ME451: Automatic Control

Lecture 24
Bode diagram of connected systems

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Course roadmap

Modeling
- ✔ Laplace transform
- ✔ Transfer function
- ✔ Models for systems
  - ✔ electrical
  - ✔ mechanical
  - ✔ electromechanical
- ✔ Block diagrams
- ✔ Linearization

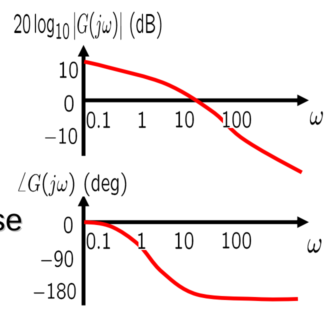
Analysis
- ✔ Time response
  - ✔ Transient
  - ✔ Steady state
- ✔ Frequency response
  - ➤ Bode plot
- ✔ Stability
  - ✔ Routh-Hurwitz
  - • Nyquist

Design
- Design specs
- ✔ Root locus
- Frequency domain
- ✔ PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

Sketching Bode plot

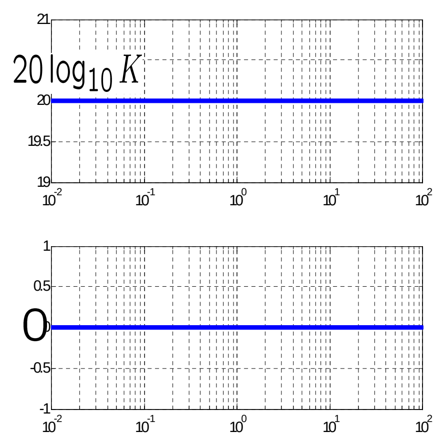
- Basic functions
  - ✔ Constant gain
  - ✔ Differentiator and integrator
  - ✔ Double integrator
  - ✔ First order system and its inverse
  - Second order system
  - Time delay
- Product of basic functions
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.



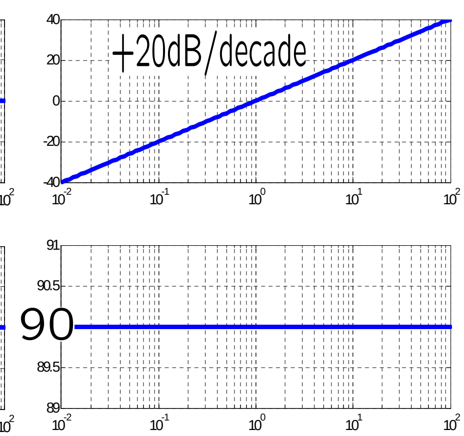
Main advantage of Bode plot!

Bode plot of basic functions (review)

- G(s)=K

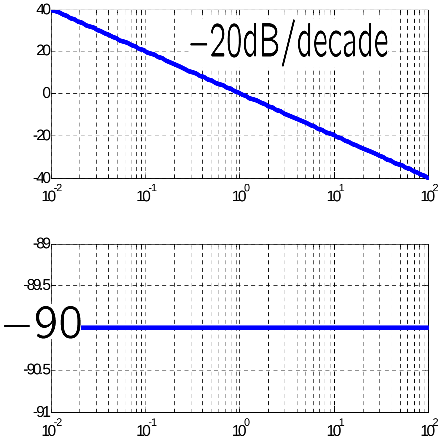


- G(s)=s

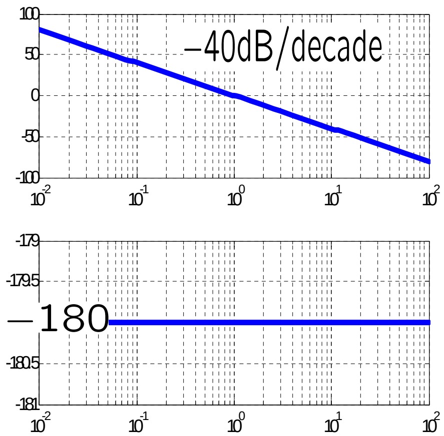


Bode plot of basic functions (review)

- G(s)=1/s

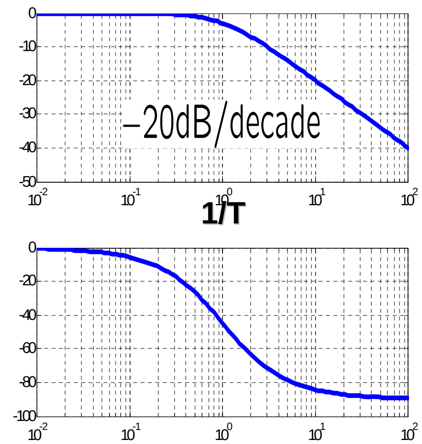


- G(s)=1/(s*s)

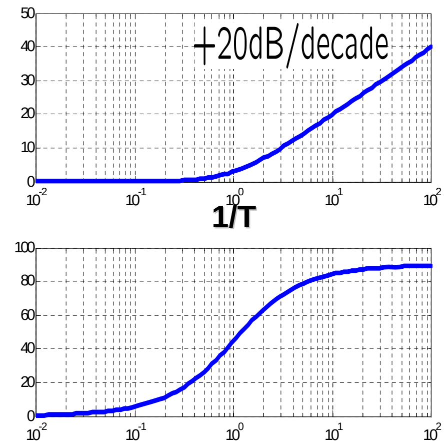


Bode plot of basic functions (review)

- G(s)=1/(Ts+1)



- G(s)=Ts+1



Bode plot of a 2nd order system

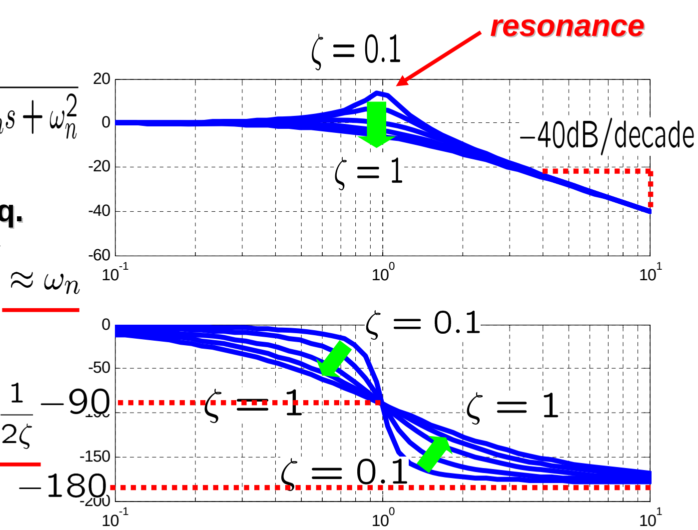
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Resonant freq.

$$\omega_n\sqrt{1-2\zeta^2} \approx \omega_n$$

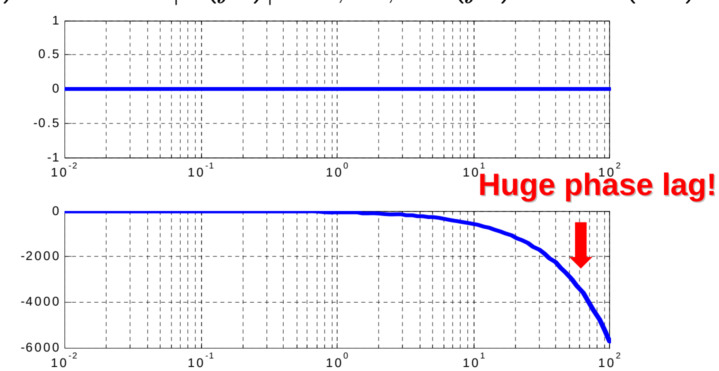
Peak gain

$$\frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta}$$



Bode plot of a time delay

$$G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall\omega,\ \angle G(j\omega) = -\omega T(\text{rad})$$



The phase lag causes instability of the closed-loop system, and thus, the difficulty in control.

An advantage of Bode plot

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2.
  - Gain

$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

  - Phase

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- We use this property to design C(s) so that G(s)C(s) has a "desired" shape of Bode plot.

Short proofs

- Use polar representation

$$G_1(j\omega) = |G_1(j\omega)|e^{j\angle G_1(j\omega)} \quad G_2(j\omega) = |G_2(j\omega)|e^{j\angle G_2(j\omega)}$$

Then,

$$\begin{aligned} G_1(j\omega)G_2(j\omega) &= |G_1(j\omega)||G_2(j\omega)|e^{j\angle G_1(j\omega)}e^{j\angle G_2(j\omega)} \\ &= |G_1(j\omega)||G_2(j\omega)|e^{j\{\angle G_1(j\omega)+\angle G_2(j\omega)\}} \end{aligned}$$

Therefore,

$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)|\cdot|G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

Example 1

- Sketch the Bode plot of a transfer function

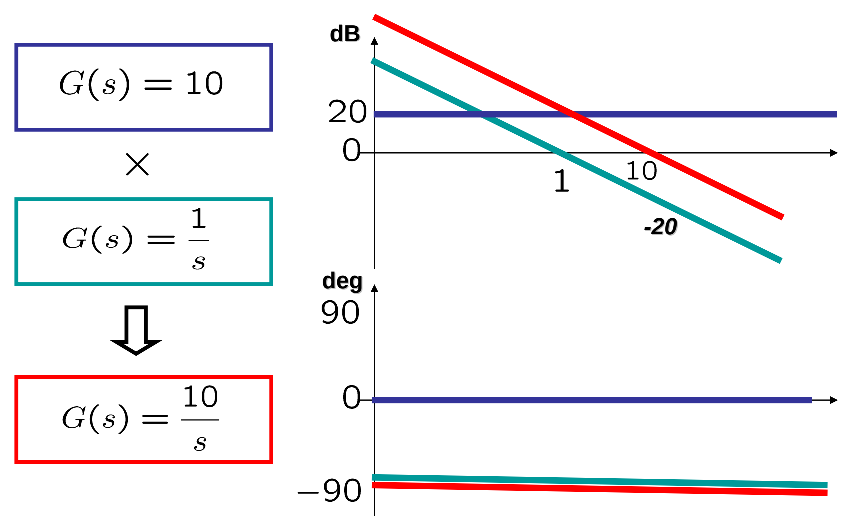
$$G(s) = \frac{10}{s}$$

1. Decompose G(s) into a product form:

$$G(s) = 10 \cdot \frac{1}{s}$$

2. Sketch a Bode plot for each component on the same graph.
3. Add them all on both gain and phase plots.

Example 1 (cont'd)



Example 2

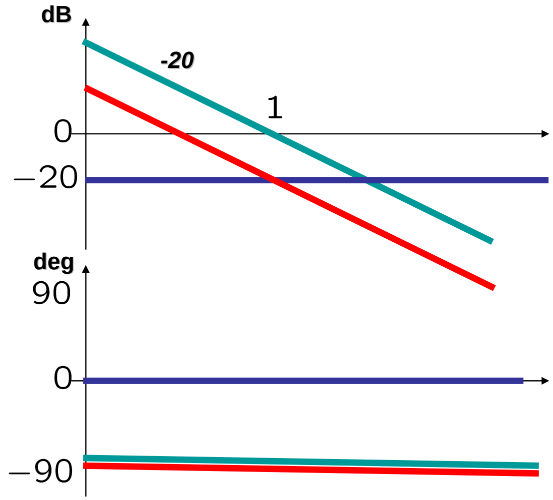
$$G(s) = 0.1$$

$\times$

$$G(s) = \frac{1}{s}$$

$\Downarrow$

$$G(s) = \frac{0.1}{s}$$



Example 3

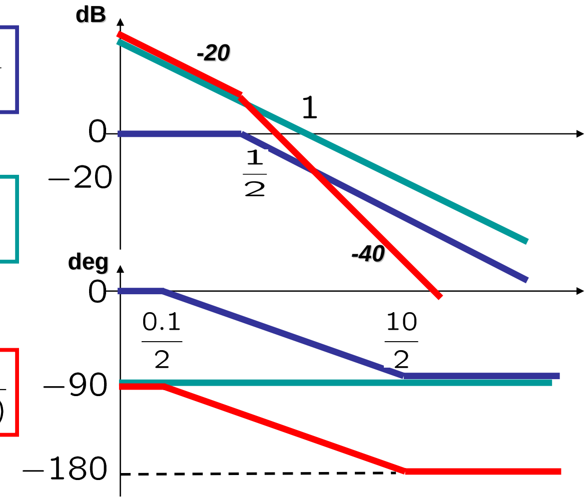
$$G(s) = \frac{1}{2s+1}$$

$\times$

$$G(s) = \frac{1}{s}$$

$\Downarrow$

$$G(s) = \frac{1}{s(2s+1)}$$



Example 4

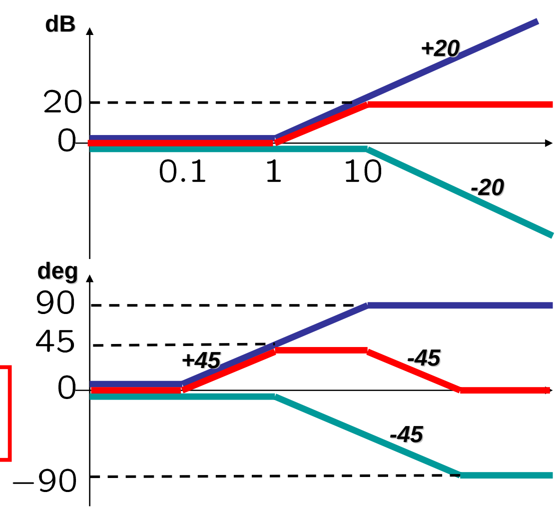
$$G(s) = s + 1$$

$\times$

$$G(s) = \frac{1}{0.1s+1}$$

$\Downarrow$

$$G(s) = \frac{10(s+1)}{(s+10)}$$



Example 5

$$G(s) = 2$$

$\times$

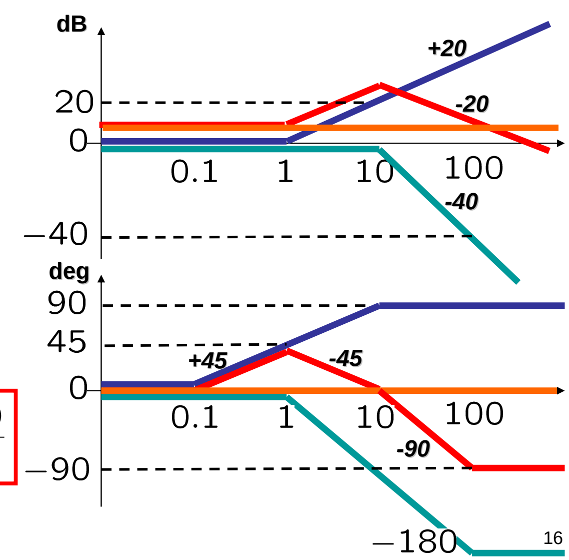
$$G(s) = s + 1$$

$\times$

$$G(s) = \frac{1}{(0.1s+1)^2}$$

$\Downarrow$

$$G(s) = \frac{200(s+1)}{(s+10)^2}$$



Summary and exercises

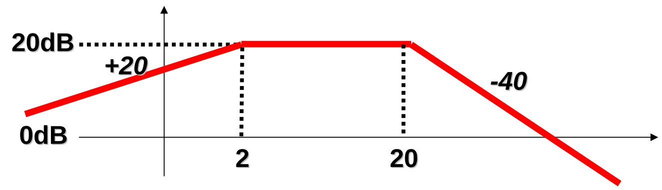
- Sketching Bode plot
  - basic functions
  - connections of basic functions
- Exercise
  - Read Section 8.2 of the textbook.

Exercises

- Sketch the Bode plot of:

$$G(s) = \frac{20}{s(s+1)^2} \quad G(s) = \frac{8s}{(s+1)^2} \quad G(s) = \frac{s+2}{s^2} \quad G(s) = \frac{2}{s^2(s+2)}$$

- Find a transfer function having the gain plot:



Ans. $G(s) = \dfrac{5s}{(\frac{1}{2}s+1)(\frac{1}{20}s+1)^2}$