(5 pts) 1. a) For the differential equation below, write the corresponding transfer function in the box.

$$5\ddot{y} + 2\dot{y} + 10y = 2f(t)$$

$$\frac{Y(s)}{F(s)} =$$

Transfer Function

(5 pts)b) For the transfer function below, write the corresponding ordinary differential equation in the box provided.

$$\frac{Y(s)}{F(s)} = \frac{4}{3s^3 + 16}$$

Differential Equation

(5 pts)c) For the state-equations below, write the transfer function $X_2(s)/U(s)$ in the box provided. Simplify your result as much as possible and put it in the box provided.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

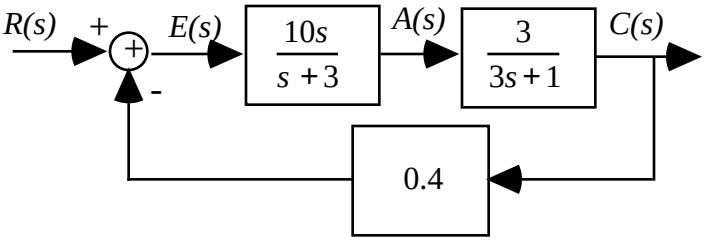
$$\frac{Y(s)}{U(s)} =$$

Transfer Function

(5 pts) d) For the block diagram below, what is the system transfer function $C(s)/R(s)$. Simplify as much as possible and express the result as the ratio of two polynomials.

$$\frac{C(s)}{R(s)} =$$

Transfer Function



2. A thermal control system has the following non-linear differential equation model for temperature T (deg C) for input fuel flow Q (kg/sec.)

$$\dot{T} = -6T^4 + 2Q$$

(5 pts) a) Find the equilibrium temperature $T = T_o$ corresponding to a fuel flow of 48000 kg/sec and enter it in the box below.

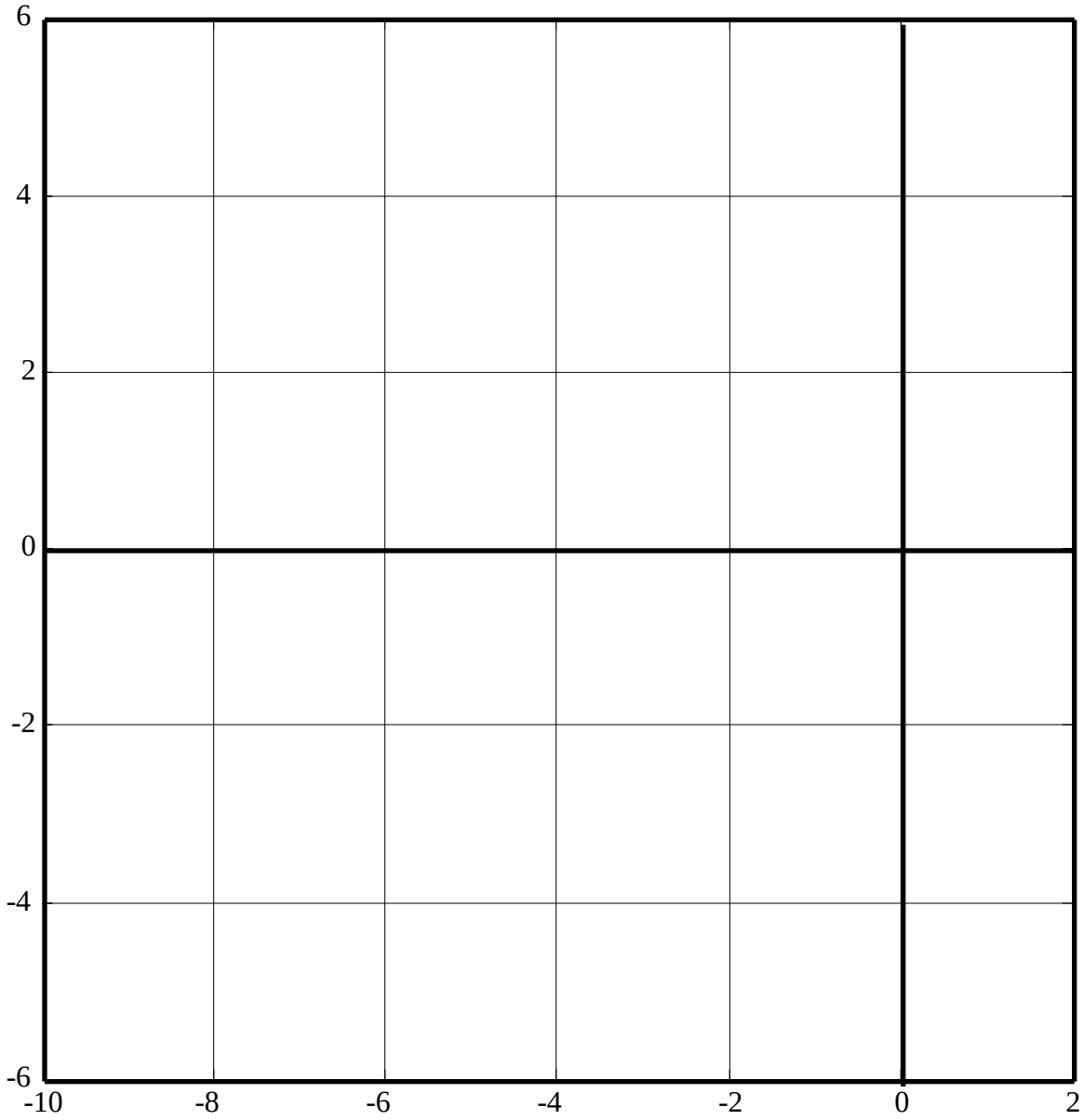
T_o = deg C

(5 pts) b) Define a new set of variables about which a linear approximation for the above equation can be derived.

(10 pts) c)Find the linearized ordinary differential equation about the equilibrium point found in part a) and enter your result in the box below

Linearized Differential Equation(s)

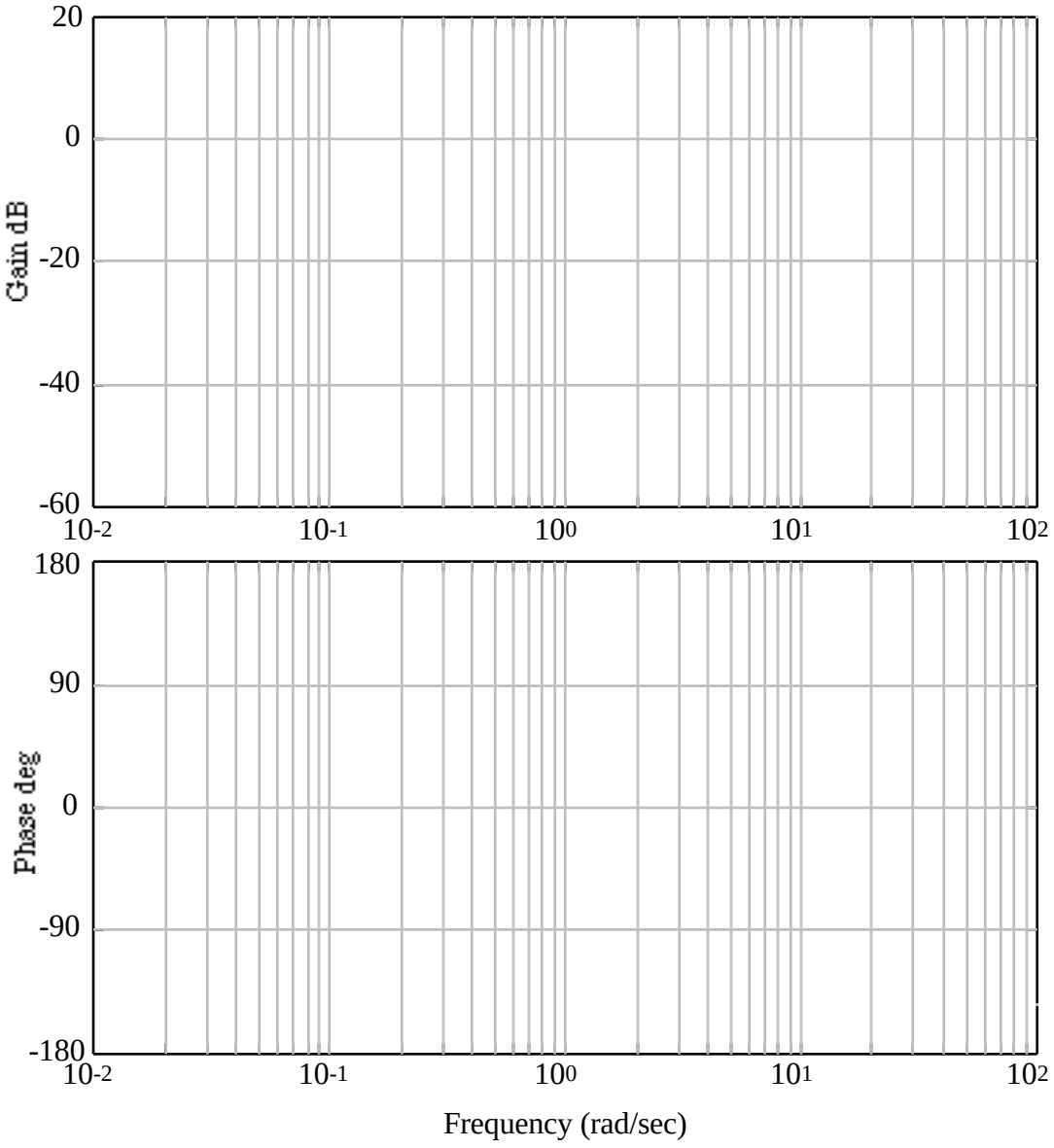
(10 pts) 3.For an open-loop system transfer function, $KGH(s) = \frac{Ks}{(s+2)(s^2+4)}$, plot the root locus diagram on the axes below for $0 < K < \infty$. When they exist, how features such as asymptotes, breakaway points, angles of departure, instability frequencies, etc.



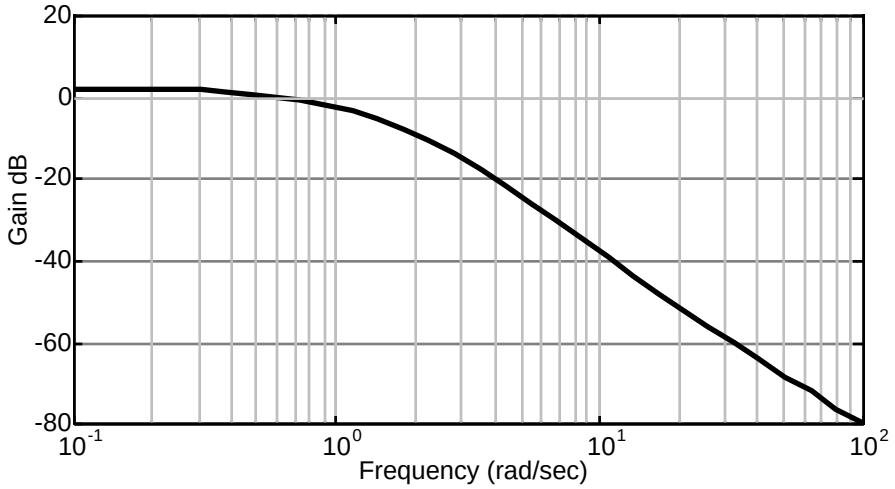
4. For the transfer function,

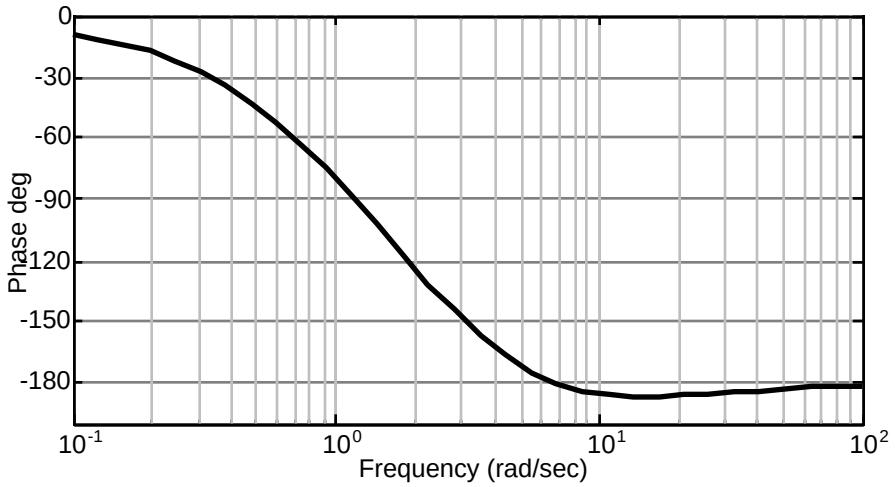
$$G(s) = \frac{10s + 1}{s + 10}$$

(10pts) Sketch the Bode Diagram on the graph below first using straight line asymptotes for magnitude and phase, then sketching the "exact " response.



5. For open-loop system Bode diagram plotted for transfer function, $KGH(s)$, with $K = 1$,.





(4 pts) a) Is the system stable at K = 1? YES or NO (circle one)

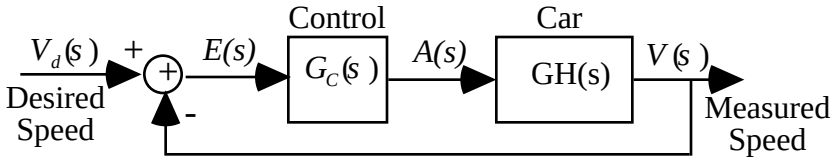
(4 pts) b) What is the Gain Margin? GM = _____ dB

(4 pts) c) What is the Phase Margin? PM = _____ deg

(4 pts) d) What is the maximum stable value of K? K_{max} = _____ dB

(4 pts) e) What value of K gives a phase margin of 45 degrees? K_{45} = _____ dB

6. The block diagram below represents an automobile speed control. The desired speed $V_d(s)$ is compared to the actual speed $V(s)$ of the car in the wash in order to develop the error $E(s)$ which is the input to the control, $G_c(s)$. The Bode Diagram in Problem 5 is measured for this vehicle



(5 pts) a) Using the Bode Diagram to analyze a proportional control, $G_C(s) = K_p = 1$, what is the steady-state automobile speed error for a unit step input?

Steady-State Speed Error

(5 pts) b) Using the Bode Diagram to analyze a proportional control, what value of $G_C(s) = K_p$, would yield either a minimum Gain Margin of 6dB and a minimum Phase Margin of 30 degrees

$G_C(s) = K_p =$

Proportional Gain

(5 pts)c) This system would benefit most from ☐ PD control or ☐ PI Control (check one) *because* (Explain in 20 words or less)

(10 pts) d) Using the Bode diagram, design the controller selected in above to provide a minimum phase margin of 30 degrees at $\omega = 1$ rad/sec and enter its transfer function in the box below

Control Transfer Function