Chapter 10: Composite Micromechanics

10.1 Problem Statement and Objectives

Given the micromechanical geometry and the material properties of each constituent, it is possible to estimate the effective composite material properties and the micromechanical stress/strain state of a composite material. The objectives of this project are (1) to determine the effective stiffness properties E_1^c, E_2^c, ν_{12}^c, ν_{23}^c of a unidirectional composite material and (2) to determine the strain concentration factor in the matrix region when the composite material is subjected to a uniform transverse normal strain in the X_2 direction.

NOTE TO ME 424 CLASS: Only do Part (2).

10.2 Background

A composite material is often defined as a combination of two or more materials fabricated in such a way that the individual constituents (materials) can still be readily identified in the final form. If designed properly, this combination of materials yields a composite material that exhibits the best properties of each constituent as well as some advantageous properties not exhibited by the individual constituents. One example of such a material is a unidirectional fiber reinforced composite, which is often used in aerospace structures. An idealized micromechanical view of a unidirectional fiber reinforced composite material is shown in Figure 10.1. In these materials, the fibers have a very small diameter and a very high length-to-diameter ratio. This geometry yields excellent stiffness and strength characteristics in the fiber, since the crystals tend to align along the fiber axis and there are fewer internal and surface defects than in the bulk material.

The properties of commonly used fiber materials are given in Table 10.1. These fibers are embedded in another material, often called the matrix material. Matrix materials may be polymers, metals, or ceramics. Some common matrix material properties are given in Table 10.2.

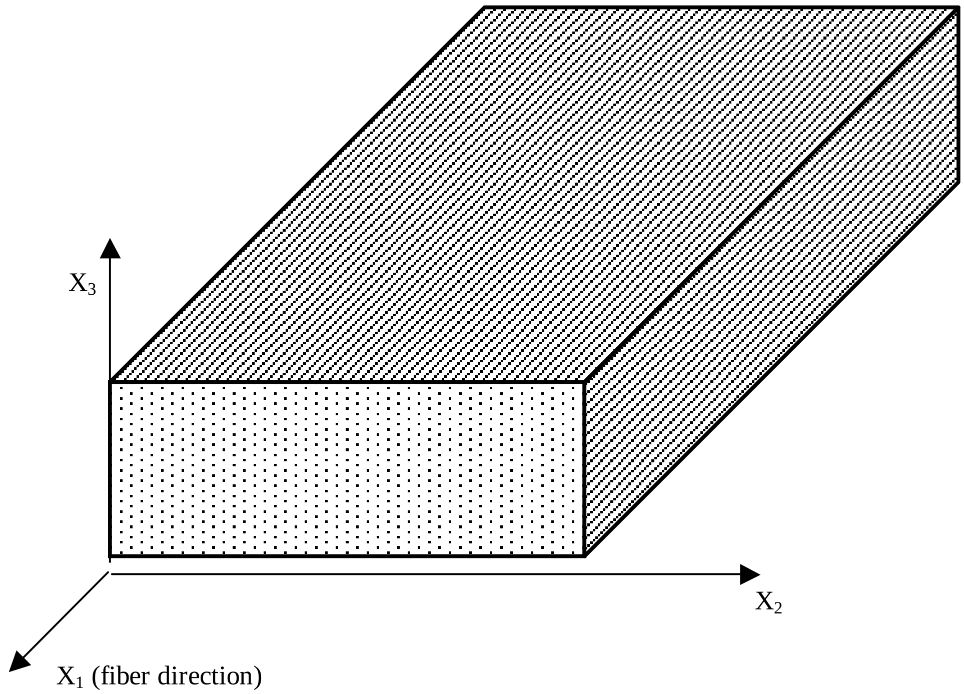

Figure 10.1 Idealized representation of a unidirectional fiber-reinforced material.

Table 10.1 Fiber and Wire Properties[1]

Fiber or Wire	Density, ρ lb/in^3 (kN/m^3)	Tensile Strength, S 10^3 lb/in^2 (GPa)	S/ρ 10^5 in (km)	Tensile Stiffness, E 10^6 lb/in^2 (GPa)	E/ρ 10^7 in (10^6 m)
Aluminum	.097 (26.3)	90 (.62)	9 (24)	10.6 (73)	11 (2.8)
Titanium	.170 (46.1)	280 (1.9)	16 (41)	16.7 (115)	10 (2.5)
Steel	.282 (76.6)	600 (4.1)	21 (54)	30 (207)	11 (2.8)
E-Glass	.092 (25.0)	500 (3.4)	54 (136)	10.5 (72)	11 (2.8)
S-Glass	.090 (24.4)	700 (4.8)	78 (197)	12.5 (86)	14 (3.5)
Carbon	.051 (13.8)	250 (1.7)	49 (123)	27 (190)	53 (14)
Beryllium	.067 (18.2)	250 (1.7)	37 (93)	44 (300)	66 (16)
Boron	.093 (25.2)	500 (3.4)	54 (137)	60 (400)	66 (16)
Graphite	.051 (13.8)	250 (1.7)	49 (123)	37 (250)	72 (18)

Table 10.2 Thermosetting Resin Matrix Properties[2]

Material	Young's Modulus, E (GPa)	Poisson's Ratio, ν	Tensile Strength (MPa)	Compressive Strength (MPa)
Polyester	3.2	0.36	65	130
Epoxy	3.0	0.37	85	130

Due to the high axial stiffness of the fibers, the stiffness of a unidirectional fiber reinforced composite material is very high in the fiber direction and relatively low in the directions perpendicular to the fibers. Hence a unidirectional fiber reinforced composite material is not isotropic. The stiffness properties are approximately the same in the X_2 and X_3 directions, but these properties are different than those in the X_1 direction. This type of material is classified as *transversely isotropic*, since there is a plane within which the properties are isotropic. A total of five material constants are required to completely define the stress-strain relations for a transversely isotropic material. The stress-strain relations for this material are:

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[1] Adapted from R.M. Jones, Mechanics of Composite Materials, Second Edition, Taylor & Francis, 1999.
[2] Adapted from R.A. Shenoi and J.F. Wellicome, "Composite Materials in Maritime Structures," Volume 1: Fundamental Aspects, Cambridge University Press, 1993.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} \\ C_{12} & C_{22} & C_{23} \\ C_{12} & C_{23} & C_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}$$

(1)

$$\tau_{23} = \frac{C_{22} - C_{23}}{2}\gamma_{23}, \qquad \tau_{13} = C_{55}\gamma_{13}, \qquad \tau_{13} = C_{55}\gamma_{13}$$

(2)

Note that the composite shear stresses and strains are uncoupled from the normal stresses and strains, and they are not considered in this study. Inversion of the normal stress-strain relations in Equations (1) yields the normal strain-stress relations:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{22} & S_{23} \\ S_{12} & S_{23} & S_{22} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}$$

(3)

where $[S] = [C]^{-1}$ and:

$$S_{11} = \frac{1}{E_1^c}, \qquad S_{22} = \frac{1}{E_2^c}, \qquad S_{12} = -\frac{\nu_{12}^c}{E_1^c}, \qquad S_{23} = -\frac{\nu_{23}^c}{E_2^c}.$$

(4)

E_1^c and E_2^c are the effective Young's moduli of the composite in the X_1 and X_2 directions, respectively. Note that $E_3^c = E_2^c$ for this material. ν_{12}^c and ν_{23}^c are the Poisson's ratios defined as $\nu_{ij}^c = -\frac{\varepsilon_j}{\varepsilon_i}$ for $\sigma_i = \sigma$ and all other stress equal to zero. Thus, four material constants are needed to completely define the normal stress-strain or strain-stress relations: E_1^c, E_2^c, ν_{12}^c, ν_{23}^c. One additional constant is needed to define the shear stress-strain relations, for a total of five material constants.

In reality, the aligned fibers in a unidirectional fiber reinforced composite material are randomly spaced within the cross section. For purposes of analysis, however, it is often assumed that the fibers are uniformly distributed in a periodic fashion. One common packing pattern is the square array, as shown in Figure 10.2. This assumption greatly simplifies the micromechanical analysis, as discussed below.

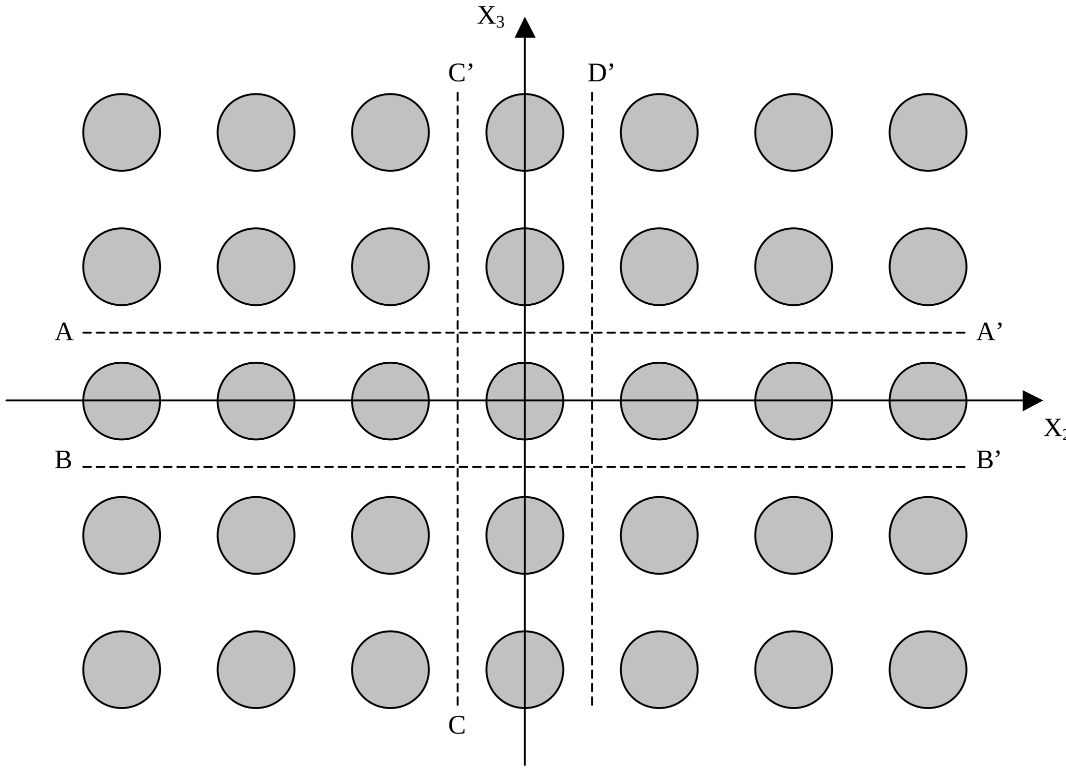


Figure 10.2 Cross section of a fiber reinforced composite material with fibers packed in a square array.

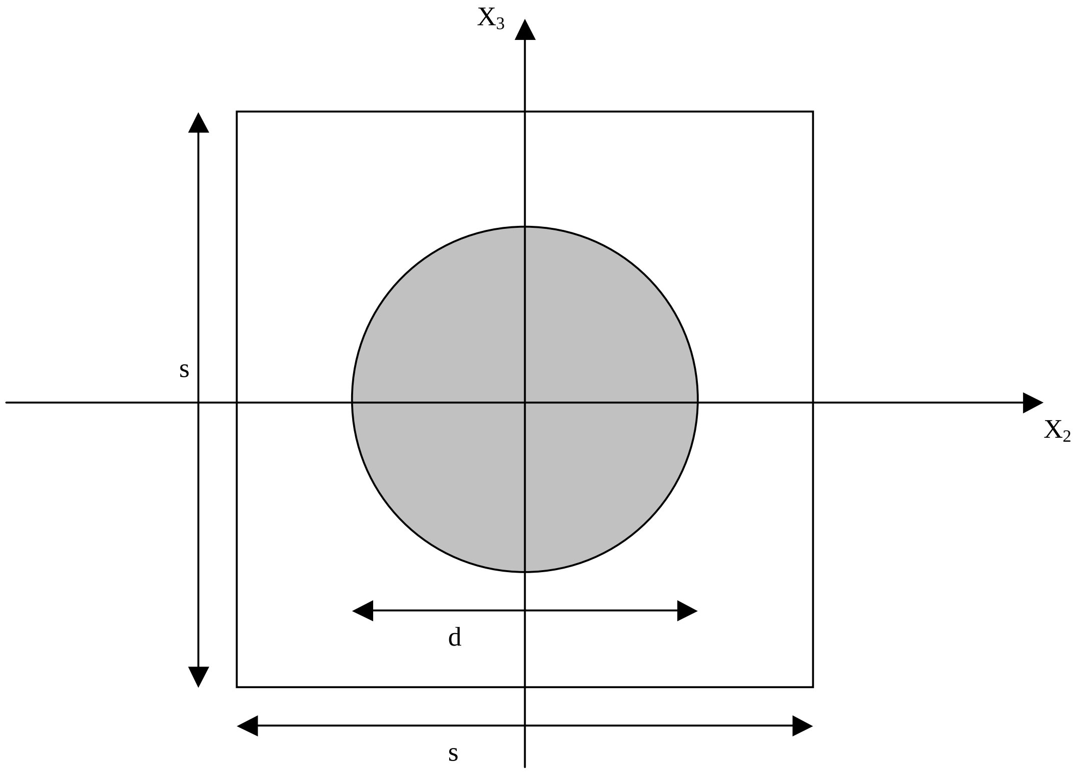


Figure 10.3 Square representative volume element (RVE) used in the micromechanical model.

10.3 Analysis Assumptions

The basic assumptions of the analytical model are as follows (see Figures 10.2 and 10.3):

1. The fibers are: (i) continuous, (ii) straight, (iii) infinitely long in the X_1 direction, (iv) perfectly aligned with X_1 axis, (v) circular in cross section, and (vi) arranged in a periodic square array.
2. The fiber and matrix materials are (i) homogeneous, (ii) isotropic, and (iii) linearly elastic.
3. The fiber and matrix are perfectly bonded at their interface.
4. Mechanical loads are applied at infinity.
5. Loads and material properties do not vary along the X_1 direction.

Assumption 1 ensures that many planes of geometric symmetry exist in the composite material. In addition, vertical and horizontal lines such as AA', BB', CC' and DD' in Figure 10.2 will remain straight during deformation.

If it is also assumed that the region of interest is a sufficient distance from the points of load application or geometric constraint (assumption 4), then every fiber and its surrounding matrix material will experience the same deformation under applied loads. Thus, only a single fiber and its surrounding matrix region need be analyzed. In other words, the response of the entire composite material may be studied by considering a single representative volume element (RVE), as shown in Figure 10.3.

10.4 Mathematical Idealization

Based on the assumptions above, a state of plane strain is assumed. Further, it is clear from Figure 10.3 that vertical and horizontal planes of symmetry exist within the RVE. Since the loads are also symmetric about these planes, it is necessary to model only one quarter of the RVE, as shown in Figure 10.4. As noted, the edges of the RVE will remain straight during all deformations, so boundary conditions should be applied in such a way as to maintain this condition. In other words, displacement boundary conditions are used instead of traction conditions on the edges.

10.5 Finite Element Model

The 2D finite element model of this structure should be developed using 2D plane strain four-noded quadrilateral finite elements. A quarter-model should be used.

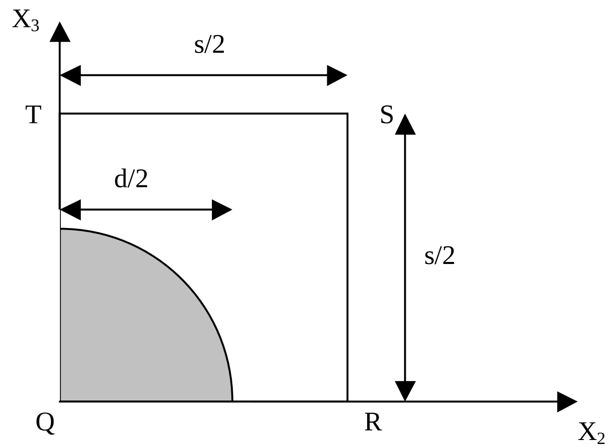


Figure 10.4 Quarter model of the RVE to be used in the finite element analysis.

10.6 Validation

In order to validate the analysis, some simple hand calculations should be performed to estimate the effective composite properties. The results of these calculations will be used to assess the validity of the finite element results (i.e., to make sure that the finite element results are reasonable and do not contain any large error due to a simple mistake in the model).

Based on mechanics of materials approaches, the effective material properties of a unidirectional fiber reinforced composite material can be estimated as follows[3]:

[3] For more details, see R.M. Jones, Mechanics of Composite Materials, Second Edition, Taylor & Francis, 1999.

$$E_1^c = E_f V_f + E_m V_m$$
(5)

$$E_2^c = \frac{E_f E_m}{V_m E_f + V_f E_m}$$
(6)

$$\nu_{12}^c = \nu_m V_m + \nu_f V_f$$
(7)

where E_f, ν_f, V_f and E_m, ν_m, V_m are the moduli, Poisson's ratios, and volume fractions of the fiber and matrix materials, respectively. The volume fractions are defined as:

$$V_f = \frac{\text{Fiber volume}}{\text{Total volume}}, \qquad V_m = \frac{\text{Matrix volume}}{\text{Total volume}}$$
(8)

E_1^c is a fiber dominated property, and it is accurately predicted by Equation (5). E_2^c and ν_{12}^c are influenced strongly by the matrix properties and deformations, and these composite properties are not always predicted accurately by the formulae in Equations (6) and (7).

10.7 Procedure for Calculating Effective Composite Properties

The effective composite properties are calculated by applying a unit strain in one direction (with all other strains set to zero) and then calculating the resulting average stresses. In this section, $\bar{\sigma}_i$ indicates the average stress in the i-th direction.

For example, if $\varepsilon_2 = 1$ and all other strains are zero, then Equation (1) yields:

$$C_{12} = \bar{\sigma}_1 \quad , \quad C_{22} = \bar{\sigma}_2 , \quad C_{23} = \bar{\sigma}_3$$
(9)

$\bar{\sigma}_2$ and $\bar{\sigma}_3$ can be calculated directly from the finite element results, as discussed below. If a three-dimensional analysis were performed, then $\bar{\sigma}_1$ could be calculated in a similar manner. In the present case, however, $\bar{\sigma}_1$ must be calculated using the known condition of plane strain. For a state of plane strain as assumed here, note that the first of Equations (3) yields (for $\varepsilon_1 = 0$):

$$0 = S_{11}\bar{\sigma}_1 + S_{12}\bar{\sigma}_2 + S_{12}\bar{\sigma}_3 \quad \Rightarrow \quad \bar{\sigma}_1 = -\frac{S_{12}}{S_{11}}(\bar{\sigma}_2 + \bar{\sigma}_3)$$

(10)

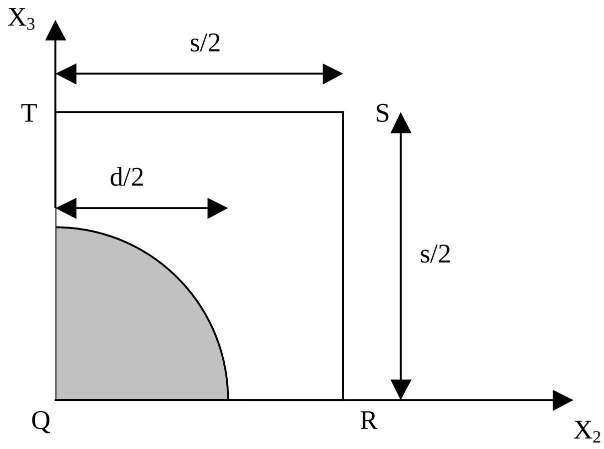
It is assumed here that the prediction of E_1^c and ν_{12}^c in Equations (5) and (7) are sufficiently accurate, so that these values can be employed in Equations (4) to calculate S_{11} and S_{12} for use in Equation (10). If E_1^c is calculated using Equation (5), then C_{11} can now be calculated as follows:

$$C_{11} = E_1^c + \frac{2C_{12}^2}{C_{22} + C_{23}}$$

(11)

Now all four of the material constants $C_{11}, C_{12}, C_{22}, C_{23}$ are known, with C_{11} and C_{12} approximated based on the use of Equations (5) and (7). The inverse of $[C]$ is now readily found as $[S] = [C]^{-1}$, and the remaining material constants E_2^c, ν_{12}^c, ν_{23}^c are obtained using Equations (4).

The above procedure requires the calculation of the average stresses $\bar{\sigma}_2$ and $\bar{\sigma}_3$ that result from the application of a uniaxial strain state $\varepsilon_2 = 1$. The boundary conditions for this case are shown in Figure 10.5. After generating the finite element solution for this problem, the reaction forces along edge TQ (or RS) can be summed to obtain the resultant force in the X_2 direction due to the applied strain state. Dividing this force by the area over which it acts will yield the average stress $\bar{\sigma}_2$. In a similar manner, the average stress $\bar{\sigma}_3$ that results from the same loading state can be obtained by summing the reaction forces in the X_3 direction along edge ST (or QR) and dividing this resultant force by the area over which it acts.



Edge QR: $u_3 = 0, \tau_{23} = 0$

Edge RS: $u_2 = s/2, \tau_{23} = 0$

Edge ST: $u_3 = 0, \tau_{23} = 0$

Edge TQ: $u_2 = 0, \tau_{23} = 0$

Figure 10.5 Boundary conditions to be used in the finite element analysis for a uniaxial strain state $\varepsilon_2 = 1$. Note that directions X_2 and X_3 may be labeled differently within the finite element software.

10.8 Calculating the Strain Concentration Factor

When a uniaxial transverse strain is applied to a composite, as shown in Figure 10.5, the strain encountered by the fiber and matrix will be different than the strain applied globally. This is due to the material stiffness mismatch and the geometry of the two constituents. By applying the strain state shown in Figure 10.5 (unit strain in the 2-direction), the strain concentration factor is the maximum (principal) strain in either the fiber or the matrix. This strain can be obtained from a finite element model of the micromechanical section shown in the figure.