Natural Convection Experiment
Measurements from a Vertical Surface

OBJECTIVE

1. To demonstrate the basic principles of natural convection heat transfer including determination of the convective heat transfer coefficient.

2. To demonstrate the boundary layer character of external natural convection.

BACKGROUND

Since the density of most fluids vary with temperature, temperature gradients within a fluid medium will give rise to density gradients. If these density gradients are such that the fluid is in an unstable situation, heavy fluid on top of light fluid, the fluid will begin to move. This motion is termed natural convection. Newton's law of cooling governs this physical process which states that the heat transfer from the surface is directly proportional to the temperature difference between the surface and the fluid far away from the surface or

\[ \dot{q}'' \propto (T_{\text{surface}} - T_{\text{fluid}}) \]  

Introducing the convective heat transfer coefficient as the constant of proportionality, we have

\[ \dot{q}'' = h_c \left( T_{\text{surface}} - T_{\text{fluid}} \right) \]

It is clear that the major obstacle in utilizing Eq. (2) for convective heat transfer calculations is the evaluation of the convective heat transfer coefficient. There are three standard methods used to evaluate \( h_c \). The first involves a mathematical solution to the conservation equations in differential form. For problems where these equations are too complicated to be solved analytically, we can employ the second method, a computational solution. Finally for problems that are so complicated that we cannot even write the appropriate conservation equations, we must go into the laboratory and make measurements in employing an experimental solution. Before we contemplate employing one of these solution methods, it is useful to use our intuition to figure out upon what the convective heat transfer coefficient depends. Our intuition tells us that the three major factors associated with the calculation of \( h_c \) should be

(i) fluid mechanics
Starting with the fluid mechanics, we recognize there are a variety of ways to characterize the flow. We can consider what is driving the flow and classify it as forced or natural convection. Next, we consider how many boundaries the fluid flow interacts with and classify it as external or internal flow. Recall that the difference between internal flows and external flows is often one of perspective. A third way to characterize the flow is by the presence or absence of turbulence.

An important consideration in the handling of convective heat transfer coefficients is the notion of dynamic similarity. It is found that certain systems in fluid mechanics or heat transfer are found to have similar behaviors even though the physical situations may be quite different. Recall the fluid mechanics of flow in a pipe. What we are able to do is to take data as shown in Fig. 1 for different fluids and pipe diameters and by appropriately scaling collapse these curves into one curve.

**Figure 1. Dynamic Similarity for Pipe Flow**

![Graphic showing velocity, pressure drop, and friction factor vs. Reynolds number](image)

In convective heat transfer we may apply dynamics scaling to make a parallel transformation.
We have defined a dimensionless convective heat transfer coefficient called the Nusselt number as

\[ \text{Nu} = \frac{h_c L}{k} \]  

(3)

where

- \( h_c \): convective heat transfer coefficient
- \( L \): characteristic length
- \( k \): thermal conductivity of the fluid.

The characteristic length is chosen as the system length that most affects the fluid flow. For flow along a flat plate our characteristic length is the plate length, \( L \), and we write

\[ \text{Nu}_D = \frac{h_c L}{k} \]  

(4)

The Rayleigh number is indicative of the buoyancy force that is driving the flow and is given by

\[ Ra = \frac{g \beta (T_s - T_f) L^3}{\nu \alpha} \]  

(5)
where

\[ g: \text{ acceleration due to gravity} \]
\[ \beta: \text{ fluid thermal expansion coefficient} \]
\[ T_s: \text{ surface temperature} \]
\[ T_f: \text{ fluid temperature} \]
\[ L: \text{ characteristic length} \]
\[ \nu: \text{ fluid kinematic viscosity} \]
\[ \alpha: \text{ fluid thermal diffusivity.} \]

Another important feature introduced by the fluid mechanics is the local nature of the convective heat transfer coefficient. If as the fluid flows over different regions of the surface, the fluid mechanics change, then the convective heat transfer coefficient will change. For natural convection over a vertical flat plate we will have the boundary layer flow shown in Fig. 3.

Figure 3. Flat Plate Boundary Layer Development

As the boundary layer thickness grows, the fluid mechanics change significantly, so that the convective heat transfer coefficient will vary along the length of plate and we will have a local convective heat transfer coefficient, \( h_c(x) \). Then we may also define local Nusselt and Rayleigh numbers as

\[ \text{Nu}_x = \frac{h_c(x) \cdot x}{k} \]  \hspace{1cm} (6)
\[ Ra = \frac{g \beta |T_s(x) - T_f| x^3}{\nu \alpha} \]  

(7)

Though local heat transfer conditions can be extremely important, an average heat transfer coefficient over the entire surface length is often desirable. By definition we have

\[ h_{c,\text{avg}} = \frac{1}{L} \int_0^1 h_c(x) \, dx \]  

(8)

with an average Nusselt Number given as

\[ Nu_{L,\text{avg}} = \frac{h_{c,\text{avg}} \cdot L}{k} \]  

(9)

The dimensionless parameter which is used to represent the affect of fluid properties is the Prandtl number

\[ Pr = \frac{\nu}{\alpha} \]  

(10)

The influence of geometry may be seen in a couple of ways. First, for those configurations that have two length dimensions, such as a cylinder, we introduce a dimensionless geometric parameter

\[ X = \frac{D}{L} \]  

(11)

The second way in which we see geometrical influences is through the functional form of the Nusselt number correlation. In general we may write

\[ Nu = fn(Ra,Pr,X) \]  

(12)

For simple situations these may often be written as power law relationships

\[ Nu = a \, Ra^n \, Pr^m \]  

(13)

where the constants \( a, m, \) and \( n \) will change for different geometries.
In this experiment the student will develop the relationship among the Nusselt number and other dimensionless parameters for natural convection from a vertical flat surface. The students will determine local heat transfer coefficients which are indicative of a boundary layer phenomena. These local measurements will then be averaged and compared to published correlations.

Natural convection heat transfer from a vertical surface will be investigated using an electrically heated flat plate. A schematic of the apparatus is shown in Figure 4.

Figure 4. Schematic of Experimental Apparatus

Assuming the heating is uniform, the heat flux at any location on the plate is given by,

\[ \dot{q}'' = \frac{V \cdot I}{2 \cdot w \cdot L} \]  \hspace{1cm} (14)

where V and I are the electric voltage and current supplied to the plate and w and L are the width and length of the heated surface. Temperature measurements are made along the length of the plate with thermocouple embedded beneath the heated surface.
The local heat transfer coefficient (the heat transfer coefficient at a certain distance from the leading edge) is then calculated as

$$h_c(x) = \frac{\dot{q}''}{T_s(x) - T_f}$$

(15)

Since in most engineering applications the interest would be in an average heat transfer coefficient for the entire surface, we use the mathematical definition

$$\bar{h}_c = \frac{1}{L} \int_0^L h_c(x) \, dx$$

(16)

Now substituting and recognizing that the thermocouple is designed to read the temperature difference directly, we have

$$\bar{h}_c = \frac{\dot{q}'' L}{\Delta T(x)} \int_0^L \, dx$$

(17)

We now apply a trapezoidal rule integration to obtain

$$\bar{h}_c = \frac{\dot{q}''}{L} \sum_{i=1}^{N-1} 0.5 \left( \frac{1}{\Delta T_i} + \frac{1}{\Delta T_{i-1}} \right) \cdot (x_i - x_{i-1})$$

(18)

where \(N\) is the number of temperature data points along the plate and \(i=0\) corresponds to the point at the leading edge. The average Nusslet number can then calculated.

**PROCEDURE**

1. Check that the power supply is unplugged and that the transformer is set at zero.

2. Plug in the power supply and turn on the transformer.

5. Set the transformer at a power level of about 5% and allow the system to stabilize. You will want to have measurements for at least five power settings. Since natural convection is a relatively poor heat transfer process you will want to monitor the maximum surface temperature to prevent burnout of the system. It is recommended that you do not exceed 25% power level on the transformer.

6. Read and record the voltage measurements associated with the electrical voltage and current supplied for heating and the thermocouples.
7. Increase the power level, allow the system to stabilize, and record appropriate measurements. Repeat the experiment as needed

8. Turn off the transformer and unplug the power supply.

**DATA ANALYSIS**

1. Each case should be recorded on the Excel spreadsheets provided in the lab.

2. From the experimentally determined values of the local heat transfer coefficient calculate the average heat transfer coefficient and surface temperature for each power setting.

3. Plot the local heat transfer coefficient versus distance from the leading edge.

4. Calculate the average Nusselt number and Rayleigh number for each power setting.

5. Plot the Nusselt number versus Rayleigh number for each case along with the published correlations.

6. Estimate the uncertainty error in your experimentally determined Nusselt number and Rayleigh number.

7. Provide one sample hand calculation of your data processing.

**SUGGESTIONS FOR DISCUSSION**

1. How does the experimental data compare to the published correlations?

2. What are some possible errors in the experiment?

3. What can you tell about the structure of the boundary layer from the local heat transfer coefficient?

4. Can you identify a transition from laminar to turbulent flow?