ME451: Control Systems

Lecture 11
Routh-Hurwitz criterion: Control examples

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Course roadmap

Modeling

- ✔ Laplace transform
- ✔ Transfer function
- ✔ Models for systems
  - ✔ electrical
  - ✔ mechanical
  - ✔ electromechanical
- ✔ Linearization

Analysis

- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot
- ✔ Stability
  - Routh-Hurwitz
  - Nyquist

Design

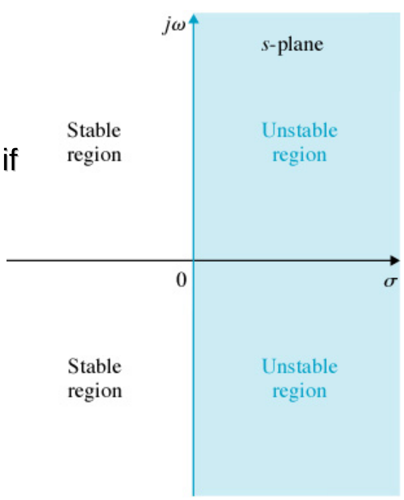
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

Stability summary (review)

Let s_i be poles of rational G. Then, G is …

- (BIBO, asymptotically) stable if $\text{Re}(s_i)<0$ for all i.
- marginally stable if
  - $\text{Re}(s_i)\le 0$ for all i, and
  - simple root for $\text{Re}(s_i)=0$
- unstable if
  it is neither stable nor marginally stable.



Routh-Hurwitz criterion (review)

$$Q(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

s^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	a_{n-7}	$\cdots$
s^{n-2}	b_1	b_2	b_3	b_4	$\cdots$
s^{n-3}	c_1	c_2	c_3	c_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$			
s^2	k_1	k_2			
s^1	l_1				
s^0	m_1				

The number of roots in the right half-plane is equal to the number of sign changes in the first column of Routh array.

Why no proof in textbooks?

An Elementary Derivation of the Routh–Hurwitz Criterion
Ming-Tzu Ho, Aniruddha Datta, and S. P. Bhattacharyya
IEEE Transactions on Automatic Control
vol. 43, no. 3, 1998, pp. 405-409.

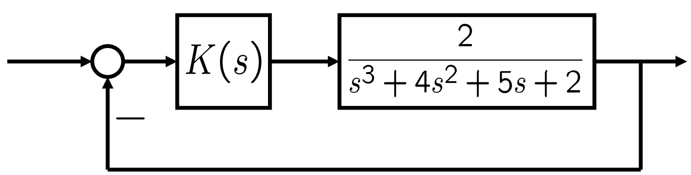
"most undergraduate students are exposed to the Routh–Hurwitz criterion in their first introductory controls course. This exposure, however, is at the purely algorithmic level in the sense that no attempt is made whatsoever to explain why or how such an algorithm works."

Why no proof in textbooks? (cont'd)

"The principal reason for this is that the classical proof of the Routh-Hurwitz criterion relies on the notion of Cauchy indexes and Sturm's theorem, both of which are beyond the scope of undergraduate students."

"Routh-Hurwitz criterion has become one of the few results in control theory that most control engineers are compelled to accept on faith."

Example 1



- Design K(s) that stabilizes the closed-loop system for the following cases.
  - K(s) = K (constant)
  - K(s) = K_P+K_I/s (PI (Proportional-Integral) controller)

Example 1: K(s)=K

- Characteristic equation

$$1+K\frac{2}{s^3+4s^2+5s+2}=0$$

$$\Rightarrow \quad s^3+4s^2+5s+2+2K=0$$

- Routh array

s^3
s^2
s^1
s^0

Example 1: K(s)=KP+KI/s

- Characteristic equation

$$1+\left(K_P+\frac{K_I}{s}\right)\frac{2}{s^3+4s^2+5s+2}=0$$

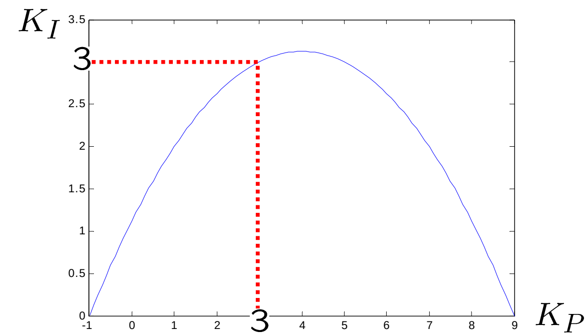
$$\Rightarrow \quad s^4+4s^3+5s^2+(2+2K_P)s+2K_I=0$$

- Routh array

s^4
s^3
s^2
s^1
s^0

Example 1: Range of (KP,KI)

- From Routh array, $K_P<9$, $K_I>0$, $(1+K_P)(9-K_P)-8K_I>0$



Example 1: K(s)=KP+KI/s (cont'd)

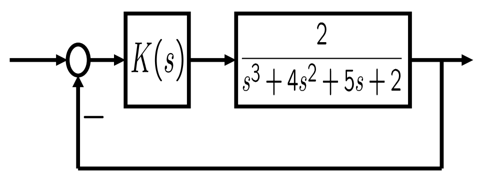
- Select KP=3 (<9)
- Routh array (cont'd)

s^4
s^3
s^2
s^1
s^0

- If we select different KP, the range of KI changes.

Example 1: What happens if KP=KI=3

- Auxiliary equation $3s^2+6=0 \Leftrightarrow s=\pm\sqrt{2}j$

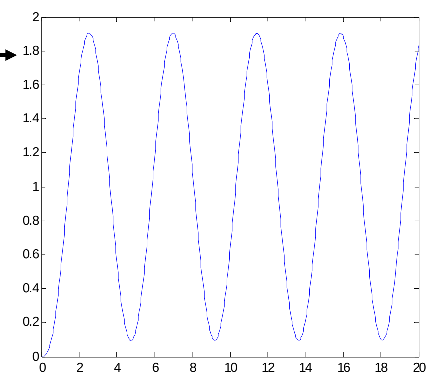


- Oscillation frequency

$$\sqrt{2}(\text{rad/sec})$$

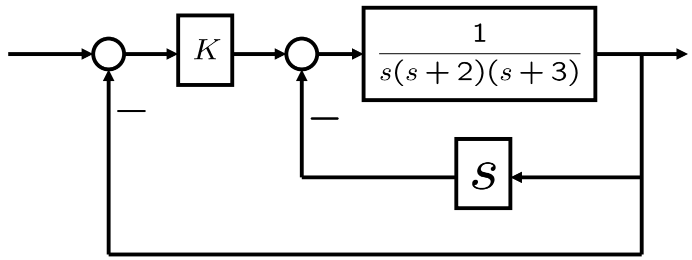
- Period

$$\frac{2\pi}{\sqrt{2}}\approx 4.4(\text{sec})$$



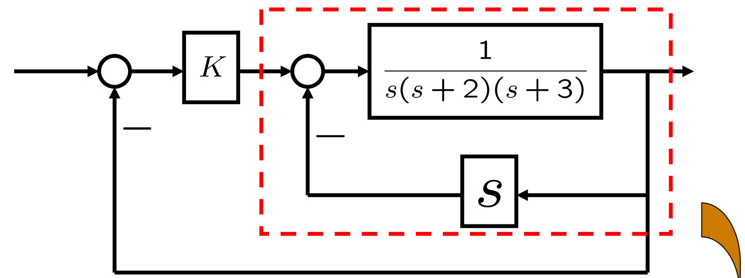
Unit step response

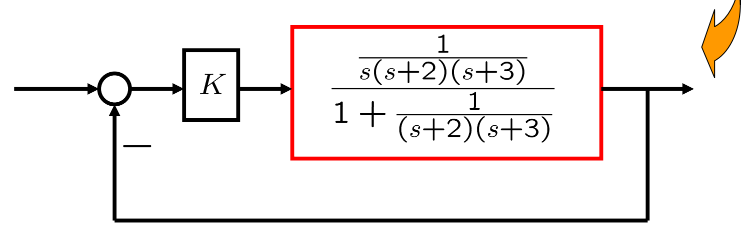
Example 2



- Determine the range of K and a that stabilize the closed-loop system.

Example 2 (cont'd)





Example 2 (cont'd)

- Characteristic equation

$$1 + K\frac{\frac{1}{s(s+2)(s+3)}}{1+\frac{1}{(s+2)(s+3)}} = 0$$

$$\Rightarrow \quad 1 + \frac{K}{s}\cdot\frac{1}{(s+2)(s+3)+1} = 0$$

$$\Rightarrow \quad s(s+2)(s+3) + s + K = 0$$

$$\Rightarrow \quad s^3 + 5s^2 + 7s + K = 0$$

Example 2 (cont'd)

- Routh array $\quad s^3 + 5s^2 + 7s + K = 0$

s^3

s^2

s^1

s^0

- If K=35, oscillation frequency is obtained by the auxiliary equation

$$5s^2 + 35 = 0 \Leftrightarrow s = \pm\sqrt{7}j$$

Summary and Exercises

- Control examples for Routh-Hurwitz criterion
  - P controller gain range for stability
  - PI controller gain range for stability
  - Oscillation frequency
  - Characteristic equation
- Next
  - Time domain specifications
- Exercises
  - Read Chapter 6 again.
  - Redo Examples 1 and 2
  - Do Problem 6.6-(a) and 6.7-(b)-Find the range of K for which the system is stable.

More example 1

$$Q(s) = s^3 + s^2 + s + 1 \ (= (s+1)(s^2+1))$$

Routh array

s^3

s^2

s^1

s^0

No sign changes in the first column → No root in OPEN(!) RHP

More example 2

$$Q(s) = s^5 + s^4 + 2s^3 + 2s^2 + s + 1 \ (= (s+1)(s^2+1)^2)$$

Routh array

s^5

s^4

s^3

s^2

s^1

s^0

More example 3

$$Q(s) = s^4 - 1 \ (= (s+1)(s-1)(s^2+1))$$

Routh array

s^4

s^3

s^2

s^1

s^0

One sign changes in the first column → One root in OPEN(!) RHP