This lab is about modeling a DC motor and estimating the parameters. Later, we will be doing control on this system.

**Review of DC motor modeling.**

Recall Lorentz force.

\[
\int_V \mathbf{J} \cdot d\mathbf{A} = \mathbf{F} \times \mathbf{B} = \mathbf{0}
\]

(current density $\mathbf{J}$, magnetic field $\mathbf{B}$, force $\mathbf{F}$)

\[\int V \mathbf{J} \cdot d\mathbf{A} \]

Recall Lorentz force.

\[\mathbf{F} = \mathbf{J}_1 \times \mathbf{B} \times \mathbf{N} = \mathbf{J}_2 \times \mathbf{B} \times \mathbf{x}
\]

Depending on radius, we apply a torque $T_m$.

\[T_m = \mathbf{J}_1 \times \mathbf{B} \times \mathbf{N}
\]

\[T_m = \mathbf{J}_2 \times \mathbf{B} \times \mathbf{x}
\]

So circuit diagram:

\[\begin{align*}
\mathbf{V_m} & \quad \mathbf{R_m} \quad \mathbf{L_m} \quad \mathbf{T_m} \quad \mathbf{J_{eq}} \quad \Theta_m = \omega_m \\
\mathbf{T_m} & \quad \mathbf{e_m} \quad \mathbf{\Theta_m}
\end{align*}\]

Let's write equations.
(circuit) \[ V_m = I_m R_m + L_m I_m + e_m \]

(rotor) \[ J_{eq} \ddot{\theta}_m = T_m. \]

The connection between circuit and rotor is through Lorentz eqns. Since that depends on geometry, we just say:

(torque) \[ T_m = K_m I_m \] since force is prop. to current density \( J \).

Also, since this is a passive interaction, if you move the rotor, you will generate a back emf:

(back emf) \[ e_m = K_m \dot{\theta}_m \] Note \( K_t \) in both.

Putting these together,

\[
\begin{align*}
V_m &= I_m R_m + L_m I_m + K_m \dot{\theta}_m \\
J_{eq} \ddot{\theta}_m &= K_m I_m
\end{align*}
\]

So \[ I_m = \frac{J_{eq}}{K_m} \dot{\theta}_m \Rightarrow V_m = \frac{R_m J_{eq}}{K_m} \dot{\omega}_m + \frac{L_m J_{eq}}{K_m} \ddot{\omega}_m + K_m \omega_m
\]\n
\[ \Rightarrow \hat{V}_m = \left( s \frac{R_m J_{eq}}{K_m} + s^2 \frac{L_m J_{eq}}{K_m} + K_m \right) \hat{\omega}_m \] (Laplace)

So \[ \hat{\omega}_m = \frac{\hat{V}_m}{L_m J_{eq} s^2 + \frac{R_m J_{eq}}{K_m} s + K_m} \]
\[ \hat{\omega}_m = \frac{k_m}{Jag s (L_m s + R_m) + k_m^2} \hat{V}_m. \]

This is a 2nd order plant. But, since \( L_m \) is from a copper wire, \( L_m \) is small.

\[ \Rightarrow \hat{\omega}_m \approx \frac{k_m}{Jag R_m s + k_m^2} \hat{V}_m \Rightarrow G(s). \]

\( G(s) \) can also be written in the more generic form,

\[ \hat{\omega}_m \approx \frac{1/k_m}{Jag R_m s + 1} = \frac{k}{\tau s + 1}, \] gain

\[ \tau \text{ time constant.} \]

You will be solving for \( K \) and \( \tau \) using several methods in the lab:

1. Static method (estim. \( k_m, R_m \)).
2. Using a bump test/step response.
3. Fine-tuning the simulation model.

If \( \dot{\Theta}_m = 0 \), then \( V_m = I_m R_m + \dot{\Phi}_m I_m \),

\[ \Rightarrow R_m = \frac{V_m}{I_m} \]

For \( k_m = \frac{e_m}{\dot{\Theta}_m} \approx 0.05 \),

\[ \frac{V_m - I_m R_m}{\dot{\Theta}_m} \approx k_m \]
2) A bump test will produce a transient that will decay with time \( \tau \). Let \( v(t) = u(t) \). Recall \( \hat{u}(s) = \frac{1}{s} \)

\[
\hat{\omega}_m = \left( \frac{K}{\tau s+1} \right) \left( \frac{1}{s} \right) \Rightarrow \omega_m(t) = K \left( 1 - e^{-\frac{t}{\tau}} \right) u(t).
\]

\( K \) is steady state gain.

when \( \tau = t \), \( \omega_m(t) \) will have reached \( (1-e^{-1}) \approx 0.63 \)

or 63\% of its final value.

3) The simulation model numerically integrates

\[
\hat{\omega}_m = \frac{k}{\tau s+1} \cdot \hat{\omega}_m \text{ for given values of } K, \tau.
\]

The theoretical values for this system are:

\[
K_m \approx 0.0502 \frac{Nm}{A} \text{ or } Vs
\]

\[
R_m \approx 10.6 \Omega
\]

\[
J_{eq} \approx 2.21 \times 10^{-5} \text{ Kgm}^2
\]

So

\[
K_{th} \approx 19.92
\]

\[
\tau_{th} \approx 0.093 \text{ s}
\]

you will compare these values to what you obtain in the lab.