ME451: Control Systems

Lecture 4
Modeling of electrical systems

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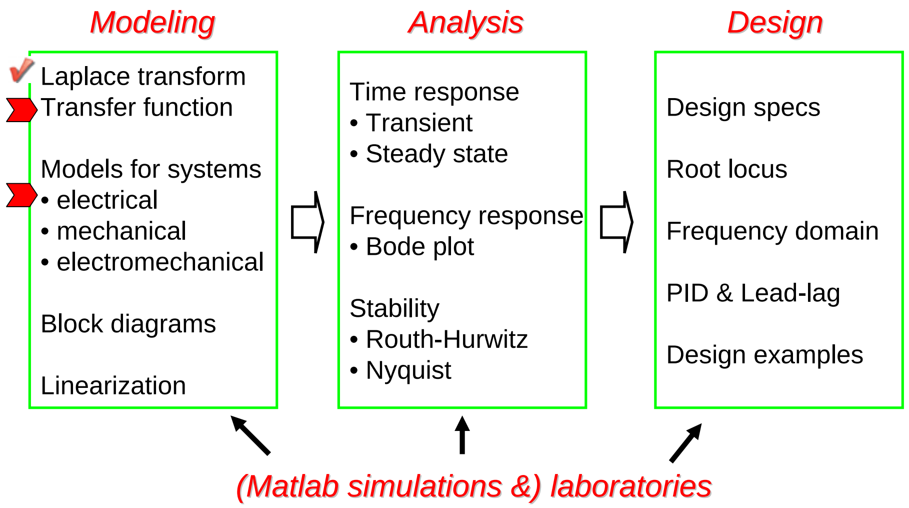
Course roadmap



Modeling

✓ Laplace transform
Transfer function

Models for systems
- electrical
- mechanical
- electromechanical

Block diagrams

Linearization

Analysis

Time response
- Transient
- Steady state

Frequency response
- Bode plot

Stability
- Routh-Hurwitz
- Nyquist

Design

Design specs

Root locus

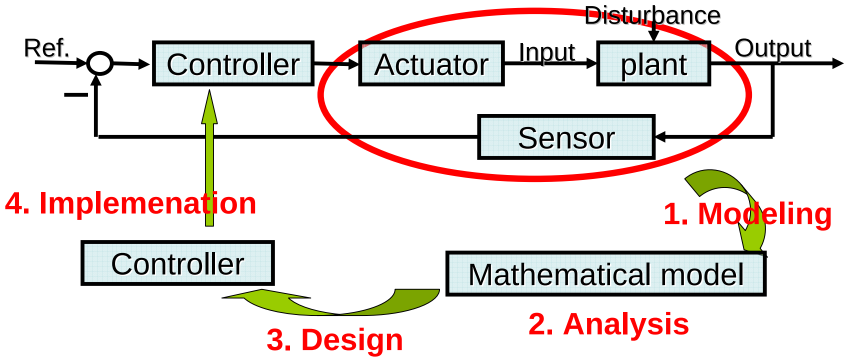
Frequency domain

PID & Lead-lag

Design examples

(Matlab simulations &) laboratories

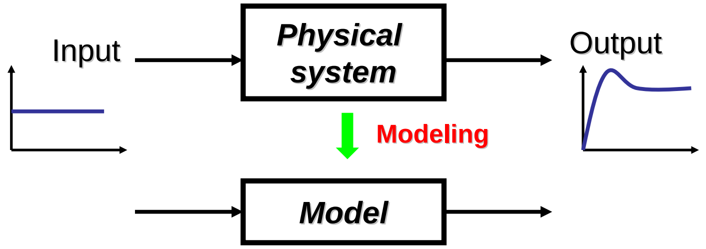
Controller design procedure (review)



- What is the “mathematical model”?
- Transfer function
- Modeling of electrical circuits

Mathematical model

- Representation of the input-output (signal) relation of a physical system



- A model is used for the **analysis** and **design** of control systems.

Important remarks on models

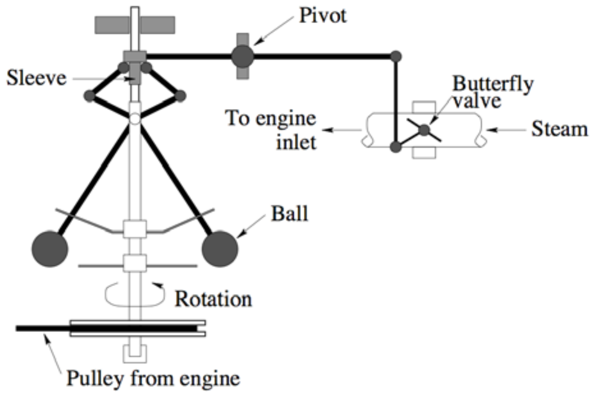
- Modeling is the most important and difficult task in control system design.
- No mathematical model exactly represents a physical system.

$$\text{Math model} \neq \text{Physical system}$$
$$\text{Math model} \approx \text{Physical system}$$

- Do not confuse models with physical systems!
- In this course, we may use the term "system" to mean a mathematical model.

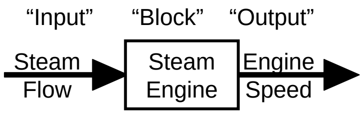
A Brief Control History

- 1788: James Watt's fly-ball governor
  - Mechanical feedback control of steam supply to an engine



The Block Diagram

- Communication tool for Engineering Systems
  - Composed of Blocks with inputs and outputs


"Input" "Block" "Output"

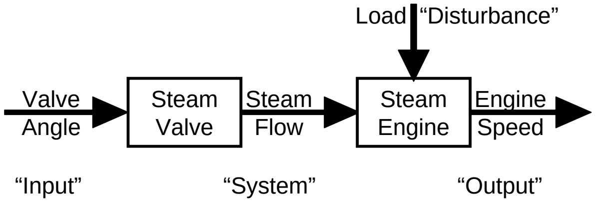
Steam Flow

Steam Engine

Engine Speed

The Block Diagram

- Blocks Connect to form systems
  - Outputs of one block becomes input to another


Load "Disturbance"

Valve Angle

Steam Valve

Steam Flow

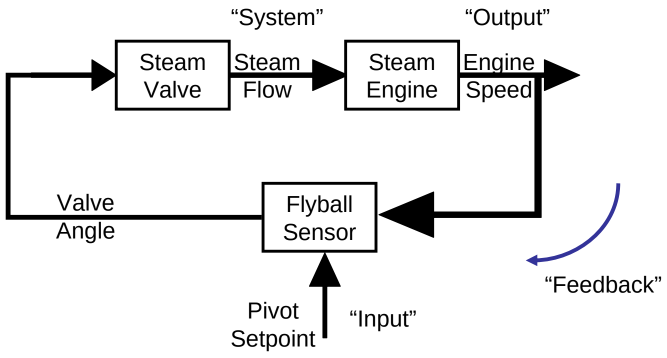
Steam Engine

Engine Speed

"Input" "System" "Output"

The Block Diagram

- Blocks Connect to form systems
  - Outputs of one block becomes input to another



"System"

"Output"

Steam Valve

Steam Flow

Steam Engine

Engine Speed

Valve Angle

Flyball Sensor

"Feedback"

Pivot Setpoint

"Input"

Transfer function

- A transfer function is defined by

$$G(s) := \frac{Y(s)}{U(s)}$$

Laplace transform of system output

Laplace transform of system input

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

- A system is assumed to be at rest. (Zero initial condition)

Impulse response

- Suppose that u(t) is the unit impulse function and system is at rest.

$$\begin{array}{l} u(t) = \delta(t) \\ U(s) = 1 \end{array} \longrightarrow \boxed{\textit{System}} \xrightarrow{g(t)}$$

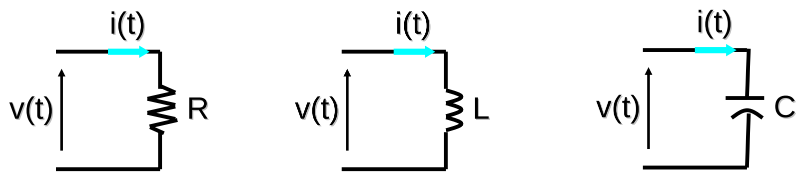
- The output g(t) for the unit impulse input is called ***impulse response***.
- Since U(s)=1, the transfer function can also be defined as the Laplace transform of impulse response: $G(s) := \mathcal{L}\{g(t)\}$

Models of electrical elements: (constitutive equations)

Resistance

Inductance

Capacitance



$$v(t) = Ri(t)$$

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = C\frac{dv(t)}{dt}$$

Laplace transform

$(i(0) = 0)$

$(v(0) = 0)$

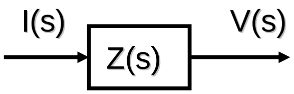
$$\frac{V(s)}{I(s)} = R$$

$$\frac{V(s)}{I(s)} = sL$$

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

Impedance

- Generalized resistance to a sinusoidal alternating current (AC) I(s)
- Z(s): V(s)=Z(s)I(s)



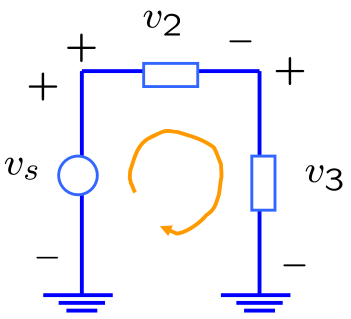
Element	Time domain	Impedance Z(s)
Resistance	$v(t)=Ri(t)$	$\frac{V(s)}{I(s)}=\boxed{R}$
Inductance	$v(t)=L\frac{di(t)}{dt}$	$\frac{V(s)}{I(s)}=\boxed{sL}$
Capacitance	$i(t)=C\frac{dv(t)}{dt}$	$\frac{V(s)}{I(s)}=\boxed{\frac{1}{sC}}$

Memorize!

Kirchhoff's Voltage Law (KVL)

- The algebraic sum of voltage drops around any loop is =0.

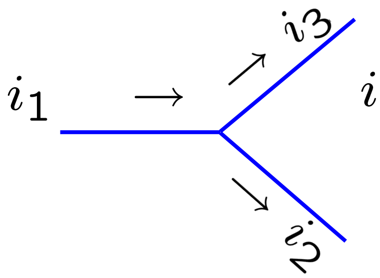
$$v_s - v_2 - v_3 = 0$$



$$v_s = v_2 + v_3$$

Kirchhoff's Current Law (KCL)

- The algebraic sum of currents into any junction is zero.

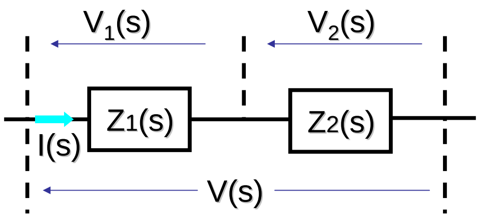


$$i_1 - i_2 - i_3 = 0$$

$$i_1 = i_2 + i_3$$

Impedance computation

- Series connection



$$Z(s) = Z_1(s) + Z_2(s)$$

  - Proof (Ohm's law)

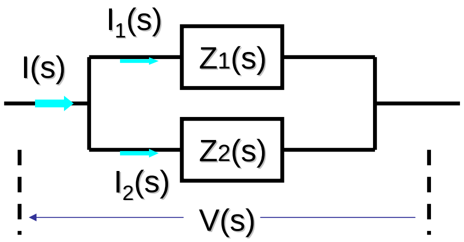
$$V_i(s) = Z_i I(s)$$

$$V(s) = V_1(s) + V_2(s) = \underbrace{(Z_1(s) + Z_2(s))}_{Z(s)} I(s)$$

Impedance computation

- Parallel connection

$$Z(s) = \frac{Z_1(s)Z_2(s)}{Z_1(s)+Z_2(s)}$$



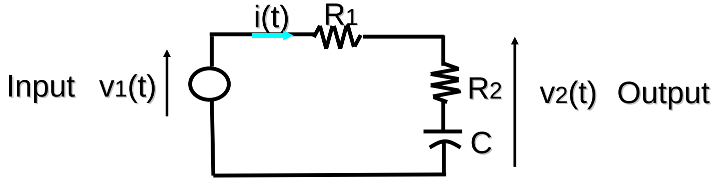
  - Proof (Ohm's law)

$$V(s) = Z_i I_i(s)$$

  - KCL $I(s) = I_1(s) + I_2(s) = \frac{V(s)}{Z_1(s)} + \frac{V(s)}{Z_2(s)}$

$$I(s) = \left(\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)}\right)V(s) = \frac{1}{Z(s)}V(s)$$

Modeling example



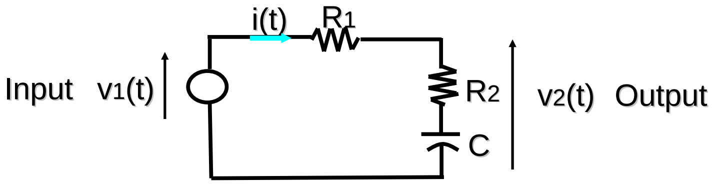
- Kirchhoff voltage law (with zero initial conditions)

$$\begin{aligned} v_1(t) &= (R_1+R_2)i(t) + \tfrac{1}{C}\int_0^t i(\tau)d\tau \\ v_2(t) &= R_2 i(t) + \tfrac{1}{C}\int_0^t i(\tau)d\tau \end{aligned}$$

- By Laplace transform,

$$\begin{aligned} V_1(s) &= (R_1+R_2)I(s) + \tfrac{1}{sC}I(s) \\ V_2(s) &= R_2 I(s) + \tfrac{1}{sC}I(s) \end{aligned}$$

Modeling example (cont'd)

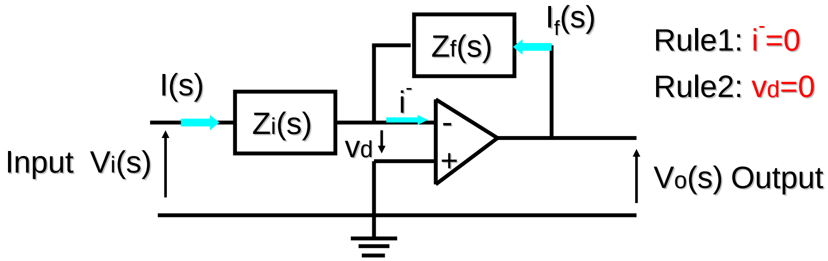


- Transfer function

$$\begin{aligned} G(s) = \frac{V_2(s)}{V_1(s)} &= \frac{R_2+\frac{1}{sC}}{(R_1+R_2)+\frac{1}{sC}} \\ &= \frac{R_2Cs+1}{(R_1+R_2)Cs+1} \end{aligned}$$

(first-order system)

Example: Modeling of op amp



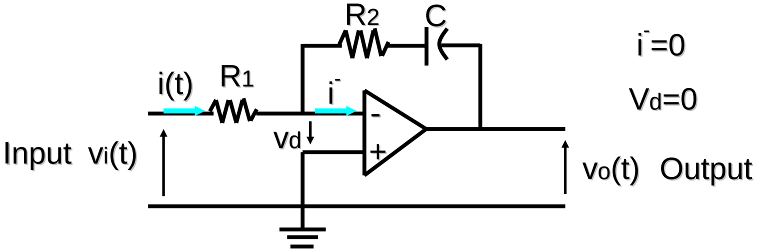
Rule1: $i^- = 0$

Rule2: $v_d = 0$

- Impedance Z(s): V(s)=Z(s)I(s)
- Transfer function of the above op amp:

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_f(s)(I_f(s) = -I(s))}{Z_i(s)I(s)} = -\frac{Z_f(s)}{Z_i(s)}$$

Modeling example: op amp



$i^- = 0$

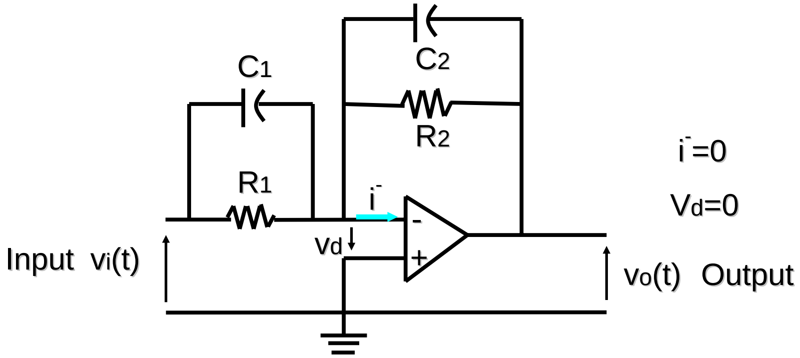
$V_d = 0$

- By the formula in previous two pages,

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{-(R_2 + \frac{1}{sC})}{R_1} = -\frac{R_2Cs + 1}{R_1Cs}$$

(first-order system)

Modeling exercise: op amp



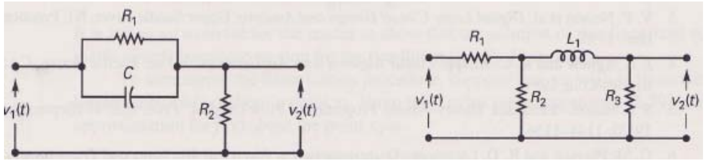
$i^- = 0$

$V_d = 0$

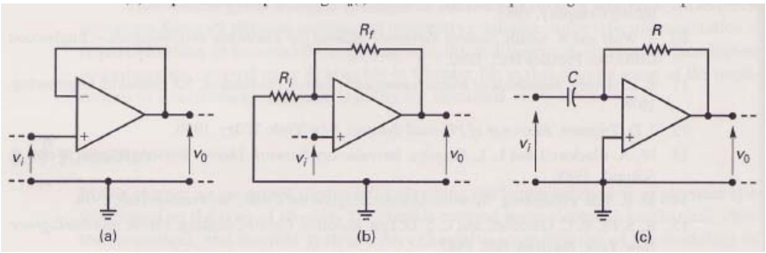
Find the transfer function!

More exercises in the textbook

- Find a transfer function from v_1 to v_2.



- Find a transfer function from v_i to v_o.



Summary & Exercises

- Modeling
  - Modeling is an important task!
  - Mathematical model
  - Transfer function
  - Modeling of electrical systems
- Next, modeling of mechanical systems
- Exercises
  - Read Sections 2.2, 2.3
  - Solve problems 2.1, 2.2, and 2.3 in page 62