Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48824-1226

ME451 Laboratory

PI CONTROL DESIGN PROJECT
WEEK 1

Bring the course textbook to the Lab
'C.L.Philips and R.D.Harbor, Feedback Control Systems'

NOTE: It maybe helpful to look at the following in the course textbook before coming to the lab:

Chapter 7:
Sections 7.1 to 7.5
Section 7.10



Figure 1: The *WindDrive* Residential Wind Powered Electric Generator Prototype

Introduction:

As stated in the Whirlwind memorandum of January 18, the Residential Wind Power Development Group is dedicated to the development of the new *WindDrive* wind powered electric generator for residential use. In the previous months we have investigated various techniques related to control systems analysis and design because we feel that the ability to accurately control a process is vital to high technology product performance. We have verified the mathematical model that governs the response of the DC servomotor, and are now ready to apply these tools to design a speed control system. We will test the control design process on the servomotor in lab because it is a good physical model to serve in place of the *WindDrive* electric generator. After you finalize your design, your design will be tested by Whirlwind technicians to see if it meets performance specifications.

The WindDrive Electric Generator

The *WindDrive* electric generator (Fig. 1) has an electrical voltage output $e_g(t)$ that is determined by a combination of wind speed $v_w(t)$ and propeller angle of attack $\alpha_g(t)$.

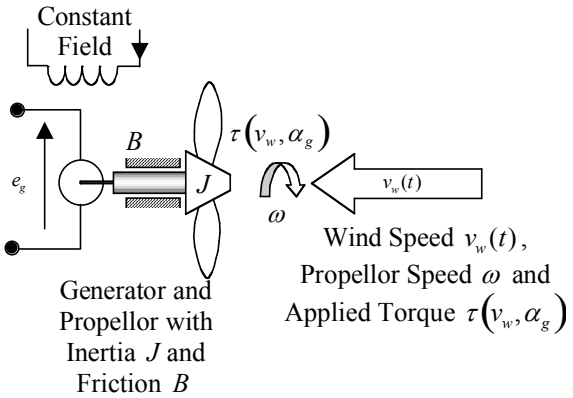


Figure 2. The WindDrive Wind Turbine

Lift Force L produces Torque

$$T = rL = k_{lift} v_w \alpha_{rel} = r k_{lift} v_w \left(\alpha_g - r\omega\right)$$

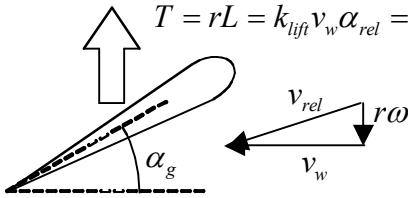


Figure 3. Angle of Attack

The turbine propeller blade airfoil rotates at speed $\omega = \dot{\theta}$ with angle of attack $\alpha_g(t)$ (Fig. 3). The propeller's effective angle of attack is reduced by the propeller's rotation speed ω and determines the Lift force L and torque T on the generator. This torque generated is proportional to the product of this angle and the wind velocity.

As the wind speed varies, we can use feedback control (Fig. 2) of propeller angle of attack $\alpha_g(t)$ to control the system's electrical voltage output $e_g(t)$.

Instead of the voltage being regulated to control motor speed, the wind turbine angle of attack α will be regulated to control the generator speed and resulting voltage. The torque applied to the generator,

$$T = rL = k_{lift} v_w \alpha_{rel} = K_{wind} \left(\alpha_g - r\omega\right) \tag{1}$$

where $K_{wind} = k_{lift} v_w$ is a wind speed dependent coefficient for the airfoil and r is the radius of the center of lift on the propeller blades. Following the derivations in the text [Phillips and Harbor section 2.72], the wind generator rotation speed ω produces the electrical potential

$$e_g = K_g \omega \tag{2}$$

through the generator's emf constant K_g. The generator's speed is modeled by

$$J\dot{\omega} + B\omega = T \tag{3}$$

Taking the Laplace Transform of (1)&(3) and substituting (1) into (3) yields the system model,

$$\left(Js + B + rK_{wind}\right)\Omega(s) = K_{wind}\alpha_g(s) \tag{4}$$

Now using this result in the Laplace transform of (2) yields the system model

$$E_g(s) = \left[\frac{K_g K_{wind}}{\left(Js + B + rK_{wind}\right)}\right]\alpha_g(s) \tag{5}$$

This model has the transfer function

$$G(s) = \frac{E_g(s)}{\alpha_g(s)} = \left[\frac{K_g K_{wind}}{(Js + B + rK_{wind})} \right] \tag{6}$$

This transfer function is a standard 1st order form

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{(\tau s + 1)} \tag{4-2) = (7}$$

where $C(s) = E_g(s)$ and $R(s) = \alpha_g(s)$ are the Laplace transforms of the output and input variables, respectively, the DC gain $K = (K_g K_{wind}) / (B + rK_{wind})$, and the time constant $\tau = J / (B + rK_{wind})$. This is the same form as the transfer function we have identified for our servo motor in previous laboratory exercises.

Objective:



The objective of this project is to design a PI (Proportional-Integral) controller to improve the transient response of the DC servomotor used in experiments 1 and 4 and demonstrate that we can control the transient response of our new product. As in experiments 1 and 4, we will represent this motor as a first-order system. This assumption has been tested in the aforementioned experiments, and found to be acceptable. The concept of root locus described in the controls textbook is essential for designing of the PID class of controllers. As such we will be relying heavily on its use.

DC Servo Motor and the PI controller:

The block diagram for the *WindDrive* electric generator speed controller is shown below where C_o is the desired speed, ΔC is the deviation from the desired speed, A_o is the nominal actuation to the plant, and ΔA is the change in actuation to compensate for ΔC.

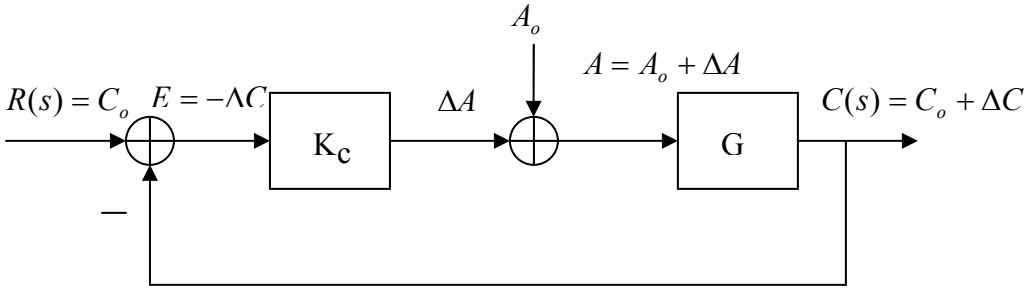


Figure 8. Block Diagram for PI Speed Control System

In an open-loop system, the nominal output $C_o = GA_o$ so that when the desired output $R = C_o$, the nominal actuation

$$A_o = \left(\frac{1}{\|G\|}\right) C_o = \left(\frac{1}{K}\right) R$$

where K is the steady-state gain of the DC motor at its operating point.

For our work here, no corrective measures would be taken for deviations from the desired speed resulting from disturbances to or variations in the system. Most block diagrams in the class textbook do not include a nominal actuation. The reason for this is that most examples in the book use an operating point of zero; hence the nominal actuation does not appear.

For small deviations from the operating point, the block diagram presented in Figure 8 can be simplified to the deviation block diagram used for our analysis and PI controller design.

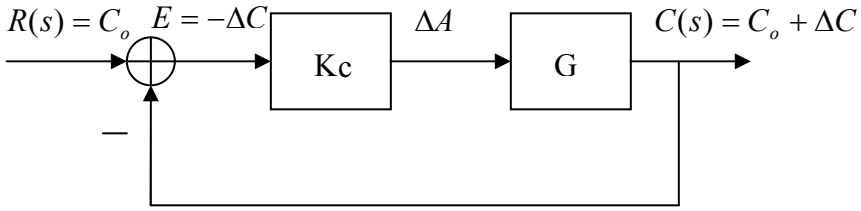


Figure 9. Simplified Block Diagram for PI Controller Design

In this block diagram R(s) is a combination of a bias level with a square wave representing changing operating points of a fixed variation about a desired mean speed (Volts). C(s) is the tachometer output of the motor (Volts), G is the plant transfer function given by (7).

From the first laboratory experiment, the model for the plant was determined to be

$$G(s) = \frac{K}{(\tau s + 1)} = \frac{0.8}{(0.3s + 1)} \tag{8}$$

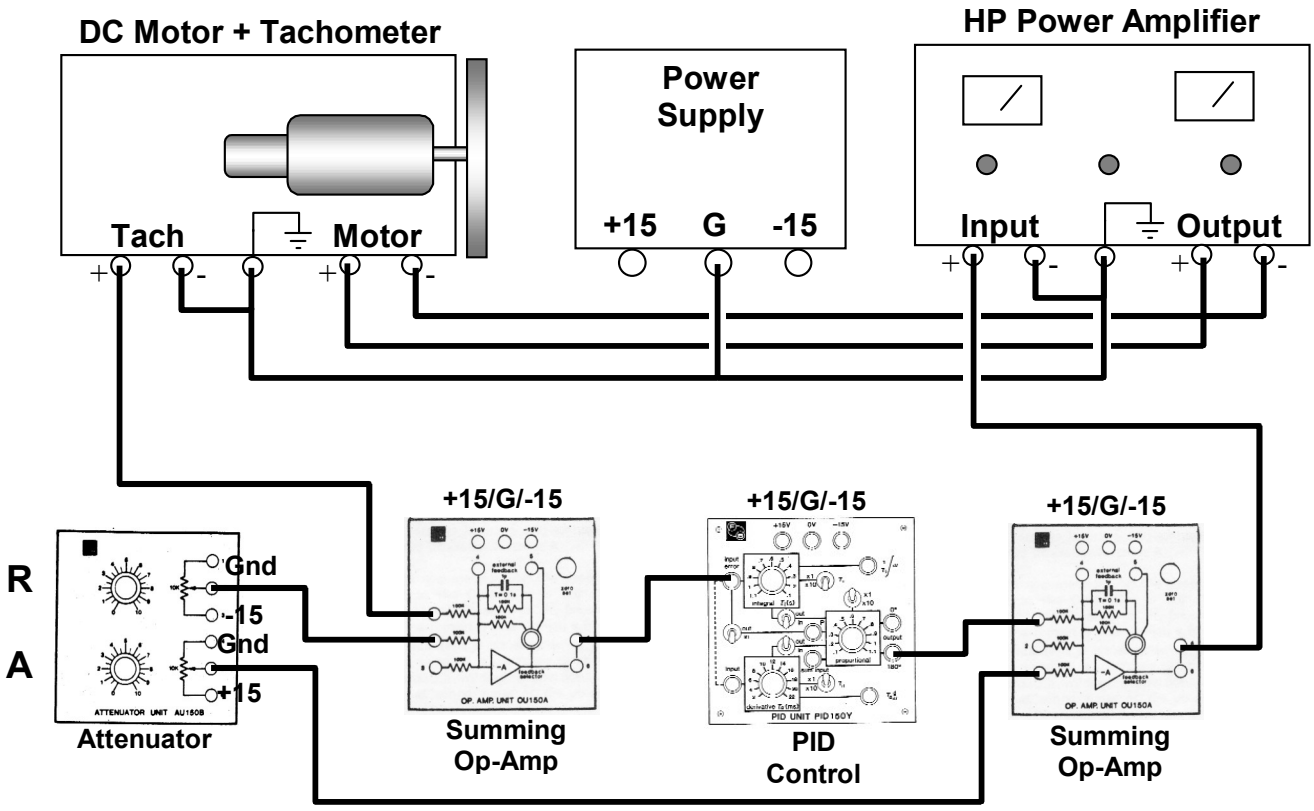
where K = 0.8±0.05 and τ = 0.3±0.05.
NOTE: The operating point is well above zero voltage. This is done in order to avoid the static friction effects determined in the first laboratory session. The operating point for which the above values were calculated is:

$$\begin{aligned} u_0 &= 4.0V \\ y_0 &= 3.25V \end{aligned} \tag{9}$$

As stated above, the design team wishes to implement a PI controller, K_C, given as,

$$K_C = K_P + \frac{K_I}{s} \tag{10}$$

Circuit Diagram :



Note : The power supply is not indicated in the diagram above to make it clearer to read.

Whirlwind Speed Control Design:

a. In designing a speed controller for the DC servo motor system, we will first investigate the case of Proportional Control (refer to section 7.10.1 of the text) and finally investigate Proportional and Integral Control (refer to section 7.10.2 of the text). The designs will be tested by Whirlwind lab consultants to see if they produce acceptable results.

Speed Control Design Project **Week 1**

Name: _____ Date: _____

Controller Limitations:

Before you begin, it is important to know controller design constraints based on available hardware. Check the controller limitations on controller gain implementation.

$$____ \leq K_P \leq ____$$

$$_____ \leq \tau_i \leq _____$$

Proportional Control:

1. We would like to improve the response time of our system (See 4-27). The desired improved settling times, group-wise, are A- 0.5s, B- 0.6s, C- 0.7s. Calculate the desired closed-loop time constant.

$$\tau_{Desired} = ____$$

2. Compute the Open-Loop Transfer Function and sketch the root locus

OL – TF =

Sketch:

What must be the closed loop pole location to achieve the desired time constant?

3. Compute the Closed-Loop Transfer Function and put it in standard form.

CL – TF =

4. What level of Proportional Control must be used to achieve the desired time constant?

$$K_P = ____$$

5. Compute the Closed-Loop DC gain and time constant

$$K = ____$$

$$\tau = ____$$

6. Calculate the steady-state error for a step input.

$$e_{SS} = ____ \%$$

7. Run a simulation of the closed-loop system's response to a 2 volt step input using the Matlab command 'step'. Print your step response and measure the theoretical DC gain, settling time, and steady-state error from the Matlab plot.

Verify with the lab technician. Implement your proportional controller and measure the experimental DC gain, settling time, and steady-state error. Compare your theoretical and experimental results for DC gain, settling time and steady-state error.

PI Control:
The steady state error and tracking properties of the proportional control are inadequate for the *WindDrive* design goals. Hence we will design a PI controller to obtain the desired performance.

1. We would now like our system to have a response that retains our new settling time and eliminates steady-state error. The new controller will have the form

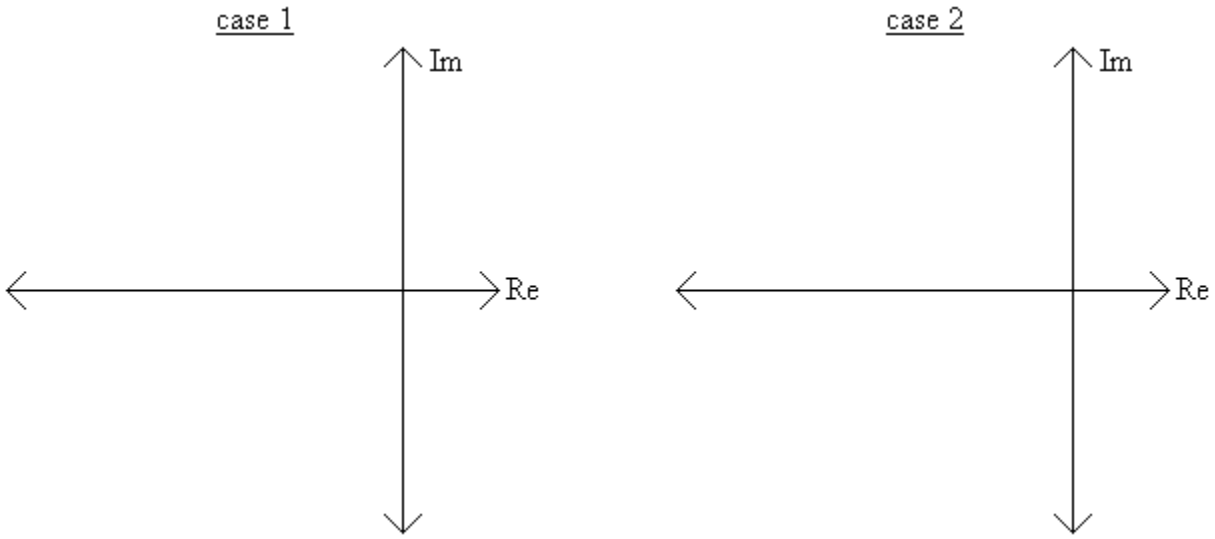
$$K_C = _____$$

2. Compute the Open-Loop TF and put it in standard form

OL-TF =

3. Sketch the root locus (Keep in mind the location of the closed loop pole) There are two possible locations where the zero can be placed. Sketch the root locus for both these possibilities.

Sketch:



4. Which zero location always gives a non-oscillatory response, for all gains?

Which zero location allows for an oscillatory response?

Which zero location can give a smaller closed loop time constant? Explain.

5. Consider the root locus with the zero placed to the left of both the open-loop poles. Where on the root locus would you obtain the smallest time constant?

Where on the root locus would you obtain a damping ratio (ζ) of 0.707?

6. Would you prefer an oscillatory response or an over-damped response for the same time constant? Explain.

7. Our response should have the following characteristics:

$e_{s.s} = 0$
$\zeta = 0.707$
T_s = (as given before for each group)

Using the knowledge you have gained thus far and the tools available, design a controller to meet these requirements. Choose a appropriate value for the damping ratio. You need to decide whether you should have a damping ratio less than or more than one. Also do not forget the design limitations which limit the value of K_p and τ_i you can select.

The following steps will guide you through the design process.

8. Where should the closed-loop roots be placed to achieve your desired settling time?

9. What is the zero location in terms of the parameters K_I and K_P? Decide on a good value for the zero location keeping in mind the specifications for your response. Explain your decision.

10. Once you have fixed the zero, you can change only the proportional gain. Use Matlab to plot the root locus with the proportional gain as the parameter being varied. From the plot select a value of proportional gain which helps you to meet the given specifications. Attach the plot to your short form.

K_P = ____ K_I = ____ τ_i = ____

11. Compute the Closed-Loop TF and put it in standard form.

CL-TF =

12. Use Matlab to plot the step response of your closed loop system to check your design. Attach the plot.

13. Compute the DC gain, percent overshoot, settling time and steady-state error.

14. Check your experimental results with the lab technician and compare your theoretical and experimental results for DC gain, Settling Time and Steady-State Error.