

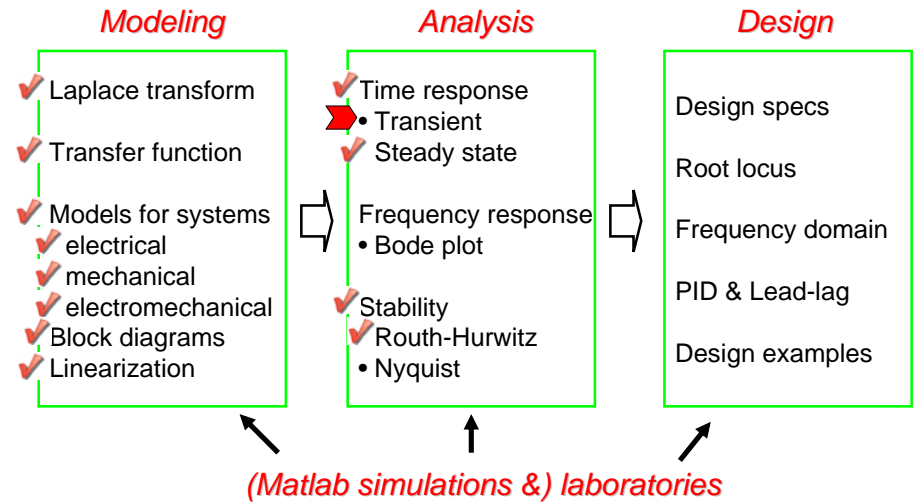
ME451: Control Systems

Lecture 14

Time response of 1st-order systems

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Course roadmap



Performance measures (review)

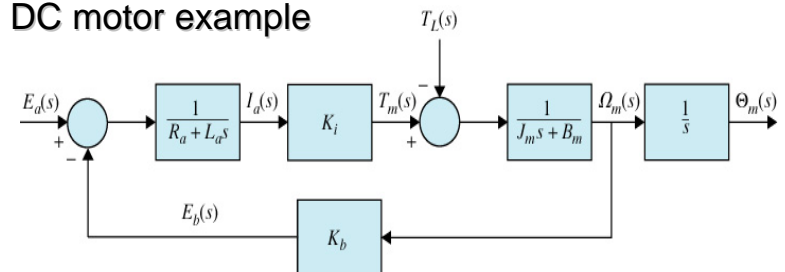
- Transient response (Today's lecture)
 - Peak value
 - Peak time
 - Percent overshoot
 - Delay time
 - Rise time
 - Settling time
 - Steady state response (Done)
 - Steady state error
- Next, we will connect these measures with s-domain.**

First-order system

- A **standard form** of the first-order system:

$$G(s) = \frac{K}{Ts + 1}$$

- DC motor example



DC motor example (cont'd)

- If $L_a \ll R_a$, we can obtain a 1st-order system

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_i}{(L_a s + R_a)(J_m s + B_m) + K_b K_i} \approx \frac{K_i}{R_a(J_m s + B_m) + K_b K_i}$$

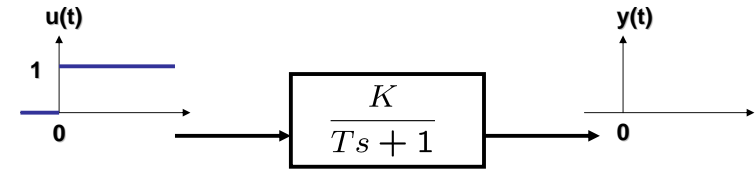
$$=: \frac{K}{T s + 1} \left(K := \frac{K_i}{R_a B_m + K_b K_i}, T := \frac{R_a J_m}{R_a B_m + K_b K_i} \right)$$

- TF from motor input voltage to
 - motor **speed** is 1st-order
 - motor **position** is 2nd-order

5

Step response for 1st-order system

- Input a **unit step function** to a first-order system. Then, what is the output?



$$Y(s) = G(s)U(s)$$

$$= \frac{K/T}{s+1/T} \cdot \frac{1}{s}$$

$$= \frac{K}{s} + \frac{-K}{s+1/T}$$

(Partial fraction expansion)

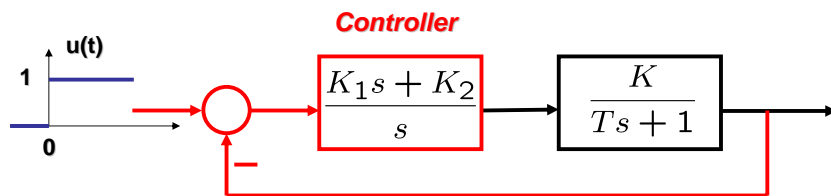
$$\xrightarrow{\mathcal{L}^{-1}} y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$= \frac{K(1 - e^{-t/T})}{(t > 0)}$$

6

How to eliminate steady-state error

- Make a feedback system with a controller having an integrator (*copy of Laplace transform of a unit step function*):



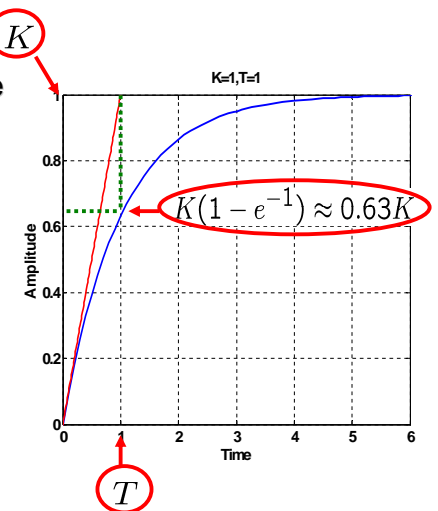
One has to select controller parameters to stabilize the feedback system.
Suppose $K=T=1$, and obtain such parameters!

7

Meaning of K and T

- K : Gain**
 - Final (steady-state) value

$$\lim_{t \rightarrow \infty} y(t) = K$$
- T : Time constant**
 - Time when response rises 63% of final value
 - Indication of **speed** of response (convergence)
 - Response is faster as T becomes smaller.



8

DC gain for a general system

- DC gain : Final value of a unit step response
 - For first-order systems, DC gain is K.
 - For a **general stable system G**, DC gain is G(0).

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sG(s) \frac{1}{s} = G(0)$$

Final value theorem

- Examples

$$G(s) = \frac{3}{2s + 5} \quad G(0) = \frac{3}{5}$$

$$G(s) = \frac{7}{s^2 + 2s + 3} \quad G(0) = \frac{7}{3}$$

9

Settling time of 1st-order systems

$$y(t) = K(1 - e^{-t/T})$$

- Relation between time and exponential decay

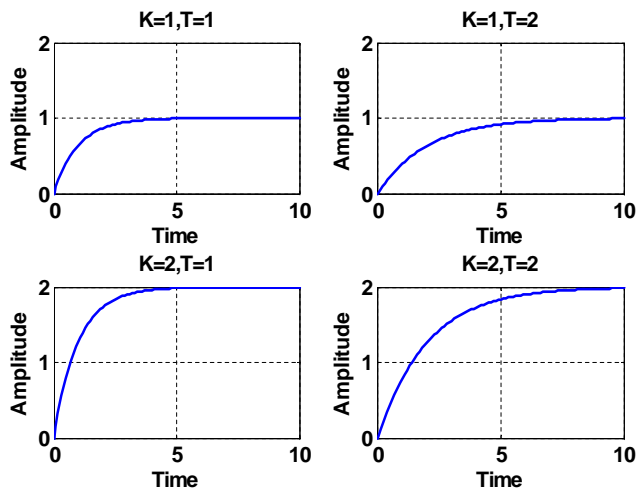
t	$e^{-t/T}$
0	1
T	0.3679
$2T$	0.1353
$3T$	0.0498
$4T$	0.0183
$5T$	0.0067

← 5% settling time is about 3T!

← 2% settling time is about 4T!

10

Step response for some K & T



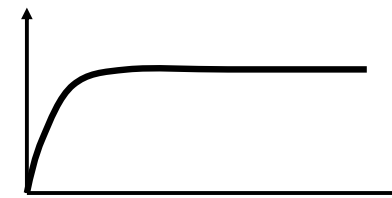
11

System identification

- Suppose that we have a “black-box” system



- Obtain step response

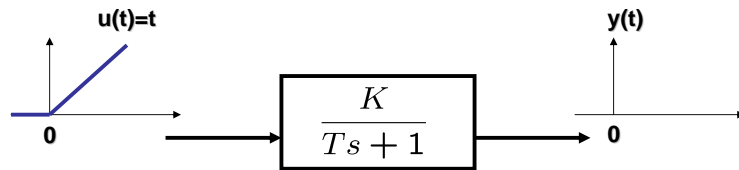


- Can you obtain a transfer function? How?

12

Ramp response for 1st-order system

- Input a **unit ramp function** to a 1st-order system. Then, what is the output?



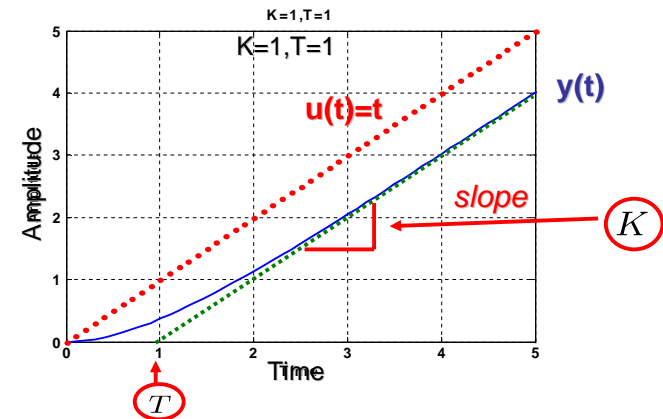
$$\begin{aligned}
 Y(s) &= G(s)U(s) \\
 &= \frac{K/T}{s+1/T} \cdot \frac{1}{s^2} \\
 &= \frac{K}{s^2} + \frac{-KT}{s} + \frac{KT}{s+1/T}
 \end{aligned}$$

(Partial fraction expansion)

$$\begin{aligned}
 \mathcal{L}^{-1} y(t) &= \mathcal{L}^{-1}\{Y(s)\} \\
 &= K(t - T + Ke^{-t/T}) \quad (t > 0)
 \end{aligned}$$

13

Ramp response for 1st-order system

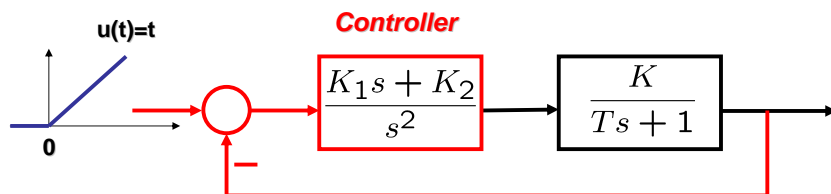


- Steady state response $y_{ss}(t) = K(t - T)$
- We may want to modify the system s.t. $y_{ss}(t) = u(t)$

14

How to eliminate steady-state error

- Make a feedback system with a controller having a double integrator (*copy of Laplace transform of ramp function*):



One has to select controller parameters to stabilize the feedback system.
Suppose $K=T=1$, and obtain such parameters!

15

Summary and exercises

- Time response for 1st-order systems
 - Step and ramp responses
 - Time constant and DC gain
 - System identification
- Next, time response for 2nd-order systems
- Exercises
 - Read Section 4.
 - Solve Problems 4.1 and 4.2.

16