an incident field causes a current to flow on the shield, a voltage drop occurs from the outer surface of the shield to the inner surface of the shield. Thus, external fields can cause noise voltages even on shielded cables. In order to calculate the noise induced by an incident field on a shielded coaxial cable, we must first calculate the current, $I_{sh}$, induced on the exterior of the shield by the impressed electromagnetic wave. Next, we must know the surface transfer impedance of the shield, which is given by

$$Z_T = R_0 \frac{\gamma t_{sh}}{\sinh(\gamma t_{sh})}$$

which has units of Ω/m and is thus a measure of impedance per unit thickness of the shield. The thickness of the shield is $t_{sh}$ and the propagation constant in the shield material is

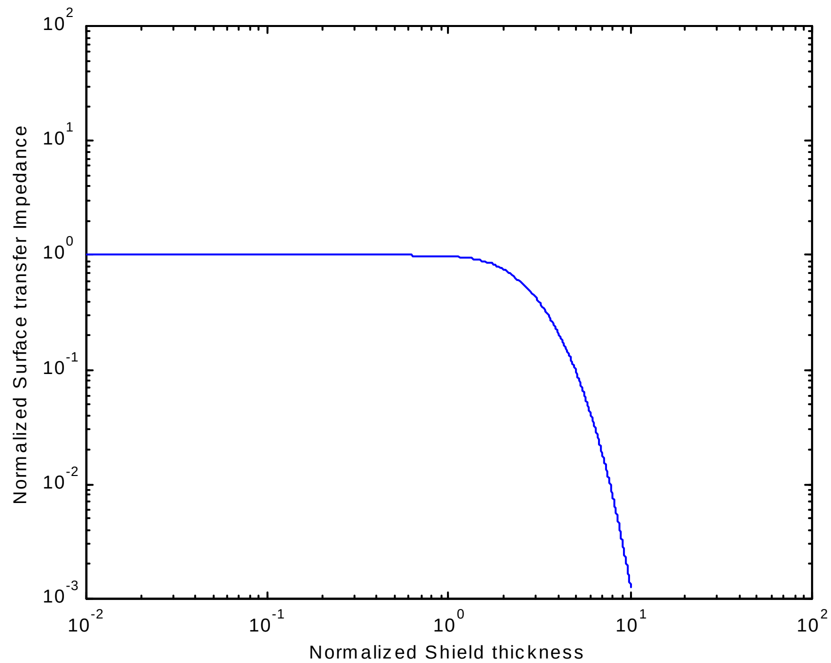
$$\gamma = \frac{1+j}{\delta}$$

where $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ is the skin depth, $f$ is the frequency, and $\mu$ and $\sigma$ are the permeability and conductivity of the shield. $R_0$ is the per-unit-length dc resistance of the shield, and is given by

$$R_0 = \frac{1}{2\pi\sigma r_{sh} t_{sh}} \text{ (in } \Omega/\text{m) for } t_{sh} << \delta$$

$r_{sh}$ is the inner radius of the shield.

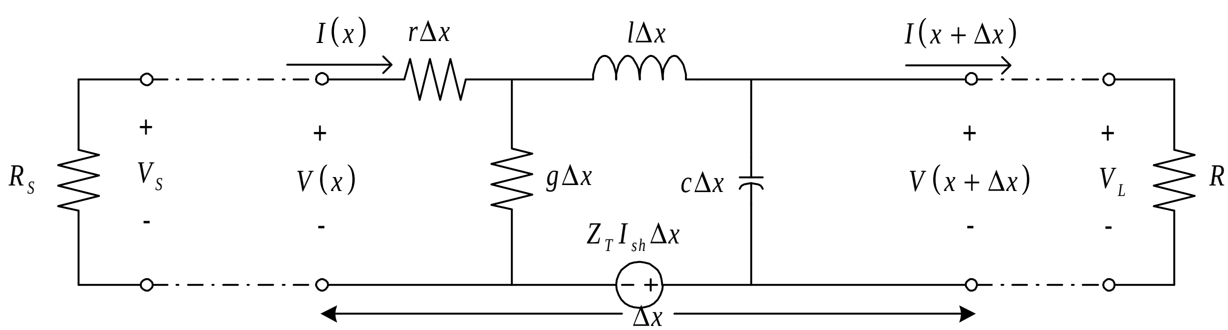
Below is a plot of the magnitude of the surface transfer impedance as a function of shield thickness. Both the surface transfer impedance and the shield thickness are normalized. The normalized surface transfer impedance is normalized to the per-unit-length dc resistance, $Z_T/R_0$ and the normalized shield thickness is normalized to the skin depth of the shielding material, $t_{sh}/\delta$. This figure illustrates that for shield thicknesses that are less than the skin depth, diffusion will occur. For shield that are greater than a skin depth thick, diffusion will not occur. Obviously, the skin depth of the material is frequency dependent, but the best way to prevent electromagnetic fields from interfering with a cable is to make sure the thickness of the shield will be greater than one skin depth of the material at frequencies of the incident fields the cable will likely encounter




If the shield thickness is less than a skin depth the shield current $I_{sh}$ diffuses through the shield wall with surface transfer impedance $Z_T$, resulting in a voltage drop on the interior of the shield

$$dV = Z_T I_{sh} dx$$

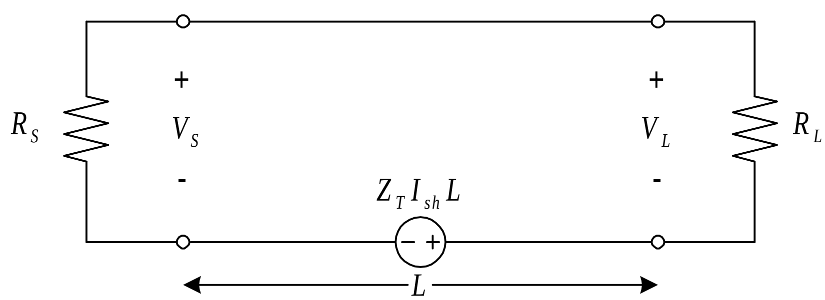
This voltage drop on the interior of the shield acts as a voltage source along the longitudinal interior surface of the shield. Thus, we can create a per-unit-length model of a shielded cable that includes per-unit-length resistance *r*, inductance *l*, capacitance *c*, and conductance *g* of the shield, as well as the induced voltage source due to the impressed EM field.




The coupled transmission line equations describing this per-unit-length equivalent circuit are

$$\frac{dV(x)}{dx} + (r + j\omega L)I(x) = -Z_T I_{sh}$$
$$\frac{dI(x)}{dx} + (g + j\omega c)V(x) = 0$$

For an electrically short line, we can lump the distributed source ZTIsh, and as long as the terminal impedance values are not extreme values, such as short or open circuits, the per-unit-length parameters of the inner-wire shield circuit can be ignored. Thus, for electrically short lines, the equivalent circuit becomes




and the transmission line equations for this circuit are

$$V_S = \frac{R_S}{R_S + R_L} Z_T I_{sh} L$$
$$V_L = \frac{-R_L}{R_S + R_L} Z_T I_{sh} L$$

These simple uncoupled equations allow us to quickly compute approximate noise voltages induced on shielded cables due to incident EM fields.

**-braided shields**

Not all shields are cylinders of unbroken conductor. In fact, shields are often constructed from braids of wire woven in a herringbone patter to give flexibility. Braided shields are slightly more susceptible to electromagnetic radiation, since the EM fields are capable of penetrating through the small gaps between braids. We will examine the EM susceptibility of cables with braided wire shields here.

For braided shields, the surface transfer impedance is given by

$$Z_T = R_0^{bw} \frac{\gamma 2 r_{bw}}{\sinh(\gamma 2 r_{bw})} \quad \text{(in } \Omega\text{/m)}$$

Note that this is very similar to the surface transfer impedance for a continuous shield cable except that the thickness of the shield is replaced by $2r_{bw}$, which is the diameter (2 times the radius) of the wires that make up the braided shield. If we consider these wires to be simply connected electrically in parallel, the per-unit-length dc resistance can be approximated as

$$R_0^{bw} = \frac{r_b}{BW\cos\theta_w} \quad \text{(in } \Omega\text{/m)}$$

where $B$ is the number of belts in the shield braid, $W$ is the number of wires per belt, $\theta_w$ is the weave angle of the belts, and $r_b$ is the per-unit-length dc resistance of the braid wires

$$r_b = \frac{1}{\sigma\pi r_{bw}^2} \quad \text{(in } \Omega\text{/m) for } r_{bw} << \delta$$

Knowing $Z_T$ for braided shields allows us to compute the diffusion source $Z_T I_{sh}$ induced by incident fields. However, the gaps between the braids in the braided shield will also allow external field penetration to the center conductor. We must also account for the noise caused by this penetration through the gaps in the braided bands.

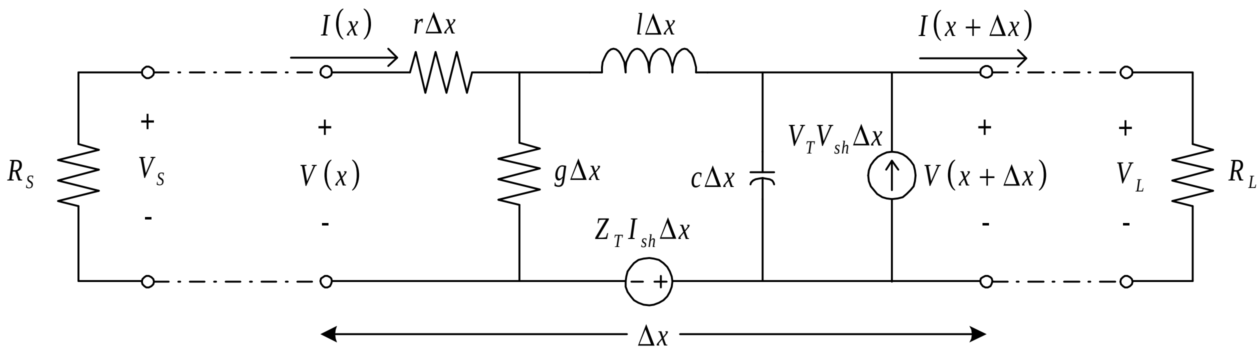
The magnetic field of the incident EM wave penetrates through the gaps in the braided belts, resulting in a per-unit-length inductance $m_{12}$. This inductance adds in series with the surface transfer impedance, so it is convenient to add it to this term. Thus, the serface transfer impedance now becomes

$$Z_T = R_0^{bw}\frac{\gamma 2r_{bw}}{\sinh\left(\gamma 2r_{bw}\right)} + j\omega m_{12} \quad \text{(in } \Omega\text{/m)}$$

The electric field of the incident EM wave also penetrates through the gaps in the bands of the braide shield, resulting in a per-unit-length mutual capacitance. This causes the equivalent circuit to also have a surface transfer admittance

$$Y_T = j\omega c_{12}$$

thus a parallel current source must be added to the per-unit-length equivalent circuit. To determine the value of this current source, the voltage $V_{sg}$ between the shield and the ground plane must be computed, much like the surface current needed to be computed in order to determine the per-unit-length voltage source $Z_T I_{sh}\Delta x$. The equivalent per-unit-length circuit for braided shield cables is then

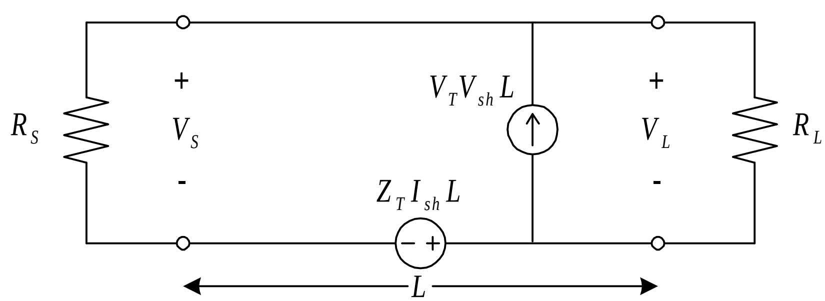



The coupled transmission line equations for this per-unit-length circuit are

$$\frac{dV(x)}{dx} + (r + j\omega L)I(x) = -Z_T I_{sh}$$

$$\frac{dI(x)}{dx} + (g + j\omega c)V(x) = Y_T V_{sh}$$

Typically, the surface transfer admittance $Y_T$ can be neglected, except for very large termination impedances. However, we will simplify this model and continue to include $Y_T$. For electrically short lines, with non-extreme terminal impedance values, the circuit model becomes




The transmisson line equations for this circuit are

$$V_S = \frac{R_S}{R_S + R_L} Z_T I_{sh} L - \frac{R_S R_L}{R_S + R_L} Y_T V_{sh} L$$

$$V_L = \frac{-R_L}{R_S + R_L} Z_T I_{sh} L - \frac{R_S}{R_S + R_L} Y_T V_{sh} L$$

## References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3. Hoeft, L.O. and Hofstra, J.S., *Measured Electromagnetic shielding Performance of commonly Used Cables and Connectors*, IEEE Trans. On Electromagnetic Compatibility, EMC-30, 260-275, 1988

4. Vance, E.F., *Shielding Effectiveness of Braided-Wire Shields*, IEEE Trans. On Electromagnetic Compatibility, EMC-17, 71-77, 1975