

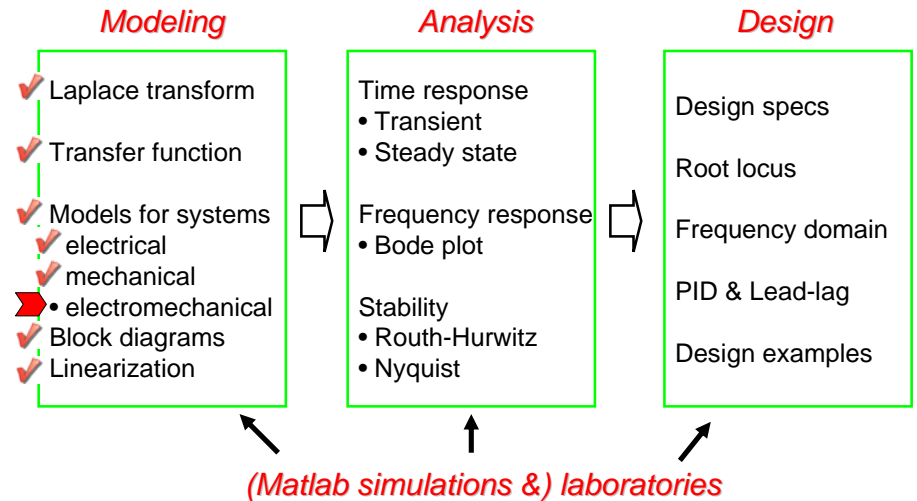
# ME451: Control Systems

## Lecture 8 Modeling of DC motors

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# Course roadmap



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## What is DC motor?

*An actuator, converting electrical energy into rotational mechanical energy*



(You will see DC motor during Lab 1 and 4.)

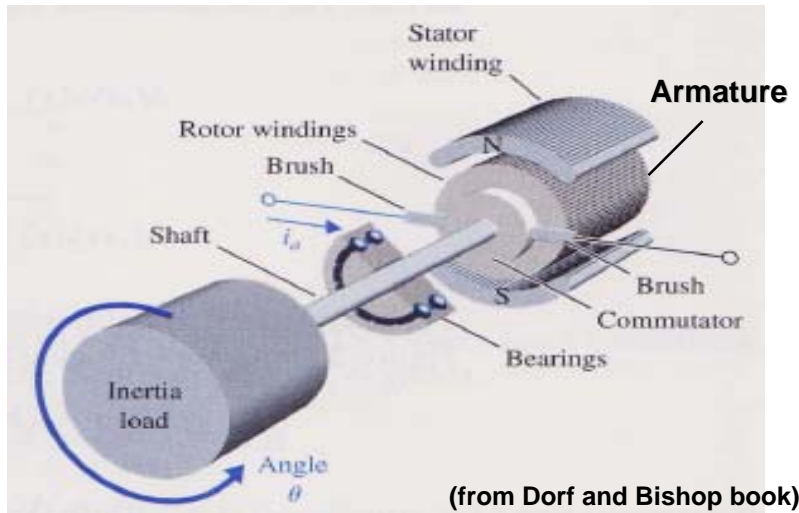
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## Why DC motor?

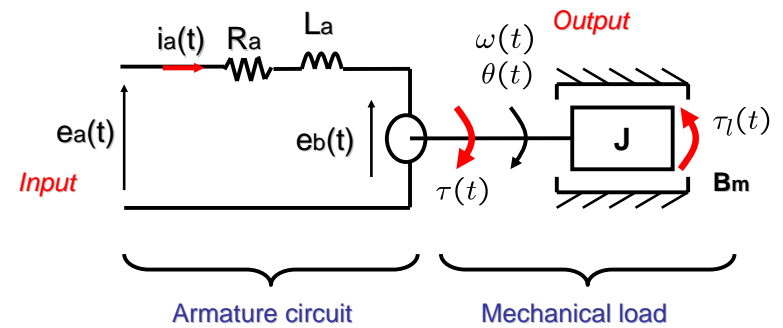
- Advantages:
  - high torque
  - speed controllability
  - portability, etc.
- Widely used in control applications: robot, tape drives, printers, machine tool industries, radar tracking system, etc.
- Used for moving loads when
  - Rapid (microseconds) response is not required
  - Relatively low power is required

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# How does DC motor work?



# Model of DC motor



- "a" : armature
- ea : applied voltage
- ia : armature current
- "b" : back EMF
- mechanical
- theta : angular position
- omega : angular velocity
- J : rotor inertia
- B : viscous friction

## Modeling of DC motor: time domain

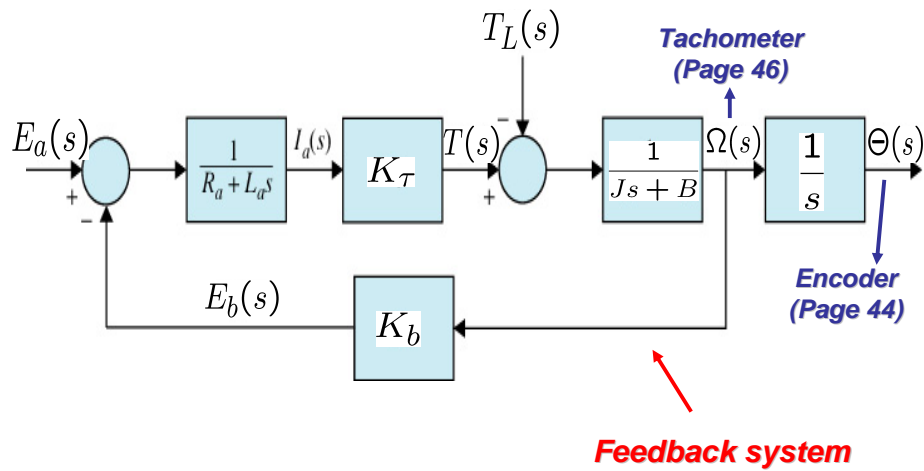
- Armature circuit  $e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$
- Connection between mechanical/electrical parts
  - Motor torque  $\tau(t) = K_\tau i_a(t)$
  - Back EMF  $e_b(t) = K_b \omega(t)$
- Mechanical load  $J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) - \tau_l(t)$ 

Load torque
- Angular position  $\omega(t) = \dot{\theta}(t)$

## Modeling of DC motor: s-domain

- Armature circuit  $I_a(s) = \frac{1}{R_a + L_a s} (E_a(s) - E_b(s))$
- Connection between mechanical/electrical parts
  - Motor torque  $T(s) = K_\tau I_a(s)$
  - Back EMF  $E_b(s) = K_b \Omega(s)$
- Mechanical load  $\Omega(s) = \frac{1}{J s + B} (T(s) - T_L(s))$
- Angular position  $\Theta(s) = \frac{1}{s} \Omega(s)$

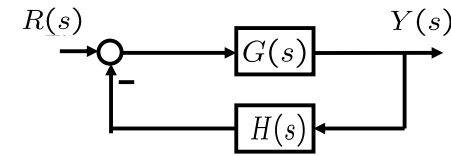
## DC motor: Block diagram



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## Useful formula for feedback

### Negative feedback system



$$Y(s) = G(s)(R(s) - H(s)Y(s)) \quad \rightarrow \quad (1 + G(s)H(s))Y(s) = G(s)R(s)$$

$$\rightarrow \frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g} = \frac{G(s)}{1 + G(s)H(s)}$$

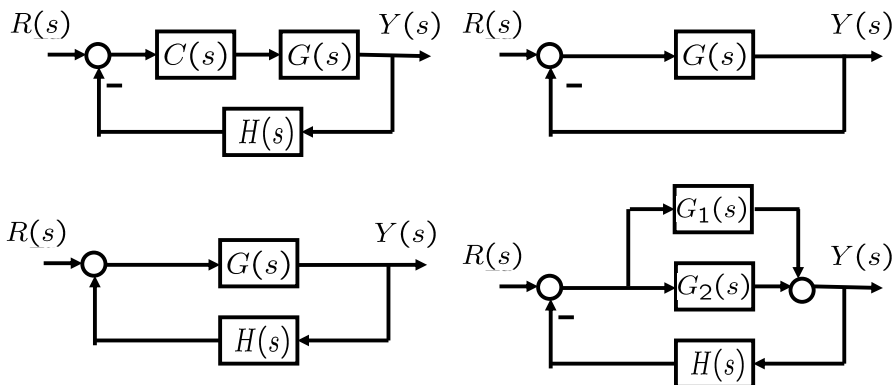
**Memorize this!**

$$\begin{pmatrix} G(s) & : \text{forward gain} \\ G(s)H(s)(-1) & : \text{loop gain} \end{pmatrix}$$

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## Ex: Derivation of transfer functions

Compute transfer functions from  $R(s)$  to  $Y(s)$ .



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## DC motor: Transfer functions (TF)

$$\frac{\Omega(s)}{E_a(s)} =$$

$$\frac{\Omega(s)}{T_L(s)} =$$

**2<sup>nd</sup> order system**

$$\rightarrow \Omega(s) = G_1(s)E_a(s) + G_2(s)T_L(s)$$

$$\rightarrow \Theta(s) = \frac{1}{s}\Omega(s) = \frac{1}{s}(G_1(s)E_a(s) + G_2(s)T_L(s))$$

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## DC motor: Transfer functions (cont'd)

**Note:** In many cases  $L_a \ll R_a$ . Then, an approximated TF is obtained by setting  $L_a=0$ .

$$\frac{\Omega(s)}{E_a(s)} = \frac{K_\tau}{(L_a s + R_a)(J s + B) + K_b K_\tau} \approx \frac{K_\tau}{R_a(J s + B) + K_b K_\tau}$$
$$=: \frac{K}{T s + 1} \quad \left( K := \frac{K_\tau}{R_a B + K_b K_\tau}, T = \frac{R_a J}{R_a B + K_b K_\tau} \right)$$

*2<sup>nd</sup> order system*  $\implies$  *1<sup>st</sup> order system*

$$\frac{\Theta(s)}{E_a(s)} = \frac{K}{s(T s + 1)}$$

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## Summary and Exercises

- Modeling of DC motor
  - What is DC motor and how does it work?
  - Derivation of a transfer function
  - Block diagram with feedback
- Next
  - **Stability** of linear control systems, one of the most important topics in feedback control
- Exercises
  - Read Section 2.7, 2.8.
  - Go over the derivation for DC motor transfer functions by yourself. Obtain  $T(s)/E_a(s)$ .

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Main message until this point

*Many systems  
can be represented  
as **transfer functions!***

Using the transfer functions, ....  
(to be continued)

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