1. (10 pts) A closed loop system has the characteristic equation

\[ T(s) = \frac{1}{\frac{s^3 + 3s^2 + 4s + 2}{1 + \frac{k}{s^3 + 3s^2 + 4s + 2}}} \]

Using the Routh-Hurwitz criterion to find the range of values of \( k \) for which the system is stable.

**Solution:**

\[ T(s) = \frac{1}{s^3 + 3s^2 + 4s + 2 + k} \]

\[ \Delta(s) = s^3 + 3s^2 + 4s + 2 + k = 0 \]

**Routh array**

\[
\begin{array}{ccc}
  s^3 & 1 & 4 \\
  s^2 & 3 & 2+k \\
  s^1 & b_1 & \\
  s^0 & b_2 & \\
\end{array}
\]

- \( b_1 = -\frac{1}{3} (2+k - 12) = \frac{10-k}{3} \)
- \( b_2 = \frac{-1}{b_1} \left| \begin{array}{cc} 3 & 2+k \end{array} \right| = 2+k \)

**Stability:** need \( b_1 > 0, \ \frac{10-k}{3} > 0 \) \( k < 10 \)

and \( b_2 > 0, \ 2+k > 0 \) \( k > -2 \)

**\( k \) range:** \(-2 < k < 10 \) for stability
2. (13 pts) The closed loop system

\[ + \quad \frac{k}{G(s)} \quad H(s) \]

has open-loop poles which are marked below by symbols x on the graph below. There are no finite open-loop zeros. Complete the following steps in forming the root locus for positive k:

- Number of root-locus asymptotes: \( \frac{3}{3} = 3 \)
- Asymptote angles: \( \pm 60^\circ, 180^\circ \)
- Asymptotes intersect the real axis at \( \sigma_a = \frac{3}{2} = -2 \)
- Break points: NONE or Occur at B = \( \text{NONE} \) (Given: there are NOT TWO break points)
- Departure angle at \( s_1 \) is \( \theta_d = 0^\circ \)

Using these quantities, make your root locus sketch on the graph below.
3. (7 pts) The closed loop system is sketched: 

The uncompensated function $G(s)$ open-loop poles which are marked below by symbols x on the graph below. There are no finite open-loop zeros. A compensator is then added to the open-loop transfer function to improve the response. The compensator is of the form $G_c(s) = k(s + a)$, meaning that it adds a finite zero to the open-loop transfer function. (It is a PD compensator). Find the location of this added zero such that the point $s_1 = -2 + 2j$ is on the root locus.

$$\theta_2 - (\theta_{p1} + \theta_{p2} + \theta_{p3}) = r 180^\circ$$
$$\theta_2 - (90^\circ + 90^\circ + 135^\circ) = -180^\circ \quad (r = -1)$$
$$\theta_2 = 135^\circ$$

$\Rightarrow a = 0 \text{ by geometry}$