ME 417

Design of Alternative Energy Systems

Wind Energy Calculations

Effect of Elevation on Wind Speed

To calculate the wind speed at one height, if it is known at another height, we can use

$$\vec{v}_2 = \vec{v}_1 \left(\frac{h_2}{h_1} \right)^n$$

where n is the ground surface friction coefficient and takes on different values according to the nature of the terrain. Some example values are

water or smooth flat ground: $n = 0.1$
tall crops: $n = 0.2$
city downtown: $n = 0.4$

Wind Power

The power of the wind can be calculated from

$$\dot{W} = \frac{1}{2} \rho A \vec{v}^3$$

where

ρ: density of the air
A: capture area of the wind
$\vec{v}$: wind speed

Wind Turbine Efficiency

The efficiency of a wind turbine is defined as

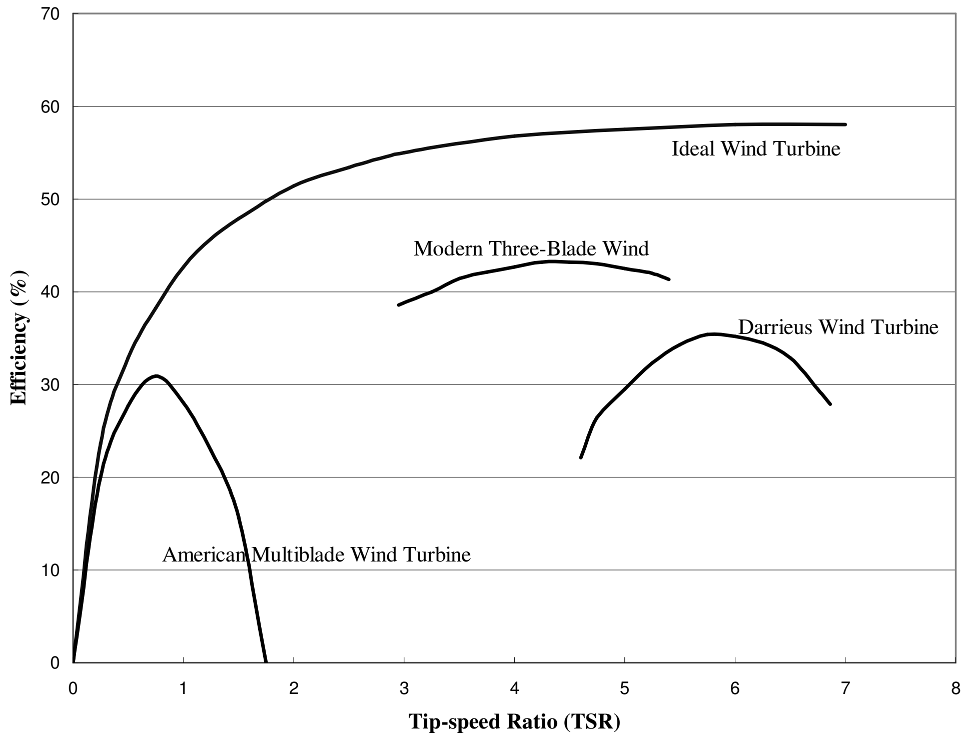
$$\eta_{wt} = \frac{\text{wind turbine power produced}}{\text{wind power}} = \frac{\dot{W}_{wt}}{(0.5)\rho A \vec{v}^3}$$

Then the power of a wind turbine is given by

$$\dot{W}_{wt} = \eta_{wt} \frac{1}{2} \rho A \vec{v}^3$$

The efficiency for several different types of wind turbines is given in the figure below.

Figure: Wind Turbine Efficiencies



The tips speed ration (TSR) is the ratio of the speed at the tip of the wind turbine blade to the wind speed and is given by

$$TSR = \frac{\omega R_{rotor}}{\vec{v}}$$

where

ω: rotational speed of the turbine rotor
R_{rotor}: radius of the rotor
$\vec{v}$: wind speed

Curve fit equations for this graph have been developed and are given below.

American Multiblade Wind Turbine

$$\text{for } TSR \le 1.75: \; \eta_{wt} = -0.39105(TSR)^2 + 0.66586(TSR) + 0.026583$$

$$\text{for } TSR > 1.75: \; \eta_{wt} = 0$$

Darrieus Wind Turbine

$$\text{for } TSR < 4.6: \; \eta_{wt} = 0$$

$$\text{for } 4.6 \le TSR \le 6.86: \; \eta_{wt} = -0.078369(TSR)^2 + 0.92146(TSR) - 2.3532$$

$$\text{for } TSR > 6.86: \; \eta_{wt} = 0$$

Modern Three-blade Wind Turbine

for $TSR < 2.95$: $\eta_{wt} = 0$

for $2.95 \le TSR \le 5.4$: $\eta_{wt} = -0.020554(TSR)^2 + 0.18327(TSR) + 0.023286$

for $TSR > 5.4$: $\eta_{wt} = 0$

Ideal Wind Turbine

for $TSR < 0.5$: $\eta_{wt} = 0.658(TSR) + 0.023833$

for $0.5 \le TSR < 1.0$: $\eta_{wt} = 0.196(TSR) + 0.23233$

for $1.0 \le TSR < 1.5$: $\eta_{wt} = 0.104(TSR) + 0.32433$

for $1.5 \le TSR < 2.5$: $\eta_{wt} = 0.055(TSR) + 0.399$

for $2.5 \le TSR < 4.0$: $\eta_{wt} = 0.022(TSR) + 0.481$

for $TSR \ge 4.0$: $\eta_{wt} = 0.0041(TSR) + 0.5532$

To calculate the rotation speed, ω, we equate the mechanical power of the turbine due to rotation with the wind power that is captured by the turbine or

$$\eta_{wt} \frac{1}{2} \rho A \vec{v}^3 = \frac{1}{2} I_{shaft} \omega^3$$

where I_{shaft} is the moment of inertia of the rotor about the rotating shaft. If we assume that blades are a parallelepiped (solid rectangle) then for our HAWT we have

$$I_{shaft} = \frac{N_B \rho_B (L_B W_B t_B) L_B^2}{3}$$

where

N_B: number of blades

ρ_B: density of blade material

L_B: length of blade

W_B: width of blade

t_B: thickness of blade

For the Darrieus wind turbine, we will have a slightly different expression

$$I_{shaft} = N_B \rho_B (L_B W_B t_B) R_{rotor}^2 + \frac{N_B \rho_B (L_B W_B t_B)\left(W_B^2 + t_B^2\right)}{12}$$

Recognizing that

$$\eta_{wt} = fn(TSR) = fn\left(\frac{\omega L_B}{\vec{v}}\right)$$

we see that we have two equations and two unknowns that must be solved iteratively for the rotational speed and wind turbine efficiency.

A possible iterative loop is

Guess η_{wt}

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Calculate ω

$$\omega = \left[\frac{\eta_{wt} \rho A \vec{v}^3}{I_{end}} \right]^{1/3}$$

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Calculate TSR

$$TSR = \frac{\omega L_B}{\vec{v}}$$

↓

Determine η_{wt} from graph

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Check η_{wt} for convergence