Answers to Selected Quizzes

Fall 1993:

1)

Operation	"$a+bi$"	"$Re^{i\theta}$"
(1+2i)+(2+2i)	**3 + 4i**	**$5e^{0.93i}$**
(1+3i)(3+4i)	**-9 +13i**	**$15.8e^{2.18i}$**
$e^{i(\pi/2)} - e^{i(3\pi/2)}$	**0 + 2i**	**$2e^{1.57i}$**
$\frac{5e^{i(\pi/2)}}{e^{i(\pi/4)}}$	**3.5 + 3.5i**	**$5e^{0.79i}$**

2) determinant of $\begin{bmatrix} 1 & 2 & 3 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = 24$

inverse of $\begin{bmatrix} 1 & 2 & 3 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 & -075 & 0 \\ 0.25 & 0.25 & -0.375 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.3\overline{3} \end{bmatrix}$

3) find *x(t)* for $3\dot{x} + 12x = 1, \quad x(0) = 1$:

Taking Laplace Transform: $[3s+12]X(s) - 3x(0) = 1/s$

Using $x(0) = 1$ and rearranging yields,

$$X(s) = \frac{3}{(3s+12)} + \frac{1}{s(3s+12)} = \left(\frac{1}{3}\right)\frac{3s+1}{s(s+4)} = \frac{A}{s} + \frac{B}{(s+4)}$$

where: $$A = \lim_{s\to 0}\left(sX(s)\right) = \left(\frac{1}{3}\right)\frac{2(0)+1}{(0+4)} = \frac{1}{12} = 0.083$$

$$B = \lim_{s\to -4}\left((s+4)X(s)\right) = \left(\frac{1}{3}\right)\frac{3(-4)+1}{-4} = \left(\frac{1}{3}\right)\frac{-11}{-4} = \frac{11}{12} = 0.917$$

so that: $X(s)\frac{0.0833}{s} + \frac{0.917}{(s+4)}$ and $x(t) = 0.0833u(t) + 0.917e^{-4t}u(t)$

Fall 1994:

1)

Operation	*"a+bi"*	*"$Re^{i\theta}$"*
(2+4i)+(3+i)	**5 + 5i**	**$7.1e^{0.785i}$**
(2+i)(3+6i)	**0 +15i**	**$15e^{1.57i}$**
$e^{i\pi} - 2$	**-3 + 0i**	**$3e^{3.14i}$**
$\frac{5e^{i\pi}}{e^{i(\pi/2)}}$	**0 + 5i**	**$5e^{1.57i}$**

2) Determinant of $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} = 1\begin{bmatrix} 4 & 2 & 0 \\ 0 & 3 & 3 \\ 0 & 1 & 2 \end{bmatrix} - 0*[\#'s] + 0*[\#'s] - 0*[\#'s]$

$$= 1\left\{4\begin{Vmatrix} 3 & 3 \\ 1 & 2 \end{Vmatrix}\right\} = 1\{4[3*2-1*3]\} = 1\{4[3]\} = 12$$

3)

$$\text{Inverse of } \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \frac{adj[\ ]}{\|\ \|} = \frac{1}{-8}\begin{bmatrix} 4 & -6 & 0 \\ -4 & 2 & 0 \\ 0 & 0 & -4 \end{bmatrix}^T = \frac{1}{-8}\begin{bmatrix} 4 & -4 & 0 \\ -6 & 2 & 0 \\ 0 & 0 & -4 \end{bmatrix}^T = \begin{bmatrix} -0.5 & 0.5 & 0 \\ 0.75 & -0.25 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

4) Find $x(t)$ for $3\ddot{x} + 15\dot{x} + 12x = 0,\ x(0) = 1,\ \dot{x}(0) = 2$

Taking the Laplace Transform:

$3[s^2X(s) - sx(0) - \dot{x}(0)] + 15[sX(s) - x(0)] + 12X(s) = 0$

substituting $x(0) = 1,\ \dot{x}(0) = 2$ yields $3[s^2X(s) - s - 2] + 15[sX(s) - 1] + 12X(s) = 0$ and

$[3s^2 + 15s + 12]X(s) = 3s + 6 + 15 = 3s + 21$ or

$$X(s) = \frac{3s+21}{3[s^2+15s+12]} = \frac{3s+21}{[3s^2+15s+12]} = \frac{s+7}{[s^2+5s+4]} = \frac{s+7}{(s+1)(s+4)} = \frac{A}{(s+1)} + \frac{B}{(s+4)}$$

where $A = \lim_{s\to -1}[(s+1)X(s)] = \frac{(-1+7)}{(-1+4)} = 2$ and $B = \lim_{s\to -4}[(s+4)X(s)] = \frac{(-4+7)}{(-4+1)} = -1$

so that (using table) $x(t) = 2e^{-t} - e^{-4t}$