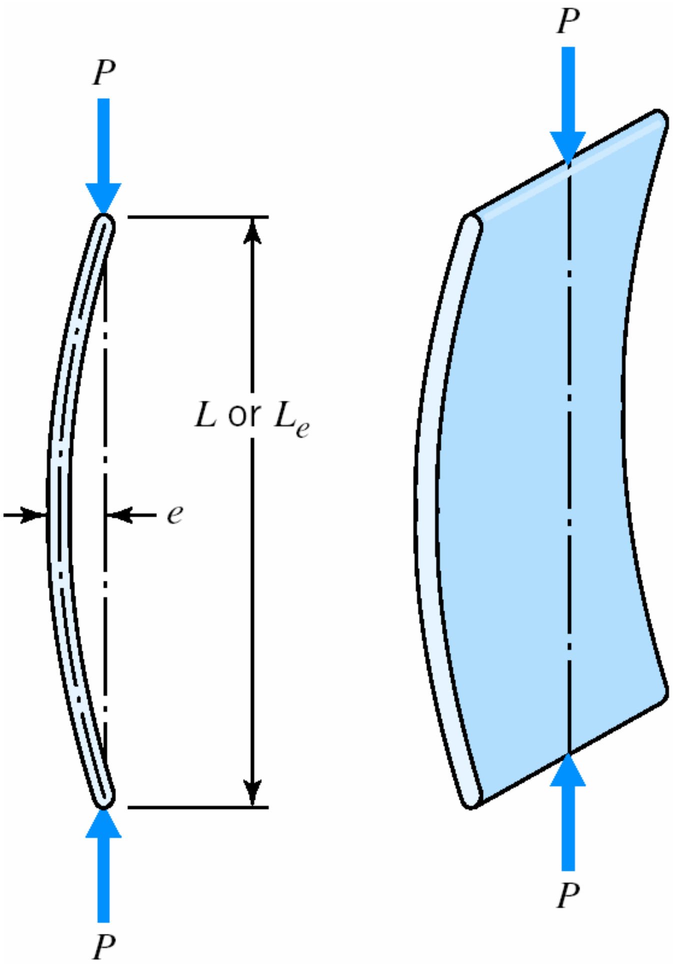
# Elastic Instability - Buckling

- *Buckling* does not vary linearly with load
  - it occurs suddenly and is therefore dangerous




*Elastically Stable*

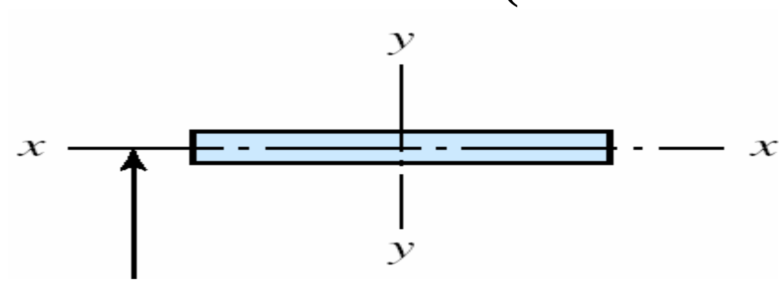
$$P < P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

***Elastically Unstable***

$$P > P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

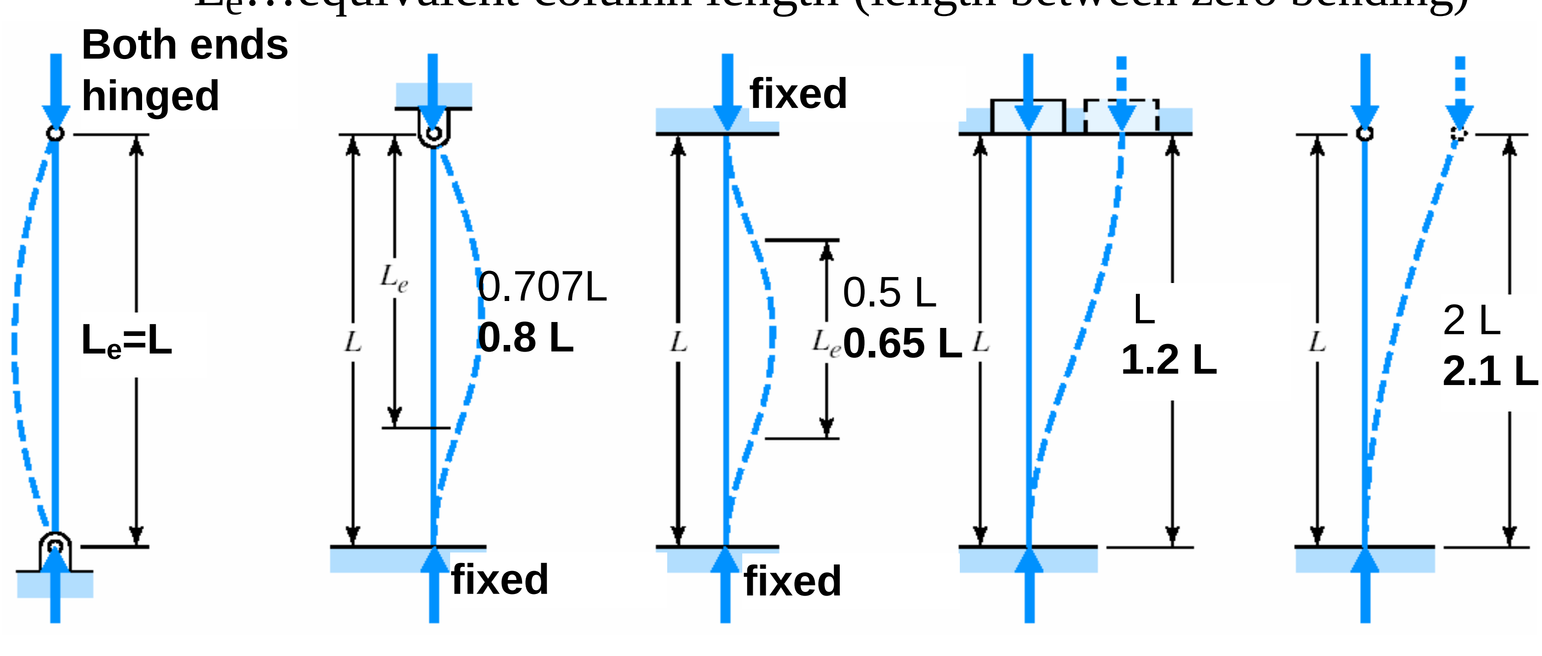
E …modulus of Elasticity

I …moment of inertia (smallest value)




Axis of least I and ρ becomes neutral bending axis

$L_e$…equivalent column length (length between zero bending)

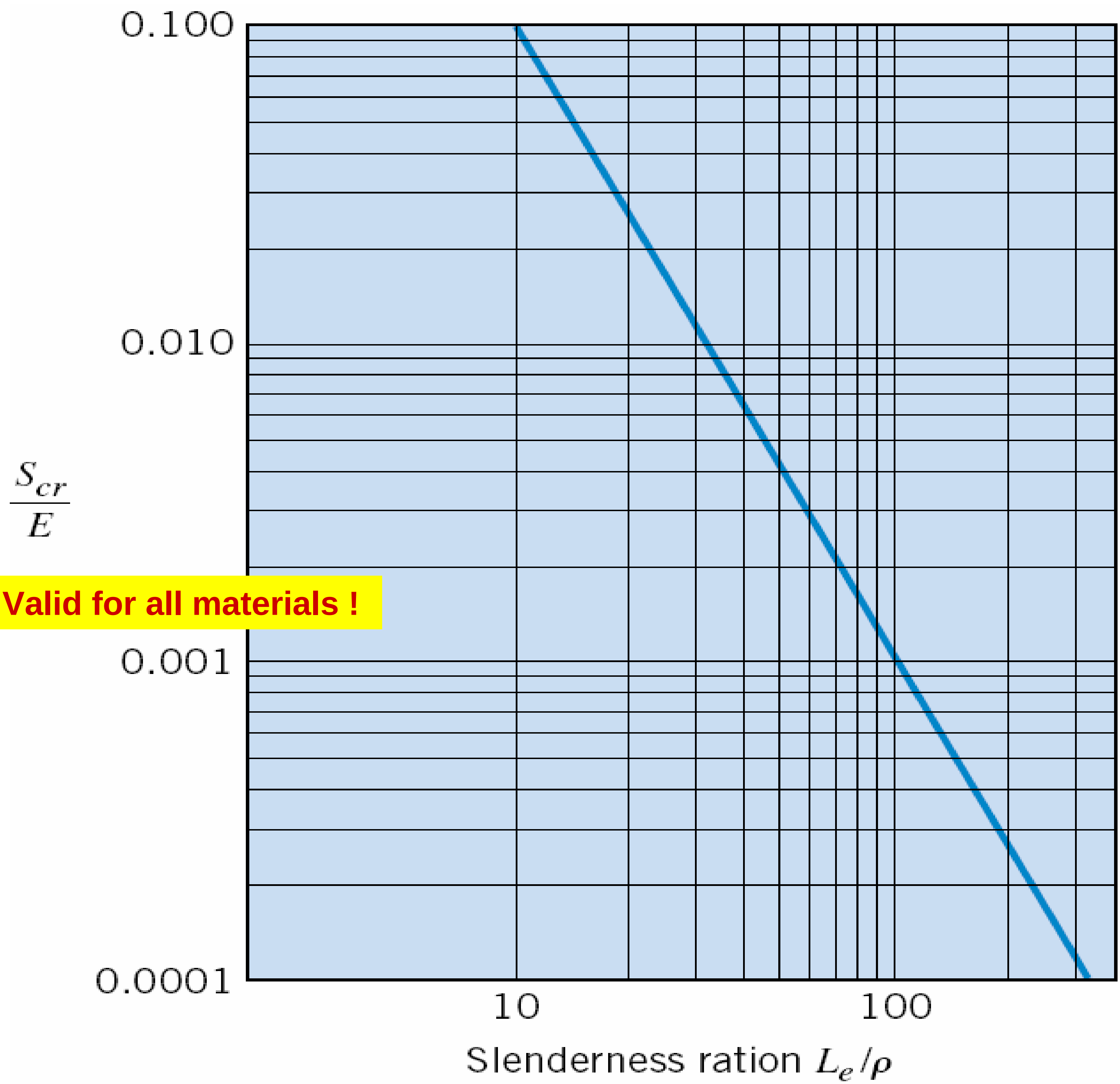


***Elastically Unstable***

$$P > P_{cr} = \frac{\pi^2 EI}{L_e^2} \qquad \text{with } I = A\rho^2 \qquad S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

$\rho$ …radius of gyration

$L_e/\rho$…slenderness ratio




**Valid for all materials !**

*J.B.* ***Johnson Parabola*** (1900)

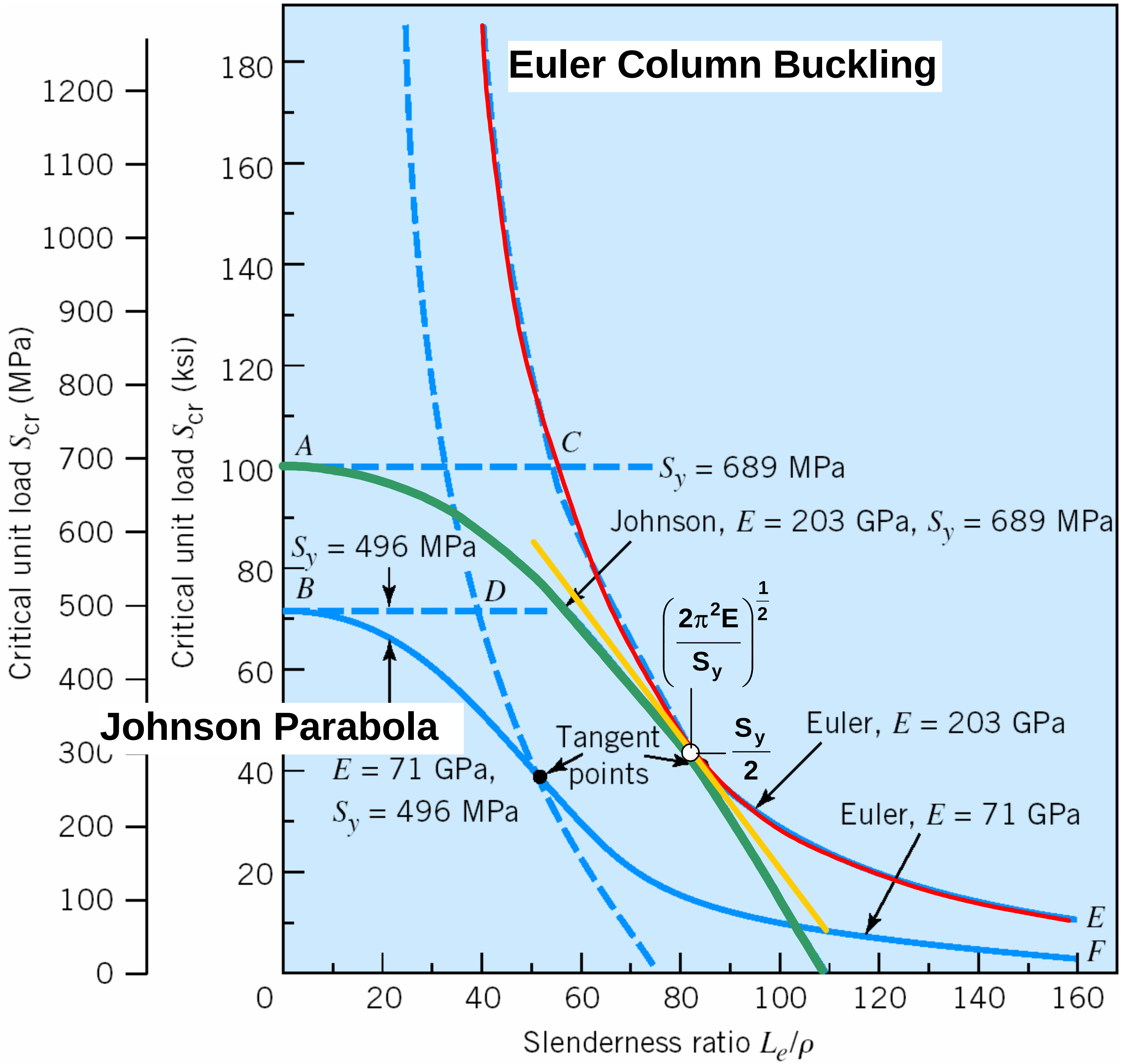
$$P > S_{cr} = S_y - \frac{S_y^{\,2}}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2$$

***Euler Buckling***

$$S_{cr} = \frac{\pi^2 E}{\left(L_e/\rho\right)^2}$$

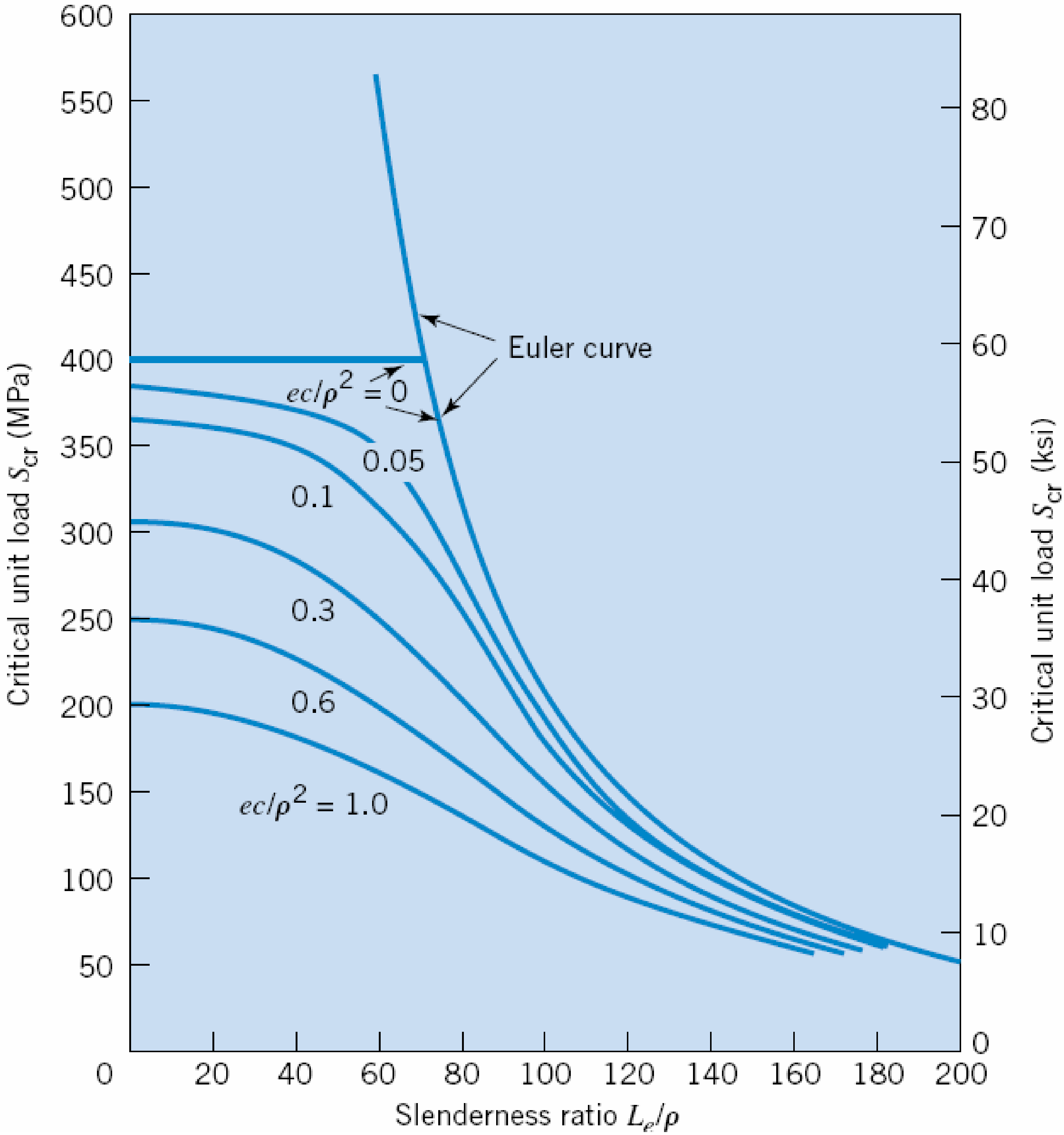
$\rho$ …radius of gyration

$L_e/\rho$…slenderness ratio



# Excentric Loaded Column

**$e=L_e/400$ or $ec/\rho^2=0.025$ is recommended**



# Equivalent Column Stress

$$S_{cr} = S_y/\alpha \qquad \alpha\text{…stress multiplier}$$

***J.B.Johnson Parabola***

$$P > S_{cr} = S_y - \frac{S_y^{\,2}}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2$$

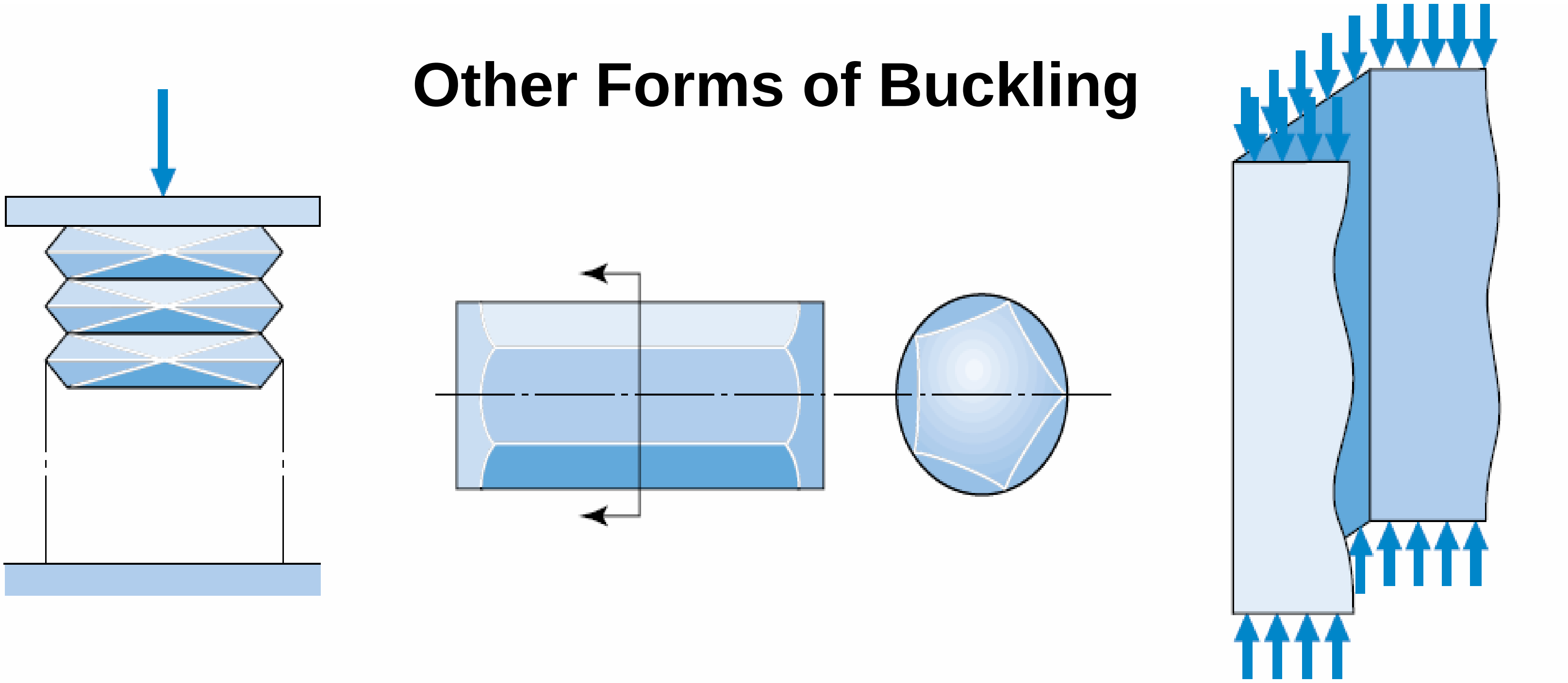
$$\alpha = \frac{S_y}{S_{cr}} = \frac{4\pi^2 E}{4\pi^2 E - S_y(L_e/\rho)^2}$$

***Euler Buckling***

$$S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

$$\alpha = \frac{S_y}{S_{cr}} = \frac{S_y(L_e/\rho)^2}{\pi^2 E}$$

# Other Forms of Buckling



(a) Wrinkling, or "accordian buckling" of thin-wall tube

(b) Typical local buckling of an externally pressurized thin-wall tube

(c) Wrinkling of thin, unsupported flanges of a channel section