

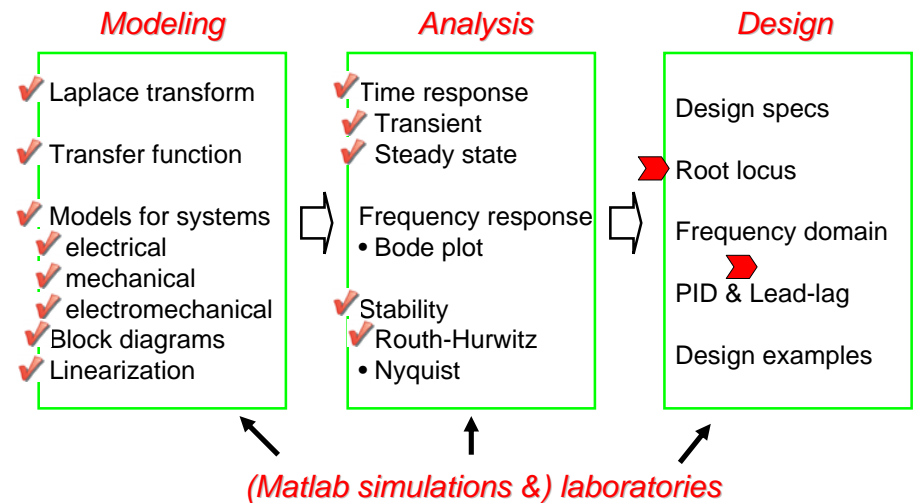
ME451: Control Systems

Lecture 20

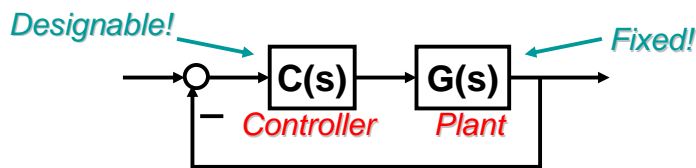
Root locus: Lead compensator design

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Course roadmap



Closed-loop design by root locus

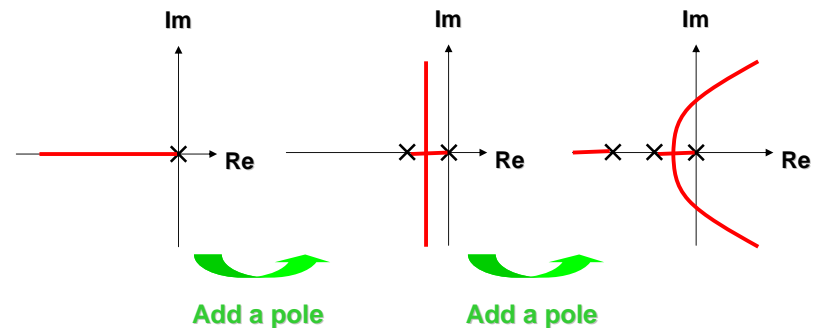


- Place closed-loop poles at desired location
 - by tuning the gain $C(s)=K$. (for time domain specs)
- If root locus does not pass the desired location, then reshape the root locus
 - by adding poles/zeros to $C(s)$. (How?)

Compensation

General effect of addition of poles

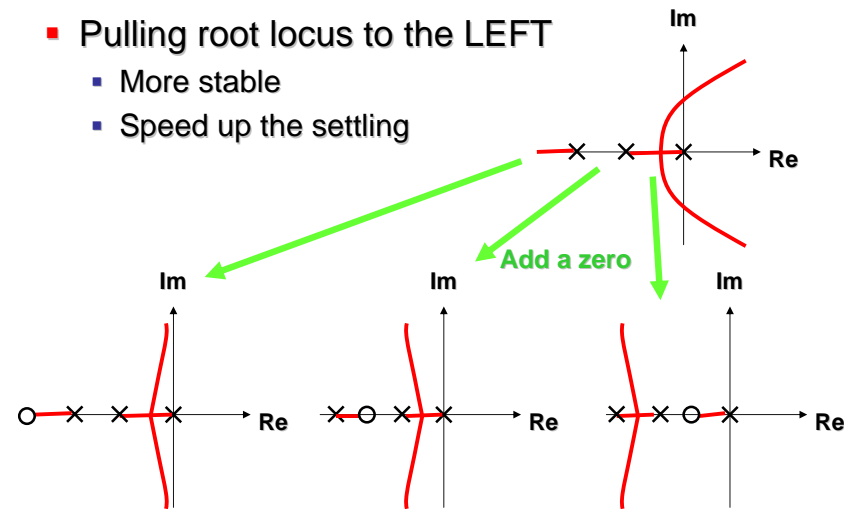
- Pulling root locus to the RIGHT
 - Less stable
 - Slow down the settling



General effect of addition of zeros

- Pulling root locus to the LEFT

- More stable
- Speed up the settling



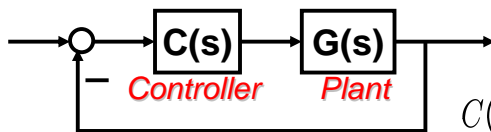
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Some remarks

- Adding only zero $C(s) = s + z, (z > 0)$
 - often problematic because such controller amplifies the **high-frequency noise**.
- Adding only pole $C(s) = 1/(s + p), (p > 0)$
 - often problematic because such controller generates a **less stable** system (by moving the closed-loop poles to the right).
- These facts can be explained by using *frequency response analysis*.
- Add both zero and pole!

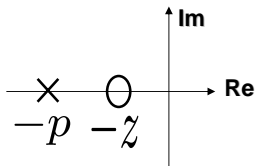
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Lead and lag compensators

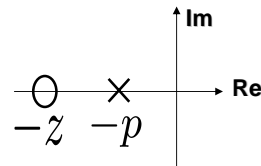


$$C(s) = K \frac{s + z}{s + p}, (z > 0, p > 0)$$

- Lead compensator



- Lag compensator



Why these are called "lead" and "lag"?

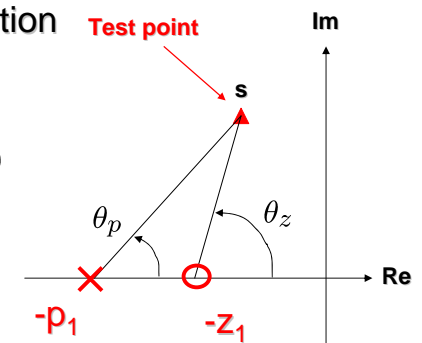
We will see that from frequency response in this class.

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Lead compensator

- Positive angle contribution

$$\angle C_{Lead}(s) = \theta_{Lead} > 0$$

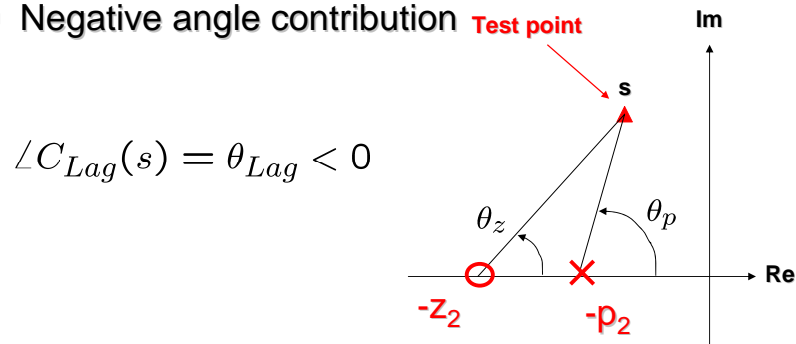


$$\begin{aligned} \angle C_{Lead}(s) &= \angle \frac{s + z_1}{s + p_1} = \angle(s + z_1) - \angle(s + p_1) \\ &= \theta_z - \theta_p = \theta_{Lead} > 0 \end{aligned}$$

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Lag compensator

- Negative angle contribution **Test point**



$$\angle C_{Lag}(s) = \theta_{Lag} < 0$$

$$\begin{aligned} \angle C_{Lag}(s) &= \angle \frac{s + z_2}{s + p_2} = \angle(s + z_2) - \angle(s + p_2) \\ &= \theta_z - \theta_p = \theta_{Lag} < 0 \end{aligned}$$

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Roles of lead and lag compensators

- Lead compensator (Today)

- Improve **transient response**
- Improve **stability**

$$C_{Lead}(s) = K_1 \frac{s + z_1}{s + p_1}$$

- Lag compensator (Next)

- Reduce **steady state error**

$$C_{Lag}(s) = K_2 \frac{s + z_2}{s + p_2}$$

- Lead-lag compensator (Next)

- Take into account all the above issues.

$$C_{LL}(s) = C_{Lead}(s)C_{Lag}(s)$$

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Radar tracking system

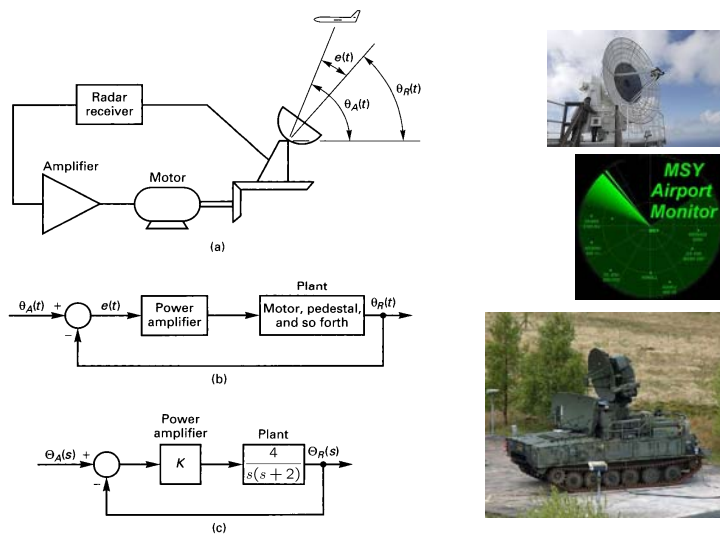


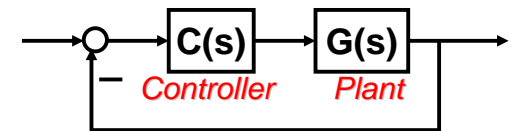
Figure 7.1 Radar tracking system.

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Lead compensator design

- Consider a system

$$G(s) = \frac{4}{s(s + 2)}$$

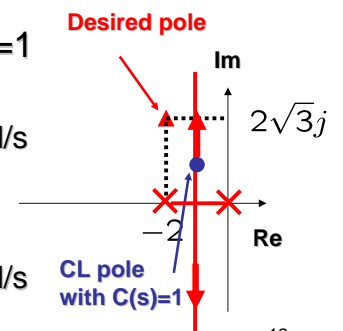


- Analysis of CL system for C(s)=1

- Damping ratio $\zeta=0.5$
- Undamped natural freq. $\omega_n=2$ rad/s

- Performance specification

- Damping ratio $\zeta=0.5$
- Undamped natural freq. $\omega_n=4$ rad/s



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Angle and magnitude conditions (review)

- A point s to be on root locus \leftrightarrow it satisfies

- Angle condition**

Odd number

$$\angle L(s) = 180^\circ \times (2k + 1), \quad k = 0, \pm 1, \pm 2, \dots$$

- For a point on root locus, gain K is obtained by

- Magnitude condition**

$$|L(s)| = \frac{1}{K}$$

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Lead compensator design (cont'd)

Evaluate $G(s)$ at the desired pole.

$$G(-2 + 2\sqrt{3}j) = \frac{4}{(-2 + 2\sqrt{3}j)2\sqrt{3}j} = \frac{-1}{3 + \sqrt{3}j}$$

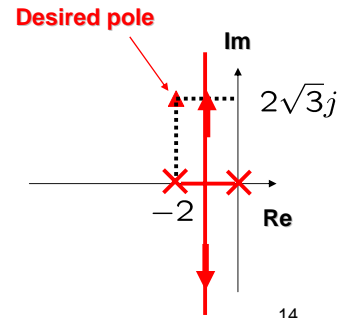
- If *angle condition* is satisfied, compute the corresponding K .

- In this example,

$$\angle G(-2 + 2\sqrt{3}j) = -210$$

Angle condition is not satisfied.

- Angle deficiency $\phi = 30$**



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Lead compensator design (cont'd)

To compensate angle deficiency, design a lead compensator $C(s)$

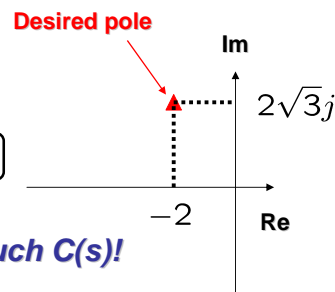
$$C(s) = K \frac{s + z}{s + p}$$

satisfying

$$\angle C(-2 + 2\sqrt{3}j) = 30 (= \phi)$$

$$\left[\rightarrow \angle GC(-2 + 2\sqrt{3}j) = -180 \right]$$

There are many ways to design such $C(s)$!

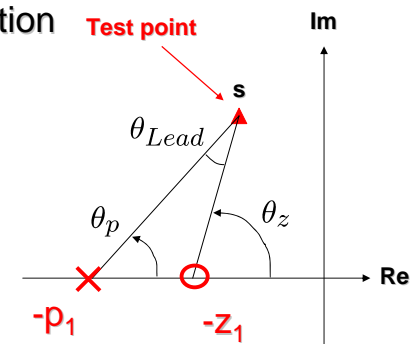


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Lead compensator

- Positive angle contribution**

$$\angle C_{Lead}(s) = \theta_{Lead} > 0$$



- Triangle**

$$\theta_p + \theta_{Lead} + (\pi - \theta_z) = \pi$$

$$\theta_z - \theta_p = \theta_{Lead}$$

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How to select pole and zero?

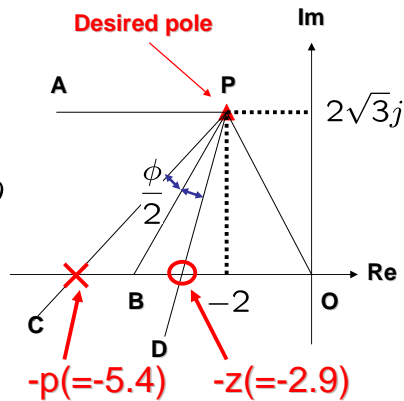
- Draw horizontal line PA
- Draw line PO
- Draw bisector PB

$$\angle APB = \angle BPO = \frac{1}{2} \angle APO$$

- Draw PC and PD

$$\angle CPB = \angle BPD = \frac{\phi}{2}$$

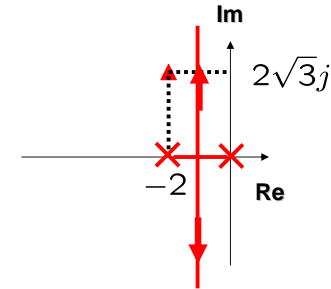
- Pole and zero of C(s) are shown in the figure.



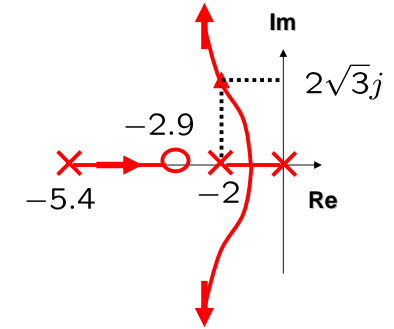
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Comparison of root locus

- G(s)



- G(s)C(s)



Improved stability!

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How to design the gain K?

- Lead compensator

$$C(s) = K \frac{s + 2.9}{s + 5.4}$$

- Open loop transfer function

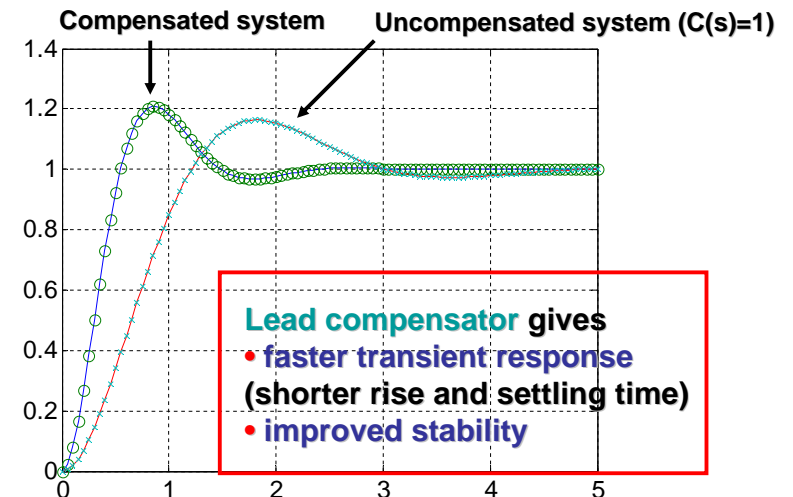
$$G(s)C(s) = K \frac{4(s + 2.9)}{s(s + 2)(s + 5.4)}$$

- Magnitude condition

$$K \left| \frac{4(s + 2.9)}{s(s + 2)(s + 5.4)} \right|_{s=-2+2\sqrt{3}j} = 1 \quad \rightarrow \quad K = 4.675$$

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Comparison of step responses



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Error constants

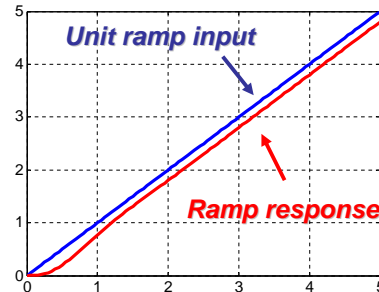
$$G(s)C(s) = \frac{4}{s(s+2)} \cdot \frac{4.675(s+2.9)}{s+5.4}$$

- Step-error constant

$$K_p := \lim_{s \rightarrow 0} G(s)C(s) = \infty$$

- Ramp-error constant

$$K_v := \lim_{s \rightarrow 0} sG(s)C(s) = 5.02$$



Lag compensator can be used to reduce steady-state error. (Next lecture)

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Summary and exercises

- Controller design based on root locus
 - General effects of addition of pole and zero
 - Lead lag compensator realization with op amp
 - Lead compensator design
 - **Lead compensator improves stability and transient response.**
- Next, lag & lead-lag compensator design

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