Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48824-1226

ME451 Laboratory

TORSIONAL CONTROL DESIGN PROJECT WEEK 2

Bring the course textbook to the Lab
'C.L.Philips and R.D.Harbor, Feedback Control Systems'

ME451 Laboratory Manual Pages, Last Revised: April 10, 2010
Send comments to: Dr. Clark Radcliffe, Professor

Introduction:

The ability to accurately control a process is vital to high technology product performance. Whirlwind Corporation is primarily interested in the potential performance increase that can be obtained in its new residential *WindDrive* electric generator product line. Reduction in vibration transmitted to the transmission shaft and increased efficiency due to lower controller effort are potential benefits Whirlwind would like to achieve. Also, precise positioning and vibration control is necessary for proper functioning of the vane angle control and subsystems. Other application examples include the automobile suspension system or a laser positioning system for machining. In the second experiment you and your colleagues verified the mathematical model that governs the response of a MSD system. Your task this week is to design a torsion control system. After finalizing the design we will have the design tested by Whirlwind technicians to see if it meets performance specifications.



Figure1: The *WindDrive*



Figure 2: The Supported *WindDrive* Generator Module

The Wind Drive Electric Generator

The structure of our *WindDrive* wind powered electric generator will be subject to torsional vibration. The generator module is supported on a tall support tube (Fig. 1). The generator module on the top of this tube is massive (Fig. 2). The support structure will be subject to torsional vibration from the interaction between the rotating generator blades and the support tube. This rotational vibration will subject the generator's support bearings to fatigue loading substantially reducing bearing life. We propose to mount an inertial torque actuator within the generator module to control torsional vibration.

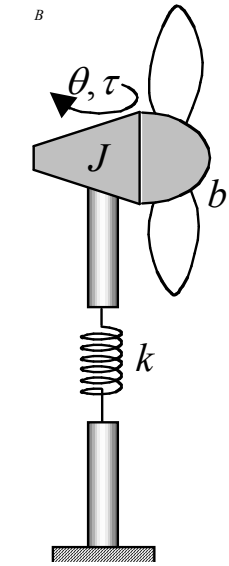


Figure 3: Schematic Diagram of *WindDrive* Support Structure

A simple schematic torsional vibration model (Fig. 3) indicates the important components contributing to structural torsional vibration. The equations of motion for this system is

$$J\ddot{\theta} + b\dot{\theta} + k\theta = \tau \qquad (1)$$

where J is the rotational inertia of the generator module, B is the small damping provided by air friction on the generator blades and body and K is the torsional stiffness of the support tube.

This equation of motion generates the same transfer function form as that used to describe the dynamics of the torsional vibration experiment you have used in the laboratory

$$G(s) = \frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + bs + k} \tag{2}$$

Because the *WindDrive* and torsional vibration apparatus have models of the same form, the research team will use the torsional vibration apparatus to investigate vibration control designs for our new2 product.

Objective:

The objective of this project is to design a PD (Proportional-Derivative) controller for improving the transient response of a second order system. The second order system in this case is the Mass Spring Damper system used in experiments 2 and 5. This system will be used to model the support dynamics of the *WindDrive* system. The concept of root locus described in the controls textbook is an effective method for designing the PID class of controllers. In this project we will focus on the PD controller.

Torsional MSD System and the PD Controller:

The torsional control system has the block diagram

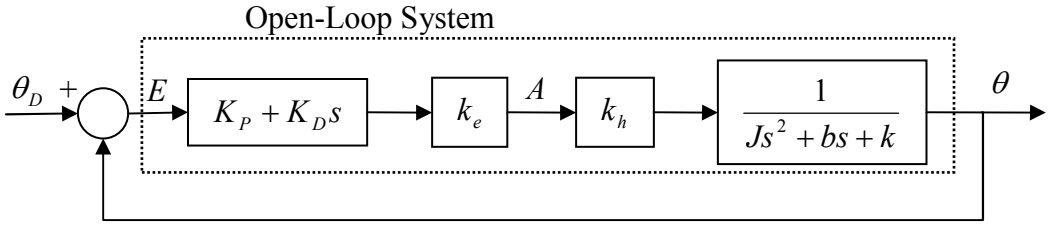


Figure 4: Closed-Loop Torsional Control Block Diagram showing the Open-Loop System Blocks as a Subsystem of the Closed-Loop System Block Diagram.

The parameters in the system are:

Table 1: Torsional Vibration System Parameters

Parameter	Value (Units)
J	0.008 ± 0.002 (kg-m^2)
b	0.025 ± 0.015 (N-m-sec.)
k	1.6 ± 0.2 (N-m/rad)
k_e	25 (volt/rad)
k_h	0.8 (N-m/volt)

The design of a controller for systems like the one above (Fig. 4) is discussed in Section 7.10 of the Phillips and Harbor *Feedback Control Systems* text. In this section, control design is performed on the open-loop transfer function defined when the comparator

connection is broken in Figure 4 above. When the parameters above are substituted into the block diagram above, the *open-loop transfer function*

$$kGH = \frac{\theta(s)}{\theta_D(s)} = \frac{\left(K_P + K_D s\right)k_e k_h}{Js^2 + bs + k} \tag{3}$$

This open-loop transfer function depends on the controller parameters, K_P and K_D, assumed to be positive numbers. The open-loop transfer function can take parameter values anywhere in the indicated ranges. Although the design of a stable control system response is performed on the open-loop transfer function, the closed-loop response of controlled system will be verified with analytical prediction of closed-loop response using the *Closed-Loop Transfer function*

$$\begin{aligned} T(s) = \frac{\theta(s)}{\theta_D(s)} = \frac{kG}{1 + kGH} &= \frac{\left(\dfrac{\left(K_P + K_D s\right)k_e k_h}{Js^2 + bs + k}\right)}{\left(1 + \dfrac{\left(K_P + K_D s\right)k_e k_h}{Js^2 + bs + k}\right)} \\ &= \frac{\left(K_P + K_D s\right)k_e k_h}{Js^2 + \left(K_D k_e k_h + b\right)s + \left(K_P k_e k_h + k\right)} \end{aligned} \tag{4}$$

This closed-loop transfer function again depends on the controller parameters, K_P and K_D, assumed to be positive numbers. The closed-loop response of controlled system will be verified with analytical prediction of closed-loop response using this *Closed-Loop Transfer function.*

Whirlwind Torsion Control Design:

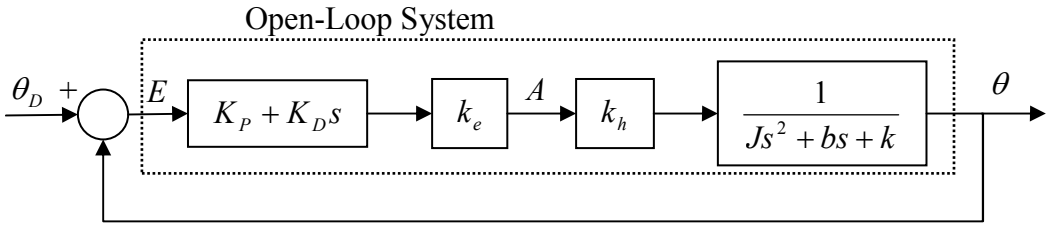
We will first investigate the case of Proportional Control for designing a controller for the mass-spring-damper system that will be used to model the *WindDrive* transmission shaft [Phillips & Harbor, section 7.10.1]. Finally, we will investigate Proportional and Derivative Control [Phillips & Harbor, 7.10.3]. Whirlwind lab consultants will test your design to see if they produce acceptable results.

Torsional Control Design Project – Week 2

Refer to the Torsional MSD system and the PD controller section for the Block Diagram and the values of the parameters of the system.

Name: **Section:** **Date:**

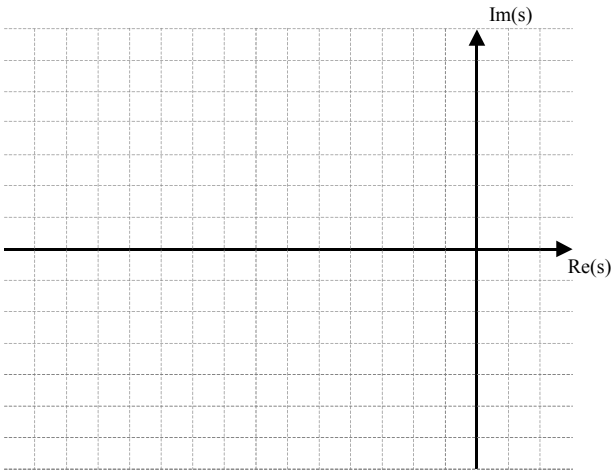
The torsional control system has the block diagram



PART I. Proportional Controller Design ($K_D = 0$)

1.1. For this system, write the "**open-loop**" transfer function (3) in the standard form for root-locus design with a single free gain K_P using the numerical parameter values in Table 1 above.

1.2. Roughly sketch the system's root locus for $0 \le K_P \le \infty$.



1.3. Is the closed-loop system stable? Does it have a reasonable closed-loop response as K_P gets large? Explain.

1.4. The Whirlwind technical staff reports that with the current system, the closed-loop system goes unstable at a frequency of 4.3 Hz or 27 rad/sec. This causes a lower mean time before failure in the current line of electric generator support structures. The instability implies that there is apparently another pole in the system, which may be due to process delay (as we have studied during the experiment 6). Assume the extra pole can be added to the denominator of the OLTF as $(\tau_{delay}s+1)$. Use the angle criterion method to find the appropriate value for τ_{delay} to meet the conditions above. Use MATLAB to confirm your results. Attach the plot to the report. Where is the extra pole located?

delay time, $\tau =$ _____sec Proportional gain K_P **at** instability = _____

1.5. With the extra pole included in the system, find the open-loop transfer function.

1.6. With the extra delay pole included in the open-loop function, use MATLAB to simulate the unit-step response of the close-loop system with the time delay τ and proportional gain K_P = .20. Is the response unstable? Attach the plot to the report.

1.7. With the extra pole included in the open-loop function, use MATLAB to simulate the unit-step response of the closed-loop system with K_P = .26. Is the response unstable? Attach the plot to the report.

CASE II: Proportional-plus-Derivative (PD) Controller Design

A PD controller stabilizes the system so that it is not easily made unstable by a process delay. Your task is to design a robust PD controller (K_P, K_D) for the given system so that it produces a step response with

settling time (T_s) = 0.2 seconds, and

damping ratio ς = 0.707.

The following steps will guide you through the design process:

2.1. For this system, write out the "**open-loop**" transfer function. Factor out the derivative gain, K_D from the open-loop transfer function and rewrite it in standard form. Note the ratio K_P/K_D becomes a parameter in this open-loop transfer function. Also remember to include the new denominator term $(\tau_{delay}s+1)$.

You should use the parameters in Table 1 to define your new Open Loop Transfer function

2.2. What is the value of the desired closed-loop root location at the specified settling time and damping ratio?

s = _____

2.3. Roughly sketch the location of the poles and zeros for this system. Also, show the position of the closed-loop root you are designing for.

2.4. By applying the angle criterion, $angle(kGH) = \pm 180^o$, compute where you should place the 'zero' associated with the ratio K_P / K_D so that the root locus passes through the specified closed-loop pole from 2.3.

2.5. How are K_P and K_D related to the specified 'zero' calculated above?

2.6. With the 'zero' location obtained above, use MATLAB to accurately draw the root locus by varying K_D. and confirm your result. Label your root locus with your name and "2.6" and attach it to this report.

2.7. From the root locus, find the value of K_D at the specified closed-loop pole location.

2.8. What are your values for K_P and K_D?

$K_P =$ _____

$K_D =$ _____

2.9. With the obtained values for K_P and K_D, use the MATLAB script design2_step.m from the class website to predict the closed-loop step response for the torsional system. Note that the measured parameters J, b, and k have measurement error. Therefore you will get a family of step responses. Attach the plot to this report.

2.10. Take your controller design values to the Lab consultant for testing. Print the measured torsional response and attach it to this report.

2.11. Compare the measured torsional system response to your predicted response. Are they comparable? Does your measured response lie within the family of step response you plotted? How do the settling time, overshoot, and DC gain compare?

2.12. From the torsional response plot obtained, measure the steady state error.

2.13. What do you think is the next step in the design of the controller, in order to eliminate the steady state error.