PRINT-HEAD MECHANISM.

(a) The system model is non-linear, because of "spring 2" (platen and hard stop).

(b) System eqns ~

\[ m \ddot{x} = F_4 - k_1 \dot{x} - b_5 \cdot \dot{v} - F_2 \]
\[ \dot{x} = \dot{v} \]
\[ F_2 = \phi (x) \quad (\text{see figure for spring 2}) \]

EP for \( F_4 = F_4 \), constant.

\[ 0 = F_4 - k_1 \dot{x} - b_5 \cdot \dot{v} - F_2 (x) \]
\[ 0 = \dot{v} \]

For \( \dot{x} = 1 \), \( F_4 = k_1 \dot{x} (x) + F_2 (x) \)

Since \( F_2 (x) = 0 \) from figure,

\[ F_4 = 2.0 \quad \leftrightarrow \text{Force} \]
(c) Let $F_\text{eq} = 0.5 \times F_\text{crit} = 1.0$, constant.

$$V = \frac{1}{2} (1.0 - 2.0 \times x - 0.2 \times v - F_2(x))$$

$$\dot{x} = v$$

If $(-1 \leq x \leq +1)$ during response, then $F_2(x) = 0$. Assume this is true.

$$E\text{P}$$

$$0 = \frac{1}{2} (1 - 2 \times x^2 - 0.2 \times v)$$

$$0 = \dot{x}$$

So $\dot{x} = 0.5$

$$\{\text{Reasonable, since } \dot{x} \text{ for } F_\text{eq} = 2 \text{ was } 1.0\}$$

$$A = \begin{bmatrix} -\frac{b_5}{m} & -\frac{b_4}{m} \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -0.1 & -1 \\ 1 & 0 \end{bmatrix} \text{ then } |2.5 A| = \begin{vmatrix} 2.5 & 0.1 \\ -1 & 0 \end{vmatrix}$$

$$\text{CP}(x) = x^2 + 0.12x + 1 \Rightarrow \lambda_{i,2} = -0.05 \pm 0.999$$

Stable, very light damping.
(d) If $F_q = 0.98 \times F_{crit} = 1.96$, then

$$\hat{x} = 0.98 \quad \text{(Very close to +1)}$$

Any motion for $x$ that exceeds $0.02$ will make the system non-linear.

Surely, with light damping and an initial state of $(x_0 = 0, v_0 = 0)$, $x$ will travel past $x = 0.98 \pm 0.02$.

So a linear model will not do for this case.

(e) For $EP$ at $(\hat{x} = 1.5, \hat{v} = 0)$, spring 2 is active.

$$0 = F_q - 2.0 \cdot \hat{x} - 20 \cdot (1.5 - 1.0)$$

So $F_q = 13$ to make $\hat{x} = 1.5$.

In the vicinity of $EP$, both springs are active. Combined $K'$ is $k_1 + k_2 = 2 + 20 = 22$.

Combined

$$m \ddot{v} = F_q - k' x - b \dot{v}$$

$$\dot{x} = v$$

$$\begin{cases} F_q = 13 \\ k' = 22 \end{cases}$$
(e) cont'd. Near $(x=0.5, v=0)$ ~ 

$$A = \begin{bmatrix} -0.1 & -1 \ 1 & 0 \end{bmatrix} \quad | \lambda I - A | = \begin{vmatrix} \lambda + 0.1 & 1 \\ -1 & \lambda \end{vmatrix}$$

$$CP(x) = \lambda^2 + 0.1\lambda + 11 \implies \lambda_{1,2} = -0.05 \pm 3.31\imath$$

Stable, lightly damped with higher frequency of oscillation than in (c).
Pin Printer example.

The state vector is $X' = \{ v, x \}$ (velocity of pin, displacement of pin).
The input is force $F_4$.

Run 1. Verify the equilibrium point at $\{v=0.0, x=1.0\}$ for $F_4=2.0$.

Run 2. Examine behavior for $F_4=1.0$, $EP$ at $\{v=0.0, x=0.5\}$, and $X_{init} = \{ 0, 0 \}$. 
Run 3. Examine behavior for $F_4 = 13.0$ (EP at $v=0, x=1.5$).
Matlab script files to simulate the pin printer system.

System equations file.

```matlab
function xdot= PrinterEqns(t,x)
% This file defines a set of system equations for the pin-printer problem.
% In vector terms: xdot= f(x,u)
% X= [ v, x ]
%
% Parameters
m= 2.0;
x1= 2.0;
% spring 2 defined by nonlinear function
if (x(2)>1.0)
    F2= 2000.0*(x(2)-1.0);
elseif (x(2)>=-1.0)
    F2= 0.0;
else
    F2= 2000.0*(x(2)+1.0);
end
b5= 0.2;
%
% Inputs
% F4= 2.0;     % critical force for xp0 at 1.0
% F5= 1.0;     % critical force for xpin at 0.5
F4= 1003.0;   % critical force for xpin at 1.0
%
xdot(1)= (1/m)*(F4 -k1*x(2) -b5*x(1) -F2);
xdot(2)= x(1);
%
xdot= xdot';    % return xdot as a column vector
```

Setup file.

```matlab
% PrinterSetup.m  setup file for pin printer problem
% x= [ v, x ]
%
% ts= [0:0.01:2.0];  % set time vector
%
x0= [ 0.0; 0.0 ];  % set initial conditions
%
[t,x]= odes23('PrinterEqns',ts,x0);  % generate solution
%
vpin= x(:,1);
xpin= x(:,2);
for i=1:201
    F2(i)=0;
    if (xp0(i)>1.0)
        F2(i)-= 2000.0*(xp0(i)-1.0);
    elseif (xp0(i)>=-1.0)
        F2(i)= 0.0;
    else
        F2(i)= 2000.0*(xp0(i)+1.0);
    end
end
% plot (t,xpin,'k-',t,vpin,'k--');
plot(t,xpin,'k-',t,F2,'k--');
ttitle ('Run 8.5, EP at xpin= 1.5, k2= 2000.0');
xlabel('time');
ylabel('xpin,F2');
```

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