A crane is used to lift a pile of logs with center of mass at point $D$. Make a free-body diagram of the logs and discuss whether or not the system is statically determinate and/or properly constrained.

Solution:

There are 3 unknown scalar forces $T_A$, $T_B$, and $T_C$, provided the geometry is given. Thus there will be 3 unknowns and potentially six equations. However, the 3 forces pass through a common point so the moment equations yield no information. Thus there are 3 equations in 3 unknowns and the system is **statically determinate** but **improperly constrained** as rotation could occur about the point of intersection of the three forces.
5.56 Sketch the free body diagram of the circus tent stake. The stake is mounted on a hard surface by a ball and socket arrangement at point C. Is this system properly constrained? Is it statically indeterminate?

![Figure P5.56](image)

Solution: From the free body diagram there are 5 unknown magnitudes: $T_1$, $T_2$, $C_x$, $C_y$ and $C_z$, which, with geometry assumed given, can be determined from the 5 equations of equilibrium

$$
\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0, \quad \sum M_x = 0, \quad \sum M_y = 0.
$$

Thus the system is **statically determinate**. The sixth equation, $\sum M_y = 0$ is automatically satisfied because all the forces intersect the y-axis, implying that the system is **improperly constrained** as nothing prevents rotation about the y-axis. You may want to point out that this is a two force member.

![Figure S5.56](image)
5.62 A winch system consists of a 0.1-m diameter drum, shaft and motor. Compute the reactions at A and C, and the motor torque $M_z$ required to keep the 100-kg mass in equilibrium. The spindle supports at A and C are thrustless bearings. Neglect any moments at A and C.

Solution: The free-body is given and the equilibrium equations are (unrestrained in z direction)

Further manipulation of the moment equation yields (continued)
\[ M_2 \dot{\mathbf{k}} - (0.05)(981) \mathbf{k} + (0.2)(981) \mathbf{i} + (0.4A_x) \mathbf{j} - 0.4A_y \mathbf{i} = 0 \]

or in scalar form

\[ k : \quad M_x = 49.05 \text{ Nm} \quad (3) \]
\[ j : \quad 0.4A_x = 0 \quad (4) \]
\[ i : \quad 196.2 - 0.4A_y = 0 \quad (5) \]

These 5 equations yield \( A_x = C_x = 0, \ A_y = 490.5 \text{ N, } C_y = 490.5 \text{ m.} \)
5.64 A mounting platform is secured in place by a frictionless support at A, a ball and socket at B and a rope at C. The A 100 N gravitational force acts at its geometric center and two boxes sit on the platform modeled by the force $F_1 = 500$ N and $F_2 = 50$ N. Calculate the components of the reaction forces at the supports.

Solution: A free body diagram yields the following equilibrium equations

\[ \sum F_x: B_x = 0 \]  (1)

(continued)
\[ \sum F_y : \quad T_c + B_y + A_y - 650 = 0 \quad (2) \]
\[ \sum F_z = B_z = 0 \quad (3) \]
\[ \sum M_0 = (0.5\hat{i}) \times T_c \hat{j} + (k \times A_y \hat{j}) + \hat{i} \times (B_y \hat{j}) + k \times (B_{z\hat{j}}) + 0.5(\hat{i} + k) \times (-100\hat{j}) \]
\[ + (0.75\hat{i} + 0.5\hat{k}) \times (-500\hat{j}) + (0.3\hat{i} + 0.6\hat{k}) \times (-50\hat{j}) = 0 \]

Multiplying out the moment terms (with \( B_x = B_z = 0 \)) yields from
\[ \hat{i} : -A_y - B_y + 330 = 0 \]
or
\[ A_y + B_y = 330 \quad (4). \]

From
\[ \hat{k} : 0.5T_c + B_y = 440 \quad (5) \]

Solving 1-5 yields
\[ T_c = 320 \text{ N, } B_z = 0, \ B_y = 280 \text{ N, } B_z = 0, \ A_y = 50 \text{ N} \]

Note that while stable for the given load, there is no support against twisting about the \( y \) axis so this is improperly constrained.
5.69 Compute the reactions at the ball and socket support at point D and the tensions in the support ropes ($T_1$ and $T_2$) for the sign support system. The weight of the sign exerts a force of 300 N in the down direction ($-y$) at point E, 0.25 m from D and at point F, 1.75 m from D. Note that $DC$ is not constrained from rotation about its axis.

![FIGURE P5.69](image)

Solution: A free body diagram of the sign support is given in the figure.

![FIGURE S5.69](image)

First note that all the forces intersect a common axis (that of the (continued)
line $DC$), thus there will be only 5 independent equations and only 5 variables can be determined ($D_x$, $D_y$, $D_z$, $T_1$ and $T_2$). The vectors $T_1$ and $T_2$ must first be written in terms of unit vectors along $CB$ and $CA$ respectively:

\[
T_1 = T_1 \frac{CB}{|CB|} = T_1 (-0.848\hat{i} + 0.424\hat{j} - 0.318\hat{k}),
\]

\[
T_2 = T_2 \frac{CA}{|CA|} = T_2 (-0.848\hat{i} + 0.424\hat{j} + 0.318\hat{k}).
\]

Equilibrium in the coordinate directions becomes

\[\sum F_x = 0 : \quad D_x - 0.848T_1 - 0.848T_2 = 0 \quad (1)\]

\[\sum F_y = 0 : \quad D_y + 0.424T_1 + 0.424T_2 = 600 \quad (2)\]

\[\sum F_z = 0 : \quad D_z - 0.318T_1 + 0.318T_2 = 0 \quad (3)\]

The moments about point $C$ yield

\[(-0.25\hat{i} \times (-300\hat{j}) + (-1.75\hat{i}) \times (-300\hat{j}) + (-2\hat{i}) \times (D_x\hat{i} + D_y\hat{j} + D_z\hat{k}) = 0\]

or by component

\[\hat{i} : \quad 0 = 0 \quad (4)\]

\[\hat{j} : \quad 2D_z = 0 \quad (5)\]

\[\hat{k} : \quad D_y = 300 \quad (6)\]

Solving equations 1, 2, 3 with

\[D_z = 0 \text{ and } D_y = 300 \text{ N}\]

yields

\[D_x = 600 \text{ N} \quad T_1 = 354 \text{ N} \quad T_2 = 354 \text{ N}\]
5.71 The total force acting on a telephone pole due to the wires attached to it is computed to be \( \mathbf{F} = 100\mathbf{i} - 50\mathbf{j} + 10\mathbf{k} \text{N} \). Compute the reaction at the fixed connection at point \( A \).

Solution: The free body diagram is given in the figure. The equations of equilibrium become simply

\[
\begin{align*}
F &= 100\mathbf{i} - 50\mathbf{j} + 10\mathbf{k} \\
&\text{(continued)}
\end{align*}
\]
\[ \sum F_x = 0 : A_x + 100 = 0 \text{ or } A_x = -100 \text{ N} \]
\[ \sum F_y = 0 : A_y - 50 = 0 \text{ or } A_y = 50 \text{ N} \]
\[ \sum F_z = 0 : A_z + 10 = 0 \text{ or } A_z = -10 \text{ N} \]
\[ \sum M_B = 0 : M_x \hat{i} + M_y \hat{j} + M_z \hat{k} + (3 \hat{j} - \hat{k})m \times (100 \hat{i} - 50 \hat{j} + 10 \hat{k})N = 0 \]
\[ = (M_x + 30 - 50) \hat{i} + (M_y - 100) \hat{j} + (M_z - 300) \hat{k} = 0 \]

or
\[ M_x = 20 \text{ N} \cdot \text{m}, \quad M_y = 100 \text{ N} \cdot \text{m}, \quad M_z = 300 \text{ N} \cdot \text{m} \]
5.73 A folding platform is used to hold parts as well as conserve floor space when not in use. The platform is supported by a hinge at $C$ which is assumed to support negligible moments, a leg, at $B$ modeled as a frictionless support and a removable pin at $A$ modeled as a thrustless bearing again with negligible moments. If the platform is loaded as illustrated compute the reaction forces at $A$, $B$ and $C$. Ignore the thickness of the platform.

FIGURE P5.73

Solution: The free body diagram is given in figure S5.73.

The equations of equilibrium become: (continued)
Force Summation:
\[ \sum F_x = 0 : C_x = 0 \]  \hfill (1)
\[ \sum F_y = 0 : A_y + C_y + B_y = 3000 \]  \hfill (2)
\[ \sum F_z = 0 : A_z + C_z = 0 \]  \hfill (3)

The moment equation is
\[ \sum M_0 = (0.5\hat{i} + 0.4\hat{k}) \times (-3000\hat{j}) + (0.5\hat{i}) \times (C_z\hat{i} + C_y\hat{j} + C_z\hat{k}) + (\hat{i} + 1.2\hat{k}) \times B_y\hat{j} \]
\[ + 1.2\hat{k} \times (A_y\hat{j} + A_z\hat{k}) = 0 \]

Which yields the 3 component equations
\[ \hat{i} : 1200 - 1.2B_y - 1.2A_y = 0 \]  \hfill (4)
\[ \hat{j} : -0.5C_z = 0 \]  \hfill (5)
\[ \hat{k} : -1500 + 0.5C_y + B_y = 0 \]  \hfill (6)

Equations (1), (3) and (5) yield
\[ C_x = A_x = C_z = 0 \]
by inspection. The remaining 3 equations (2, 4, 6) (solved in Mathcad) yield
\[ A_y = 500 \text{ N}, \]
\[ B_y = 500 \text{ N} \]
and
\[ C_y = 2000 \text{ N} \]