EXAM 4. Solutions.

(a) Bond graph model.

\[ A_b = A_a - \pi \left( \frac{dr}{2} \right)^2. \]

(b) System eqns. Inputs: \([Q_a, P_{\text{atm}}]\). States: \([P_a, v, P_b]\).

\[ Q_a = Q_{fa} + A_a \cdot v \]
\[ M_L v = A_a P_a - A_b P_b - b_L \cdot v \]
\[ Q_b = A_b v = Q_{fb} + Q_v \]
\[ \Delta P_v = P_b - P_{\text{atm}} \]
\[ R_v = \Delta P_v / \dot{Q}_v \]
\[ C_a = Q_{fa} P_a = Q_{fa} \]
\[ C_b = Q_{fb} P_b = Q_{fb} \]

7 Eqs

7 Unknowns
(4-1.2) State equation format.

\[ M_L \ddot{V} = A_a P_a - A_b \dot{P}_b - b_L \dot{V} \quad \text{div. by } M_L \]
\[ C_a \dot{P}_a = \dot{Q}_a - A_a \cdot \dot{V} \quad \text{div by } C_a \]
\[ C_b \dot{P}_b = A_b \dot{V} - \frac{1}{R_V} (P_b - P_{atm}) \quad \text{div by } C_b \]

(d) 3 eigenvalues.

Solve \( |SI - A| = 0 \) to find them.
Stable if all real parts negative.

(e) \[ 0 = A_a \dot{P}_a - A_b \dot{P}_b - b_L \dot{V} \]
\[ 0 = \dot{Q}_a - A_a \dot{V} \]
\[ 0 = A_b \dot{V} - \frac{1}{R_V} \dot{P}_b + \frac{1}{R_V} P_{atm} \]
\[ \frac{1}{R_V} \dot{P}_b = R_V A_b \left( \frac{1}{A_a} \right) \dot{Q}_a + \frac{1}{R_V} P_{atm} \]
\[ \frac{\dot{P}_a}{A_a} = \left( \frac{1}{A_a} \right) \frac{A_b \dot{P}_b + b_L \dot{V}}{A_a} \quad \text{subst. from (1) and (2).} \]
EXAM 4. Solution. (4.2)

\[ \begin{align*}
& \quad (S_p) \quad \frac{P_{hi}}{Q_{hi}} \downarrow \quad \frac{\delta P}{D_p \omega} \quad (S_p) \quad \frac{T_p}{R_m} \quad \frac{T_m}{\omega} \quad (S_w) \\
& \quad (S_p) \quad \frac{P_{lo}}{Q_{lo}} \\
\end{align*} \]

**Inputs:** \( P_{hi}, P_{lo}, w \)

**Outputs:** \( Q_{hi}, Q_{lo}, T_m \)

(a)
1. \( P_{hi} - P_{lo} = \Delta P \)
2. \( Q_{hi} = D_p w \)
3. \( Q_{lo} = D_p w \)

**TF:**
1. \( T_p = D_p \cdot \Delta P \)
2. \( T_p = R_m w + T_m \)

SEES

NO state variables.
Equations are linear algebraic.

(b)
1. \( Q_{hi} = D_p \cdot w \)
2. \( Q_{lo} = D_p \cdot w \)
3. \( \frac{T_m}{T_p} = \frac{P_{hi} - P_{lo}}{D_p} - R_m w \)
(4.2c) If $P_{lo} = 0$, then $T_n = D_p \cdot \Phi_i - R_{mw} \cdot w$.

$$
\eta = \frac{(T_m \cdot w)}{\Phi_i \cdot \Phi_i - P_{lo} \cdot P_{lo}} = \frac{T_m \cdot w}{\Phi_i \cdot \Phi_i}
$$

$$
\eta = \frac{(D_p \cdot \Phi_i - R_{mw} \cdot w) \cdot w}{\Phi_i \cdot D_p \cdot w} = \frac{D_p \cdot \Phi_i - R_{mw} \cdot w}{\Phi_i \cdot D_p}
$$

$$
\eta = 1 - \frac{R_{mw} \cdot w}{\Phi_i \cdot D_p} < 1 \quad \text{for} \ w > 0.
$$

as expected.
EXAM 4.

(4-3) SOLUTION

(a) \[ C_L \cdot \dot{q}_1 = Q_A - Q_1 \]

\[ I_L \cdot \dot{q}_1 = P_1 - P_L \cdot Q_1 - P_B \]

\[ I_L \cdot \dot{q}_1 = 0 \cdot P_1 - 1 \cdot Q_1 + 1 \cdot Q_A \]

\[ I_L \cdot \dot{q}_1 = 1 \cdot P_1 - R_L \cdot Q_1 - 1 \cdot P_B \]

(b) \[ C_L \cdot \dot{q}_1 = O - 1 \]

\[ I_L \cdot \dot{q}_1 = C_1 \]

\[ I_L \cdot \dot{q}_1 = C_2 \]

\[ I_L \cdot \dot{q}_1 = C_3 \]

\[ R_L = R_1 = R_2 = R_3 \]

(c) 3-lump model has 6 evls.
If \( R_L \rightarrow 0 \), then no real parts of evls.
Get \( \pm j\omega_1, \pm j\omega_2, \pm j\omega_3 \).