Frequency Response Example:

For the Open-Loop Transfer Function $KGH = K\left[\dfrac{12}{(s+1)(s+2)(s+3)}\right]$

At zero frequency, this system has a DC (steady-state) gain of

$$GH(0) = \left(\frac{12}{1*2*3}\right) = 2$$

In a decibel (dB) scale, this gain is expressed.

$$GH(0) = 20\log_{10}(2) = 20(0.3010) = 6.02\ \text{dB}$$

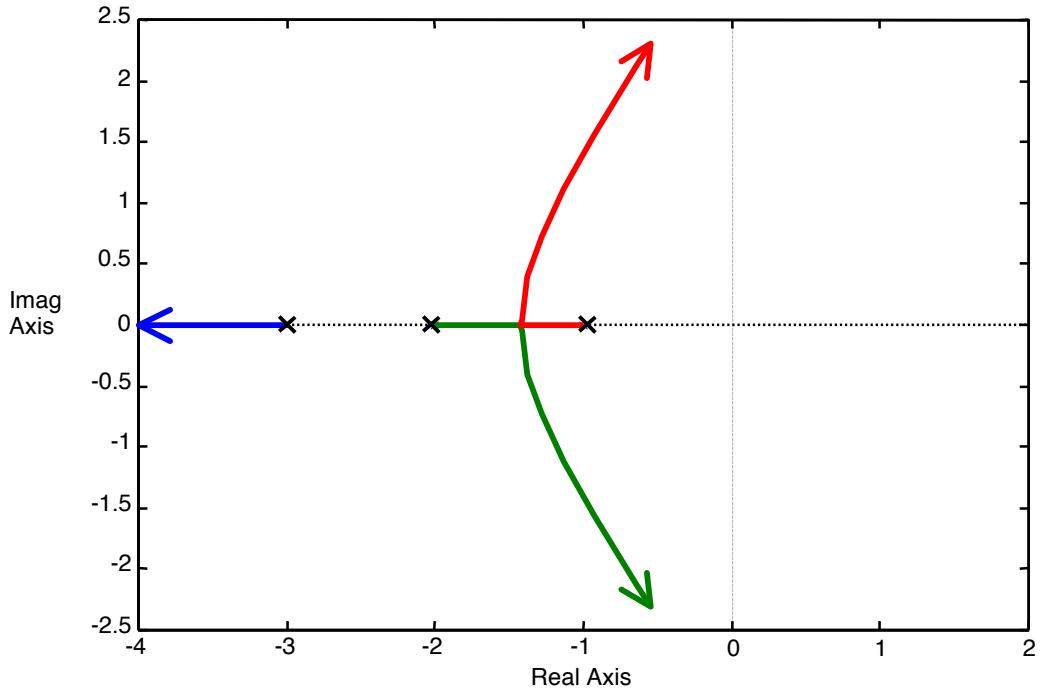
The two are equivalent but one is on a linear scale and the other on a logarithmic scale

Note: For the record, "deci-Bel" dB is the 10*log(power ratio)

$$dB = 10\log_{10}\left(\frac{P}{P_{ref}}\right) = 10\log_{10}\left(\frac{ky^2}{ky_{ref}^2}\right) = 10\log_{10}\left(\frac{y^2}{y_{ref}^2}\right) = 10\log_{10}\left[\left(\frac{y}{y_{ref}}\right)^2\right]$$

$$= 20\log_{10}\left(\frac{y}{y_{ref}}\right) = 20\log_{10}(y) \quad \text{for} \quad y_{ref} = 1 \quad \text{because} \quad \log_{10}(1) = 0$$

The Root Locus showing the closed-loop pole locations is



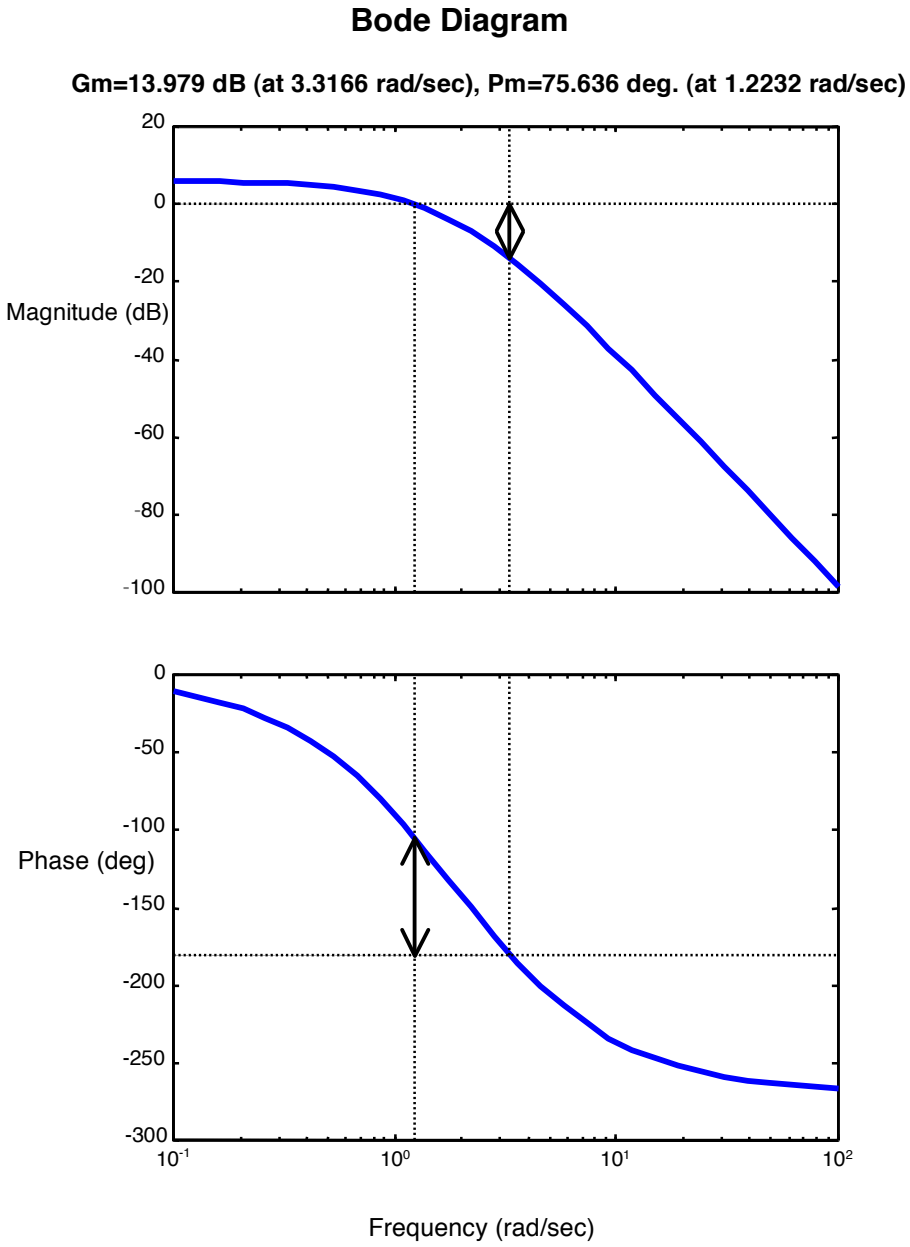
The Open-Loop Frequency Response (Bode Diagram) is plotted with the command

```
EDU» num=[12]; den=conv([1 1],conv([1 2], [1 3])); G=tf(num,den)
EDU» bode(G)
```

And the Gain and Phase margins are computed with the command

```
EDU» margin(G)
```

Yielding the plot...



This plot show a Gain Margin (GM) = 13.979 dB and a Phase Margin of 75.636 degrees.

The Gain Margin indicates that the control gain can be increased by 13.979 dB ($10^{(14/20)} = 5$) from $K = 1 = 0$dB before the system will go unstable.

The Phase Margin indicates that at the current gain (K = 1), the system can absorb an addition phase lag of 75.636 before it will go unstable.

For the system

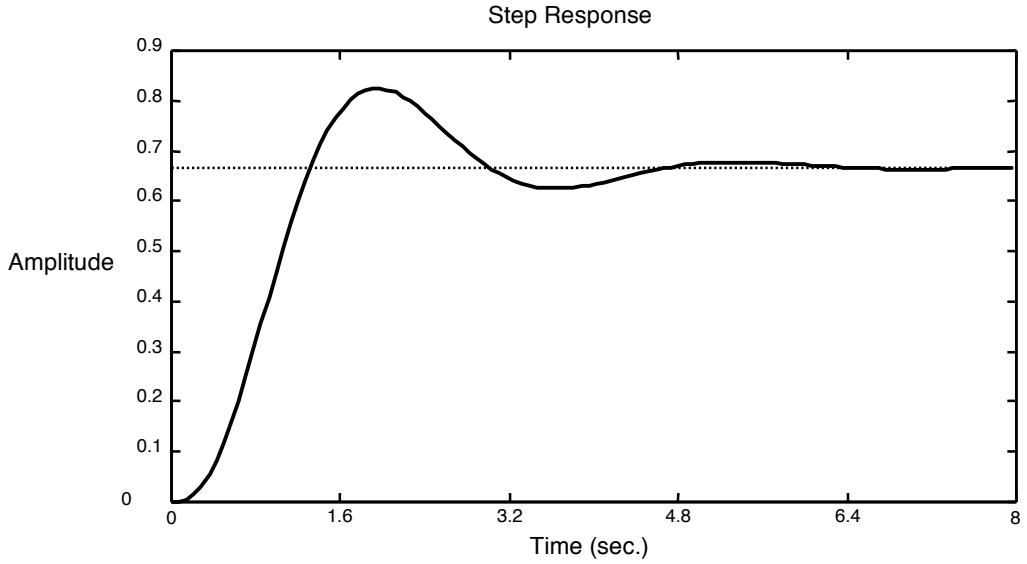
$$KGH = K\left[\frac{12}{(s+1)(s+2)(s+3)}\right]$$

the GM=14. dB and Phase Margin = 76 degrees with K=1indicate a very stable system.

The step response for the Closed-Loop system

$$T = \frac{12}{s^3 + 6s^2 + 11s + 18}$$

confirms this prediction.



Although the step response is stable, the steady state error is 33%. Could we reduce it by increasing the gain K from $K = 1\,(0\text{ dB})$ to $K = 2\,(6.02\text{ dB})$? Yes, just add 6.02 dB to the frequency response shift it up 6.02 dB at all frequencies. Let's plot it with the Matlab commands

```
EDU» num=[24]; den=[1 6 11 6]; G=tf(num,den);margin(G)
```

The new open-loop transfer function

$$G_2(s) = \frac{24}{s^3 + 6s^2 + 11s + 6}$$

has a DC gain of $G_2(s) = 4 \quad (= 12\text{ dB})$ reducing the error to $e(\infty) = 1/(1+4) = 0.2 = 20\%$ but sacrifices stability with the new GM=8 dB and PM = 35 degrees.

The system's closed-loop transfer function

$$T_2 = \frac{24}{s^3 + 6s^2 + 11s + 30}$$

has the predicted, less stable response with better steady-state error.

