ME451: Control Systems

Lecture 9
Modeling of DC motors

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Course roadmap

Modeling	*Analysis*	*Design*
✔ Laplace transform	Time response • Transient • Steady state	Design specs
✔ Transfer function		Root locus
✔ Models for systems ✔ electrical ✔ mechanical • electromechanical ✔ Block diagrams Linearization	Frequency response • Bode plot	Frequency domain
	Stability • Routh-Hurwitz • Nyquist	PID & Lead-lag
		Design examples

(Matlab simulations &) laboratories

What is DC motor?

An actuator, converting electrical energy into rotational mechanical energy

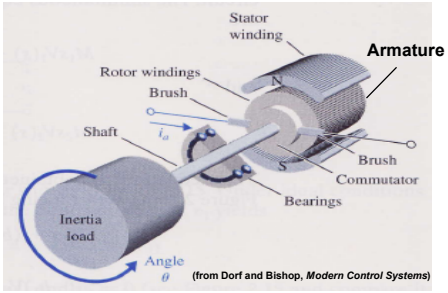


(You will see DC motor during Lab 1 and 4.)

Why DC motor?

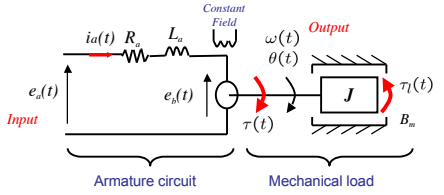
- Advantages:
  - high torque
  - speed controllability
  - portability, etc.
  - Linear model for fixed field
- Widely used in control applications: robot, tape drives, printers, machine tool industries, radar tracking system, etc.
- Used for moving loads when
  - Rapid (microseconds) response is not required
  - Relatively low power is required

How does DC motor work?



(from Dorf and Bishop, *Modern Control Systems*)

Model of DC motor



"a" :armature
e_a :applied voltage
i_a :armature current
"b" :back EMF

mechanical
θ :angular position
ω :angular velocity
J : rotor inertia
B : viscous friction

Modeling of DC motor: time domain

- Three Energy Domains:
  - Electrical (Armature circuit)

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

  - Magnetic Connection between mechanical & electrical
    - Electrically produced torque
    - Mechanically produced back emf

$$\tau(t) = K_\tau i_a(t)$$
$$e_b(t) = K_b \omega(t)$$

  - Mechanical Rotation

$$J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) - \tau_l(t)$$
$$\omega(t) = \dot{\theta}(t)$$

Load torque

Modeling of DC motor: time domain

- Substitute magnetic connection into Electrical and Mechanical ODE's
  - Electrical (armature circuit)

$$e_a = R_a i_a(t) + L_a \frac{di_a}{dt} + K_b \dot{\theta}(t)$$

  - Mechanical Rotation

$$J\ddot{\theta}(t) = K_\tau i_a(t) - B\dot{\theta}(t) - \tau_l(t)$$
$$J\ddot{\theta}(t) + B\dot{\theta}(t) - K_\tau i_a(t) = -\tau_l(t)$$

Load torque

Modeling of DC motor: Laplace domain

- Substitute magnetic connection into Electrical and Mechanical ODE's
  - Electrical (armature circuit)

$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + K_b s\Theta(s)$$
$$= (R_a + L_a s) I_a(s) + K_b s\Theta(s)$$

  - Mechanical Rotation

$$Js^2\Theta(s) + Bs\Theta(s) - K_\tau I_a(s) = -\mathrm{T}_l(s)$$
$$(Js^2 + Bs)\Theta(s) - K_\tau I_a(s) = -\mathrm{T}_l(s)$$

Modeling of DC motor: Laplace domain

- Rearranging ODE's
  - Electrical (armature circuit)

$$(R_a + L_a s) I_a(s) + K_b s\Theta(s) = E_a(s)$$

  - Mechanical Rotation

$$(Js^2 + Bs)\Theta(s) - K_\tau I_a(s) = -\mathrm{T}_l(s)$$

  - Matrix Form

$$\begin{bmatrix} (R_a + L_a s) & K_b s \\ K_\tau & -(Js^2 + Bs) \end{bmatrix} \begin{bmatrix} I_a(s) \\ \Theta(s) \end{bmatrix} = \begin{bmatrix} E_a(s) \\ \mathrm{T}_l(s) \end{bmatrix}$$

Modeling of DC motor: Laplace domain

- Using rotation velocity $\Omega(s)=s\Theta(s)$

$$\begin{bmatrix} (R_a + L_a s) & K_b \\ K_\tau & -(Js + B) \end{bmatrix} \begin{bmatrix} I_a(s) \\ \Omega(s) \end{bmatrix} = \begin{bmatrix} E_a(s) \\ \mathrm{T}_l(s) \end{bmatrix}$$

- Notice that the system is *"not quite"* symmetric
  - There is a reason… look at energy conservation across the magnetic field

Power in $= E_b(s) I_a(s) = \mathrm{T}(s)\Omega(s) =$ Power out

But $E_b(s) = K_b\Omega(s)$ and $\mathrm{T}(s) = K_\tau I_a(s)$

so $K_b\Omega(s) I_a(s) = K_\tau I_a(s)\Omega(s)$

and for power conservation $K_b = K_\tau = K_m$ (defined)

Transfer Functions

- Two inputs $[E_a(s), \mathrm{T}_l(s)]$ and two outputs $[I_a(s), \Omega(s)]$
- Solving for $[I_a(s), \Omega(s)]$ given $[E_a(s), \mathrm{T}_l(s)]$

$$\begin{bmatrix} I_a(s) \\ \Omega(s) \end{bmatrix} = \begin{bmatrix} (R_a + L_a s) & K_m \\ K_m & -(Js + B) \end{bmatrix}^{-1} \begin{bmatrix} E_a(s) \\ \mathrm{T}_l(s) \end{bmatrix}$$

- Where $[\,]^{-1}$ is

$$\begin{bmatrix} (R_a + L_a s) & K_m \\ K_m & -(Js + B) \end{bmatrix}^{-1} = \frac{adj()}{\det()} = \frac{\begin{bmatrix} -(Js + B) & -K_m \\ -K_m & (R_a + L_a s) \end{bmatrix}^T}{-\left[(R_a + L_a s)(Js + B) + K_m^2\right]}$$

Transfer Functions

- Simplifying, the TF function matrix equations …

$$\begin{bmatrix} I_a(s) \\ \Omega(s) \end{bmatrix} = \frac{\begin{bmatrix} (Js+B) & K_m \\ K_m & -(R_a+L_a s) \end{bmatrix} \begin{bmatrix} E_a(s) \\ T_l(s) \end{bmatrix}}{\left[(R_a+L_a s)(Js+B)+K_m^2\right]}$$

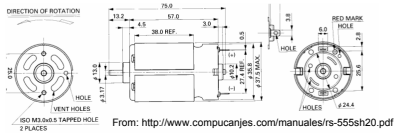
Model Parameters from Manufacturer Data

- How do you compute parameters given …

MABUCHI BRUSHED DC MOTOR MODEL RS-555SH

- Round, carbon brushed dc motor
- Incorporating an anisotropic magnet
- Output approximately 3.0W-65W
- Suitable for various applications including office equipment and automotive products
- Weight approximately 215g





From: http://www.compucanjes.com/manuales/rs-555sh20.pdf

Model Parameters from Manufacturer Data

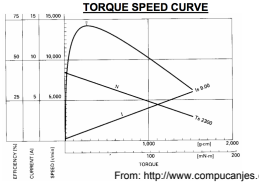
- How do you compute parameters given …

MABUCHI BRUSHED DC MOTOR MODEL RS-555SH

MODEL	VOLTAGE		NO LOAD		AT MAXIMUM EFFICIENCY				STALL	
	OPERATING RANGE	NOMINAL	SPEED	CURRENT	SPEED	CURRENT	TORQUE	OUTPUT	TORQUE	CURRENT
			R/MIN	A	R/min	A	g.cm	mN.m	W	g.cm	mN.m	A
RS-555SH	9.6-30	24V CONSTANT	8500	0.18	7400	1.27	385	27.9	21.4	2300	225	9.0

*Performance and dimension characteristics are average values

TORQUE SPEED CURVE



From: http://www.compucanjes.com/manuales/rs-555sh20.pdf

Manufacturer specs

- Steady State so we need Steady State Model
  - Set all time derivatives to zero

$$\begin{bmatrix} (R_a+L_a s) & K_m \\ K_m & -(Js+B) \end{bmatrix} \begin{bmatrix} I_a(s) \\ \Omega(s) \end{bmatrix} = \begin{bmatrix} E_a(s) \\ T_l(s) \end{bmatrix}$$

  - becomes

$$\begin{bmatrix} R_a & K_m \\ K_m & -B \end{bmatrix} \begin{bmatrix} I_a(s) \\ \Omega(s) \end{bmatrix} = \begin{bmatrix} E_a(s) \\ T_l(s) \end{bmatrix}$$

Manufacturer specs

MODEL	VOLTAGE	
	OPERATING RANGE	NOMINAL
RS-555SH	9.6-30	24V CONSTANT

$$\begin{bmatrix} R_a & K_m \\ K_m & -B \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} = \begin{bmatrix} e_a \\ \tau_l \end{bmatrix}$$

STALL	
TORQUE	CURRENT
g.cm / mN.m	A
2300 / 225	9.0

- Clamp shaft ($\omega = 0$) and "stall" motor

$$R_a i_a = e_a \;\Rightarrow\; R_a = e_{stall}/i_{stall} = 24v/9A = 2.7 \text{ (Ohm)}$$

$$K_m i_a = \tau_l \;\Rightarrow\; K_m = \tau_{stall}/i_{stall} = 0.225 \text{ N-m}/9A$$
$$= 0.025 \text{ N-m/A}$$

(Note consistent units!!)

Manufacturer specs

MODEL	VOLTAGE	
	OPERATING RANGE	NOMINAL
RS-555SH	9.6-30	24V CONSTANT

$$\begin{bmatrix} R_a & K_m \\ K_m & -B \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} = \begin{bmatrix} e_a \\ \tau_l \end{bmatrix}$$

NO LOAD	
SPEED	CURRENT
R/MIN	A
8500	0.18

- Apply no load:

$$K_m i_{noload} - B\omega_{noload} = 0 \;\Rightarrow\; B = K_m i_{noload}/\omega_{noload}$$
$$B = K_m(0.18A)/8500 \text{ rpm}$$
$$= (0.226 \text{ N-m/A})(0.18\text{A})/\left(8500 \text{ rpm}(1\text{min}/60\text{sec})(2\pi \, rad/1 \text{ rev})\right)$$
$$= (0.226 \text{ N-m/A})(0.18A)/(890 \text{ rad/sec})$$
$$= 5.2 \text{ x } 10^{-6} \text{ (N-m-sec/rad)}$$

- And to check…

$$K_m = (e_a - R_a i_a)/\omega = (24v - (2.67)(0.18A))/(890 \text{ rad/sec})$$
$$= 0.026 \text{ v-sec/rad} \quad \text{It independently agrees with 0.025 N-m/A !}$$

Check model with other Specs

AT MAXIMUM EFFICIENCY				
SPEED	CURRENT	TORQUE		OUTPUT
R/min	A	g.cm	mN.m	W
7400	1.27	285	27.9	21.6

$$\begin{bmatrix} R_a & K_m \\ K_m & -B \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} = \begin{bmatrix} e_a \\ \tau_l \end{bmatrix}$$

- At max efficiency,

$K_m i_a - B\omega \overset{?}{=} \tau_l$ where $\tau_l = 0.0279\,(\text{N-m})$

$K_m = 0.0255\,(\text{N-m/A}), \quad B = 46.\text{x}10^{-6}\,(\text{N-m-sec/rad}),$

$i_a = 1.27A, \quad \omega = 7400\,(\text{rpm}) = 775\,(\text{rad/sec})$

$K_m i_a - B\omega = (0.0255)(1.27) - (46.\text{x}10^{-6})(775) = 0.0287\ (\text{N-m})$

$R_a i_a - K_m \omega \overset{?}{=} e_a$ where $e_a = 24\ (\text{v})$

$R_a = 2.67\ (\text{Ohm}), \quad K_m = 0.0255\,(\text{N-m/A}),$

$i_a = 1.27A, \quad \omega = 7400\,(\text{rpm}) = 775\,(\text{rad/sec})$

$R_a i_a - K_m \omega = (2.67)(1.27) - (0.0257)(775) = 23.0\ (\text{v})$

Check model with other Specs

AT MAXIMUM EFFICIENCY				
SPEED	CURRENT	TORQUE		OUTPUT
R/min	A	g.cm	mN.m	W
7400	1.27	285	27.9	21.6

$$\begin{bmatrix} R_a & K_m \\ K_m & -B \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} = \begin{bmatrix} e_a \\ \tau_l \end{bmatrix}$$

- At max efficiency,

Model predicts $\tau = 0.0287$ (N-m)

Measured $\tau = 0.0279$ (N-m)

Difference: 3.99%

Looks Good.
As accurate as the measured values!

Model Predicts $e_a = 23.0$ (v)

Measured $e_a = 24.0$ (v)

Difference: -2.79%

Summary and Exercises

- Modeling of DC motor
  - What is DC motor and how does it work?
  - Derivation of a transfer function
- Next
  - ***Linearization*** of nonlinear models, one of the most important topics in engineering analysis
- Exercises
  - Read Section 2.7, 2.8.
  - Go over the derivation for DC motor transfer functions by yourself. Obtain $T(s)/E_a(s)$.

Main message until this point

Many systems can be represented as transfer functions!

Using the transfer functions, ….
(to be continued)