Torsional pendulum system

Ex. 2.12

\[
\tau(t) = J\dot{\theta}(t) = B\dot{\theta}(t) + K\theta(t)
\]

\[
J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau(t)
\]

By Laplace transform (with zero ICs),

\[
\mathcal{L}\{\ddot{\theta}(t)\} = s^2 \mathcal{L}\{\theta(t)\} = \frac{1}{s^2 + \frac{B}{J} + \frac{K}{J}} = \frac{1}{s^2 + \frac{B}{J} + \frac{K}{J}} \quad (2^nd \text{ order system})
\]

\[\tau(t) = \begin{cases} \text{friction between bob and air} \\ \text{(system used for mechanical watches)} \end{cases}\]

Course roadmap

Modeling
- Laplace transform
- Transfer function
- Models for systems
- Mechanical
- Electromechanical
- Block diagrams
- Linearization

Analysis
- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot
  - Frequency domain
- Stability
  - Routh-Hurwitz
  - Nyquist
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag Design

Design examples

(Matlab simulations & laboratories)

Rotational mechanical elements

(constitutive equations)

- Moment of inertia
- Rotational spring
- Friction

Equation of Motion

\[
J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau(t)
\]

Example

FBD
Example

By Newton’s law
\[ J_m \dot{\theta}_m(t) = \tau_m - \tau_k - K(\theta_m - \theta_L) \]
\[ J_L \dot{\theta}_L(t) = \tau_k - K(\theta_m - \theta_L) \]

Example (cont’d)

From second equation:
\[ \Theta_L(s) = \frac{K}{s^2 + K} \Theta_m(s) \]  
(2nd order system)

From first equation:
\[ \Theta_m(s) = \frac{J_m s^2 + K}{\left( J_m s^2 + B_m s + K(\Theta_m - \Theta_L) + B_m K \right) G(s)} \Theta_L(s) \]  
(4th order system)

Example

By Laplace transform (with zero ICs),
\[ J_m s^2 \Theta_m(s) + B_m \Theta_L(s) + K(\Theta_m - \Theta_L) = T_m \]
\[ J_L s^2 \Theta_L(s) - K(\Theta_m(s) - \Theta_L(s)) = 0 \]

By Laplace transform (with zero ICs),
\[ J_m s^2 \Theta_m(s) + B_m \Theta_L(s) + K(\Theta_m - \Theta_L) = T_m \]
\[ J_L s^2 \Theta_L(s) - K(\Theta_m(s) - \Theta_L(s)) = 0 \]

Example

Matrix Form

\[
\begin{pmatrix}
J_m s^2 + B_m s + K
& -K \\
-K
& J_L s^2 + K
\end{pmatrix}
\begin{pmatrix}
\Theta_m(s) \\
\Theta_L(s)
\end{pmatrix}
= \begin{pmatrix}
T_m \\
0
\end{pmatrix}
\]

Example

From first equation:
\[ \Theta_m(s) = \frac{J_m s^2 + K}{\left( J_m s^2 + B_m s + 2K(\Theta_m + J_L s) + B_m K \right) G(s)} \Theta_L(s) \]  
(4th order system)
Rigid satellite Ex. 2.13

- Broadcasting
- Weather forecast
- Communication
- GPS, etc.

\[ \tau(t) = J\dot{\theta}(t) \]
\[ G(s) = \frac{\Theta(s)}{\tau(s)} = \frac{1}{Js^2} \]

Double integrator

Summary & Exercises
- Modeling of mechanical systems
  - Translational
  - Rotational
- Next, Electromechanical systems

Exercises (Franklin et al.)
- Quarter car model: Obtain a transfer function from \( R(s) \) to \( Y(s) \).

\[ Y(s) = \frac{-K_w B s}{s + \frac{B + M_2 K_w}{M_1}} \]

Answer