7.6 Compute the internal moment and force at the midpoint for the system.

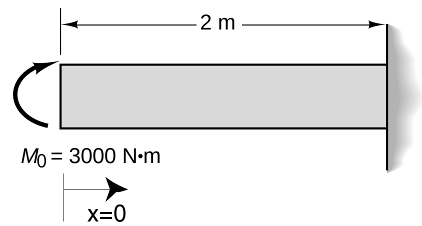


FIGURE P7.6

Solution: A free-body diagram of the section to the left of a cut through $x = 1$m is illustrated

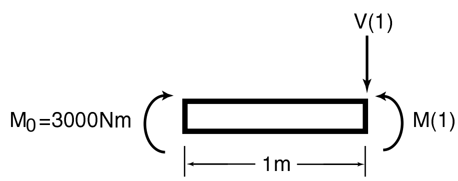


FIGURE S7.6

$$\sum F_y = 0$$

yields

$$\underline{V(1) = 0}.$$

$$\sum M_0 = 0$$

yields

$$-3000 \text{ Nm} + M(1) - (1)V(1) = 0$$

or

$$\underline{M(1) = 3000 \text{ Nm}}$$

7.19 A pipe is simply supported at one end and effectively pinned at the other. Compute the reaction forces, the internal forces, plot the shear and moment diagram and state the maximum bending moment generated by hanging a mass at the point indicated.

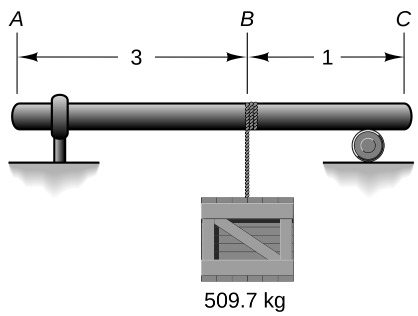


FIGURE P7.19

Solution: The appropriate free body diagrams are

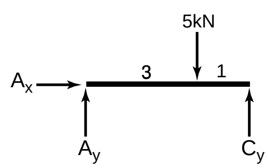


FIGURE S7.19a

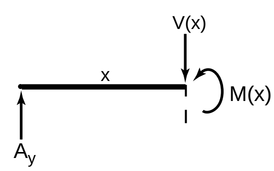


FIGURE S7.19b

Equilibrium from figure a is used to compute the reaction forces:

$$\sum F_y = A_y + C_y - 5 = 0$$

$$\sum M_A = -(3 \text{ m})(5 \text{ kN}) + 4C_y = 0$$

so that

$$\underline{C_y = 3.75 \text{ kN and } A_y = 1.25 \text{ kN}}.$$

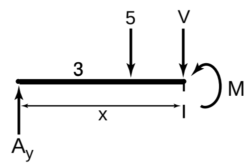


FIGURE S7.19c

Next consider equlibrium of the section made best a cut at $x < 3$

$$\sum F_x = 0 : \quad A_y - V(x) = 0$$

or

$$\underline{V(x) = 1.25 \text{ kN}, \ 0 \leq x < 3}$$

$$\sum M_A = 0 : \quad -xV(x) + M(x) = 0$$

or

$$\underline{M(x) = 1.25x \text{ kNm}, \ 0 \leq x < 3}$$

Finally consider the section formed by a cut at $3 < x \leq 4$. Equlibrium becomes

$$\sum F_x = 0 : \quad 1.25 - 5 - V(x) = 0$$

or

$$\underline{V(x) = -3.75 \text{ kN}, \ 3 < x \leq 4}$$

$$\sum M_A = 0 : \quad -(3)(5) - x(-3.75) + M(x) = 0$$

or

$$\underline{M(x) = 15 - 3.75x \text{ kNm}, \ 3 < x \leq 4}$$

Last plot the functions $V(x)$ and $M(x)$

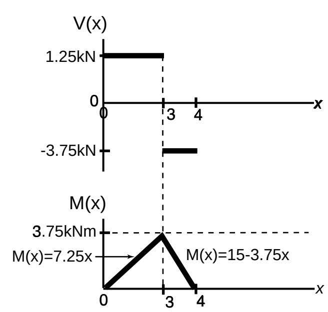


FIGURE S7.19d

Note the maximum value of the moment is 3.75 kN$\cdot$ m

compared to 5 kNm generated by the same magnitude of load acting at the center (as computed in the previous problem).

7.26 Determine the reaction forces, and the internal forces and moments. Plot the shear and bending moment function versus the distance x.

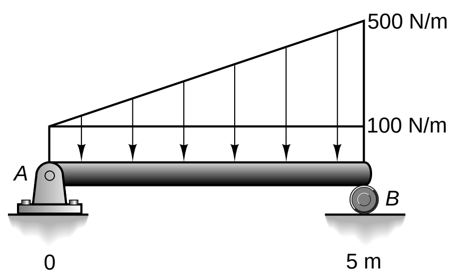


FIGURE P7.26

Solution: Following the solution of problem 4.53, the equivalent force is 1500 N applied at 3.06 m from 0. The free-body diagram for the reaction forces is illustrated.

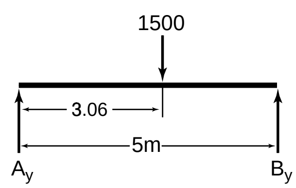


FIGURE S7.26a

$$\sum F = 0 : A_y + B_y - 1500 = 0$$

$$\sum M_A = 0 : 5B_y - (3.06)(1500) = 0$$

$$\underline{B_y = 916.67 \text{ N}, \ A_y = 583.33}$$

Next consider a cut at x and a free body of the section between A and x. Note only one section needs to be analyzed because the load does not change abruptly any where. From the solution to 4.53:

$$x_1 = 2x/3$$

and from similar triangles

$$h = 80x$$

and

$$P_1 = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)(80x) = 40x^2.$$

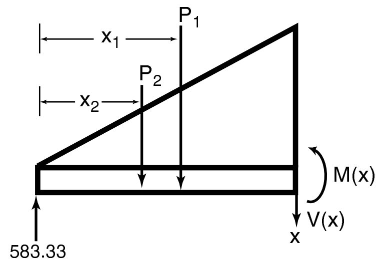


FIGURE S7.26b

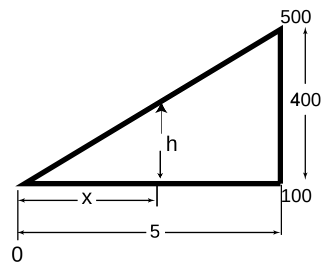


FIGURE S7.26b2

From the rectangular load,

$$x_2 = \frac{x}{2},\ P_2 = 100x.$$

$$\sum F = 0 : \quad 583.33 - 40x^2 - 100x - V(x) = 0$$

or

$$\underline{V(x) = 583.33 - 40x^2 - 100x \text{ N}}$$

$$\sum M_A = 0 = -\frac{2x}{3}(40x^2) - \frac{x}{2}(100x) - x(583.33 - 40x^2 - 100x) + M(x) = 0$$

$$\underline{M(x) = \frac{-40x^3}{3} - 50x^2 + 583.33x \text{ Nm}}$$

See computer solutions for plots.

6.3 Use the method of joints to calculate the force in each member of the truss illustrated as well as the reaction forces at the pin B and roller C.

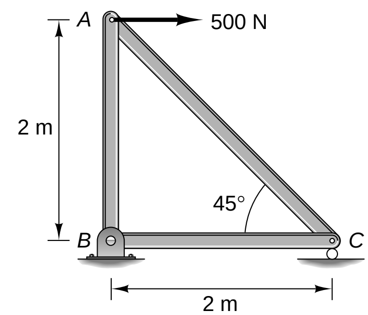


FIGURE P6.3

Solution: A free body diagram of each joint, assuming tension in each member is illustrated.

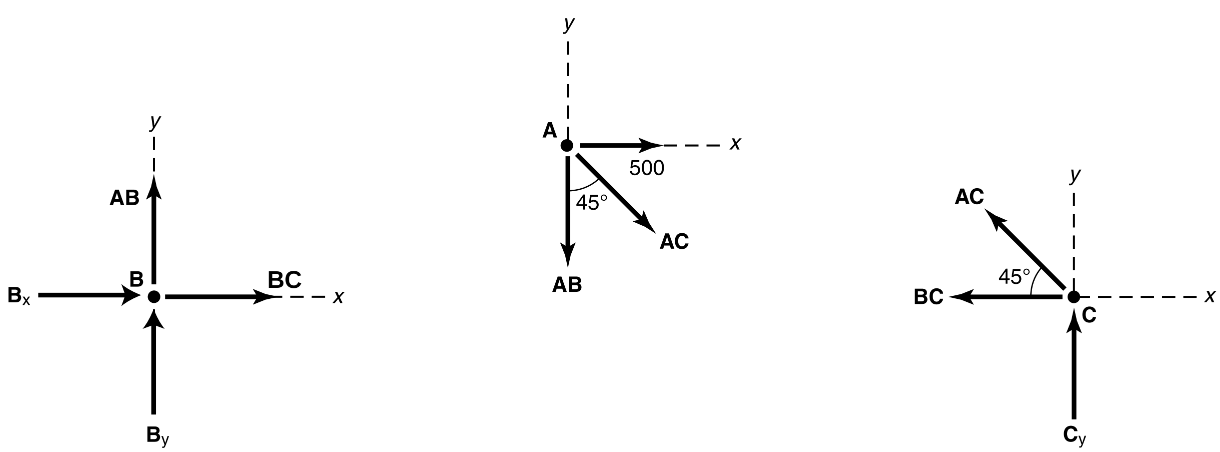


FIGURE S6.3

The equilibrium equation for joint A in the x direction is

$$500 + AC\cos 45^\circ = 0$$

or

$$\underline{AC = -707\text{N (compression)}}.$$

The equilibrium equation for joint A in the y direction ins

$$-(-707)\cos 45^\circ - AB = 0$$

or

$$\underline{AB = 500 \text{ N (tension)}}.$$

Next consider the joint at C:

$$\sum F_x = 0$$

implies

$$-BC - (-707)(\cos 45^\circ) = 0$$

or

$$\underline{BC = 500 \text{ N (tension)}}$$

$$\sum F_y = 0$$

implies

$$C_y + (-707)\cos 45^\circ = 0$$

or

$$\underline{C_y = 500\text{N (up)}}.$$

Finally at point A,

$$B_x + 500 = 0$$

or

$$\underline{B_x = -500\text{N (left)}}$$

and

$$B_y + 500 = 0$$

or

$$\underline{B_y = -500\text{N (down)}}.$$

6.6 a) Calculate the force in each member of the truss illustrated as well as the reaction forces of the pin at C and the roller at A. b) Add an additional 100 N force to the pin at D in the vertical direction pointing downward and recalculate the forces in the members. What changes?

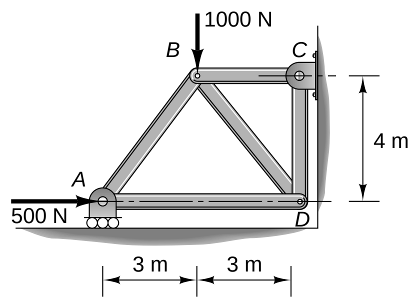


FIGURE P6.6

Solution: a) The free body diagrams of each joint and of the entire body are first constructed assuming each member to be in tension. These are given in the figure.

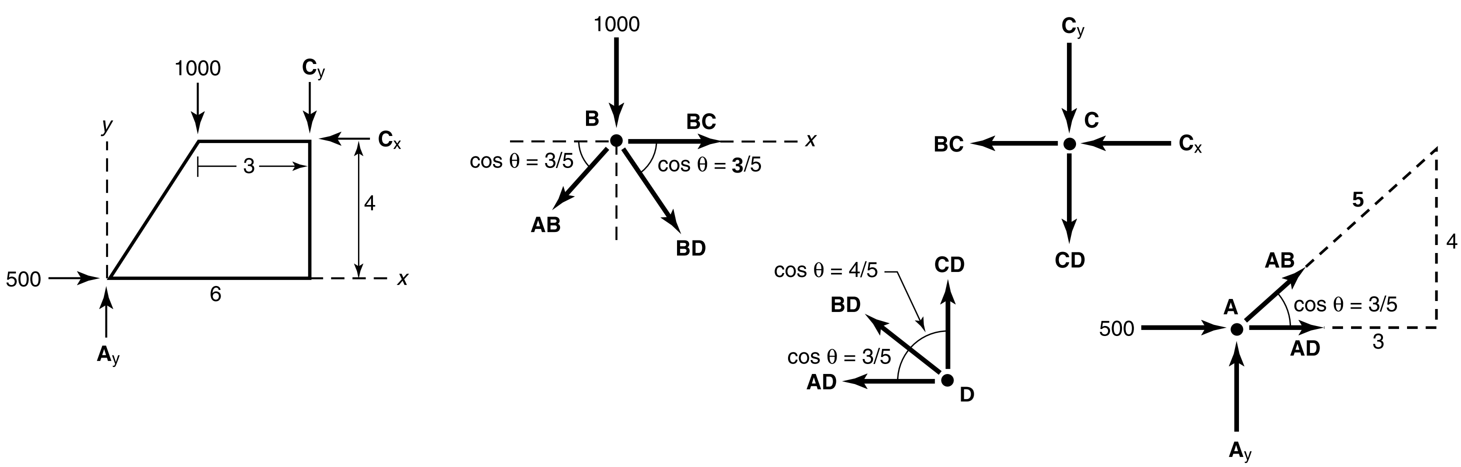


FIGURE S6.6

Note that the geometry is such that each angle is defined by a 3-4-5 triangle. The joint free body diagrams all contain more than two unknowns, so starting with the FBD's of the whole body yields

$$\sum M_c = 0 : (-6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) \times (500\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (-3\hat{\mathbf{i}}) \times (-1000\hat{\mathbf{j}}) = 0$$

or

$$-6A_y\hat{\mathbf{k}} + 2000\hat{\mathbf{k}} + 3000\hat{\mathbf{k}} = 0 \text{ or } \underline{A_y = 833 \text{ N (up)}}$$

$$\sum F_x = 0 : -C_x + 500 = 0 \text{ or } \underline{C_x = 500 \text{ (left)}}$$

$$\sum F_y = 0 : -C_y = 1000 + 833 = 0 \text{ or } \underline{C_y = -167 \text{ N (up)}}$$

Next consider the equilibrium equation for joint C

$$\sum F_x = 0 : -BC - C_x = 0 \text{ or } \underline{BC} = -C_x = \underline{-500 \text{ N (compression)}}$$

$$\sum F_y = 0 : -CD - C_y = 0 \text{ or} \underline{CD} = -C_y = \underline{167 \text{ N (tension)}}$$

The FBD for D yields

$$\sum F_x = 0 : -AB - BD3/5 = 0 \text{ or } AD = -3/5BD$$

$$\sum F_y = 0 : CD + BD4/5 = 0$$

or

$\underline{BC} = 5/4(CD) = \underline{-209 \text{ N (compression) so that } AD = 125 \text{ N (tension)}}$

The FBD for A yields

$\sum F_x = 0 : 500{+}AD{+}AB(3/5) = 0$ or $\underline{AB} = -5/3(625) = \underline{-1042 \text{ N (compression)}}$

$\sum F_y$ can be used as a check and again yields $A_y = 833$ N. The FBD for B can be used as a check.

Alternately, the matrix approach can be used. In this case only the free body equations for joints A, B, C and D are needed.

From A:

$$\begin{array}{ll} (3/5)AB + AD + 500 = 0 & (x) \\ (4/5)AB + A_y = 0 & (y) \end{array}$$

From B:

$$\begin{array}{ll} -(3/5)AB + (3/5)BD + BC = 0 & (x) \\ -(4/5)AB - (4/5)BD = 1000 & (y) \end{array}$$

From C:

$$\begin{array}{ll} -C_x - BC = 0 & (x) \\ -C_y - CD = 0 & (y) \end{array}$$

From D:

$$\begin{array}{ll} -AD - BD(3/5) = 0 & (x) \\ CD + BD(4/5) = 0 & (y) \end{array}$$

or in matrix form:

$$\begin{bmatrix} 3/5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4/5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -3/5 & 0 & 1 & 3.5 & 0 & 0 & 0 & 0 \\ -4/5 & 0 & 0 & -4/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & -3/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4/5 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} AB \\ AD \\ BC \\ BD \\ CD \\ A_y \\ C_x \\ C_y \end{bmatrix} = \begin{bmatrix} -500 \\ 0 \\ 0 \\ 1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving via Mathcad matrix inversion yields

$$\begin{array}{ll} AB = -1042 \text{ N (compression)} & A_y = 833 \text{ N (up)} \\ AD = 125 \text{ N (tension)} & C_x = 500 \text{ N (left)} \\ BC = -500 \text{ N (compression)} & C_y = -166.7 \text{ N (up)} \\ BD = -208 \text{ N (compression)} & \\ CD = 166.7 \text{ N (tension)} & \end{array}$$

b) This is exactly the same as part a except that the y equation for joint D becomes $CD + (4/5)BD - 100 = 0$. Thus the matrix equation used in part A can be copied and the last element in the load vector changed from 0 to 100. The new solution is

$$\begin{array}{ll} AB = -1042 \text{ N (compression)} & A_y = 833 \text{ N}(up) \\ AD = 125 \text{ N (tension)} & C_x = 500 \text{ N}(left) \\ BC = -500 \text{ N (compression)} & C_y = -267 \text{ N}(up) \\ BD = -208 \text{ N (compression)} & \\ CD = 267 \text{ N (tension)} & \end{array}$$

The only elements that changed are the tension in CD and the reaction C_y.

7.19 A pipe is simply supported at one end and effectively pinned at the other. Compute the reaction forces, the internal forces, plot the shear and moment diagram and state the maximum bending moment generated by hanging a mass at the point indicated.

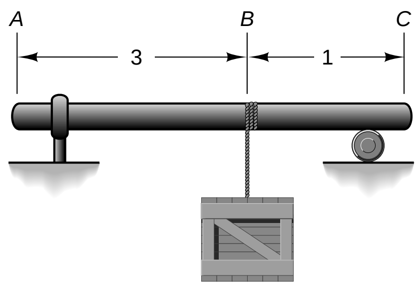


FIGURE P7.19

Solution: The appropriate free body diagrams are

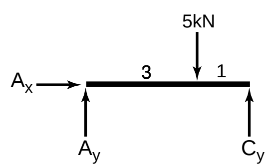


FIGURE S7.19a

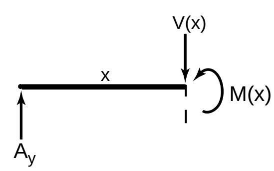


FIGURE S7.19b

Equilibrium from figure a is used to compute the reaction forces:

$$\sum F_y = A_y + C_y - 5 = 0$$

$$\sum M_A = -(3\text{ m})(5\text{ kN}) + 4C_y = 0$$

so that

$$\underline{C_y = 3.75\text{ kN and } A_y = 1.25\text{ kN}}.$$

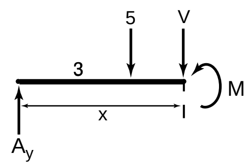


FIGURE S7.19c

Next consider equlibrium of the section made best a cut at $x < 3$

$$\sum F_x = 0: \quad A_y - V(x) = 0$$

or

$$\underline{V(x) = 1.25 \text{ kN}, \ 0 \leq x < 3}$$

$$\sum M_A = 0: \quad -xV(x) + M(x) = 0$$

or

$$\underline{M(x) = 1.25x \text{ kNm}, \ 0 \leq x < 3}$$

Finally consider the section formed by a cut at $3 < x \leq 4$. Equlibrium becomes

$$\sum F_x = 0: \quad 1.25 - 5 - V(x) = 0$$

or

$$\underline{V(x) = -3.75 \text{ kN}, \ 3 < x \leq 4}$$

$$\sum M_A = 0: \quad -(3)(5) - x(-3.75) + M(x) = 0$$

or

$$\underline{M(x) = 15 - 3.75x \text{ kNm}, \ 3 < x \leq 4}$$

Last plot the functions $V(x)$ and $M(x)$

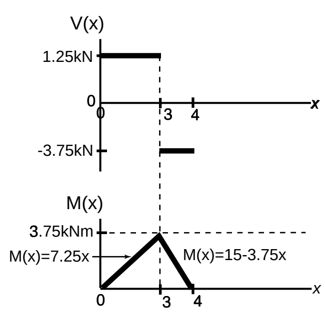


FIGURE S7.19d

Note the maximum value of the moment is 3.75 kN· m

compared to 5 kNm generated by the same magnitude of load acting at the center (as computed in the previous problem).

7.26 Determine the reaction forces, and the internal forces and moments. Plot the shear and bending moment function versus the distance x.

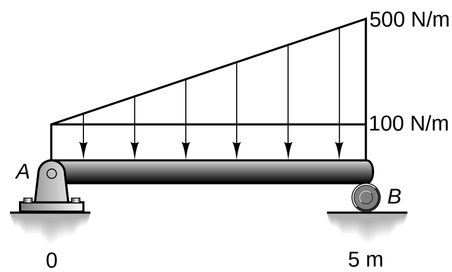


FIGURE P7.26

Solution: Following the solution of problem 4.53, the equivalent force is 1500 N applied at 3.06 m from 0. The free-body diagram for the reaction forces is illustrated.

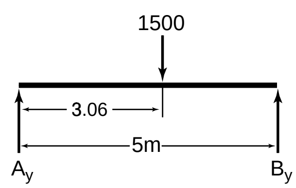


FIGURE S7.26a

$$\sum F = 0 : A_y + B_y - 1500 = 0$$

$$\sum M_A = 0 : 5B_y - (3.06)(1500) = 0$$

$$\underline{B_y = 916.67 \text{ N}, \ A_y = 583.33}$$

Next consider a cut at x and a free body of the section between A and x. Note only one section needs to be analyzed because the load does not change abruptly any where. From the solution to 4.53:

$$x_1 = 2x/3$$

and from similar triangles

$$h = 80x$$

and

$$P_1 = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)(80x) = 40x^2.$$

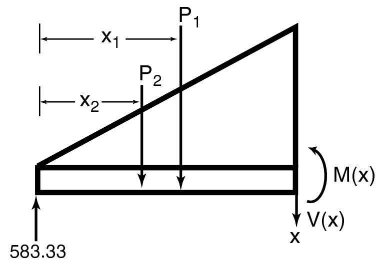


FIGURE S7.26b

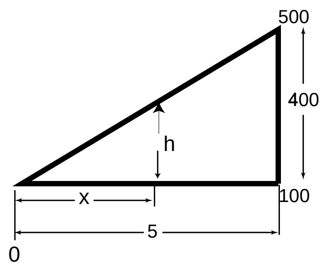


FIGURE S7.26b2

From the rectangular load,

$$x_2 = \frac{x}{2},\ P_2 = 100x.$$

$$\sum F = 0: \quad 583.33 - 40x^2 - 100x - V(x) = 0$$

or

$$\underline{V(x) = 583.33 - 40x^2 - 100x \text{ N}}$$

$$\sum M_A = 0 = -\frac{2x}{3}(40x^2) - \frac{x}{2}(100x) - x(583.33 - 40x^2 - 100x) + M(x) = 0$$

$$\underline{M(x) = \frac{-40x^3}{3} - 50x^2 + 583.33x \text{ Nm}}$$

See computer solutions for plots.

6.15 A lifting mechanism is mounted on the back of a truck. Calculate the forces in each member and the reactions at A and B for a) $\alpha = 0$, b) $\alpha = 15°$ and c) $\alpha = 30°$. Ignore the diameter of the pulley at point D.

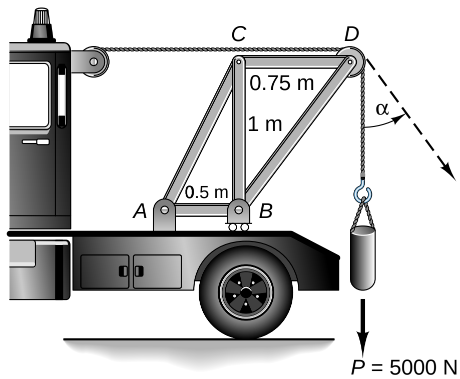


FIGURE P6.15

Solution: Using the method of joints combined with the matrix approach requires a free body diagram of each joint, followed by the equations of equilibrium for each joint. These are written in tabular form as follows

$$(< CAB = \tan^{-1}(\frac{1}{.5}) = 63.43°, < CBD = \tan^{-1}(\frac{.75}{1}) = 36.87°)$$

From A: $$\sum F_x = -A_x + AC\cos 63.43° + AB = 0 \quad (1)$$

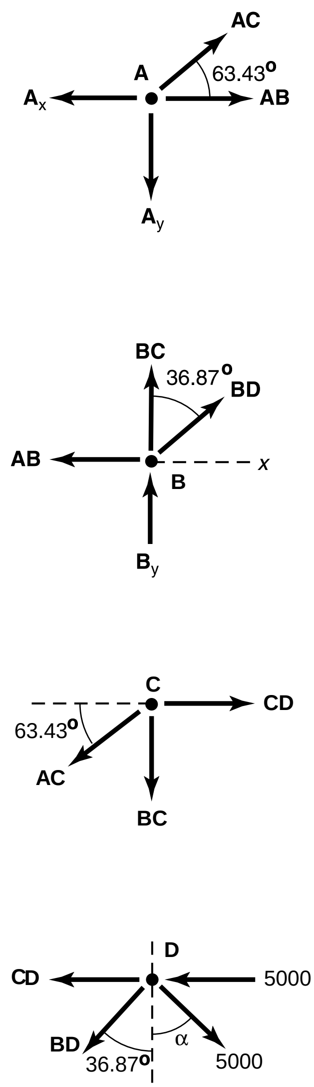
$$\sum F_y = -A_y + AC\sin 63.43° = 0 \quad (2)$$

From B: $$\sum F_x = -AB + BD\sin 36.87° = 0 \quad (3)$$

$$\sum F_y = B_y + BC + BD\cos 36.87° = 0 \quad (4)$$

From C: $$\sum F_x = -AC\cos 63.43° + CD = 0 \quad (5)$$

$$\sum F_y = -AC\sin 63.43° - BC = 0 \quad (6)$$


AC
A
63.43o
Ax
AB
Ay
BC
36.87o
BD
AB
B
x
By
C
CD
63.43o
AC
BC
D
CD
5000
BD
α
36.87o
5000
FIGURE S6.15

From D: $\sum F_x = -BD\sin 36.87^\circ - CD + 5000\sin\alpha - 5000 = 0 \quad (7)$

$$\sum F_y = -BD\cos 36.87^\circ - 5000\cos\alpha = 0 \qquad (8)$$

which is 8 independent equations in 8 unknowns. Writing the matrix form yields ($\beta = 63.43, \gamma = 36.87^\circ$)

$$\begin{bmatrix} -1 & 0 & 0 & 1 & \cos\beta & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & \sin\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & \sin\gamma & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & \cos\gamma & 0 \\ 0 & 0 & 0 & 0 & -\cos\beta & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\sin\beta & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sin\gamma & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\cos\gamma & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ B_y \\ AB \\ AC \\ BC \\ BD \\ CD \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5000(1-\sin\alpha) \\ 5000\cos\alpha \end{bmatrix}$$

which when solved for $\alpha = 0^\circ, 15^\circ$ and 30° yield a

	$\alpha = 0$	$\alpha = 15^\circ$	$\alpha = 30^\circ$
A_x	5000 N(R)	3706 N(R)	2500 N(R)
A_y	2500 N(u)	167 N(u)	1495 N(u)
B_y	2500 N(u)	4662 N(u)	5825 N(u)
AB	3750 N(c)	3622 N(c)	3248 N(c)
AC	2795 N(c)	187 N(c)	1671 N(T)
BC	2500 N(T)	167 N(T)	1495 N(c)
BD	6250 N(c)	6037 N(c)	5413 N(c)
CD	1250 N(c)	84 N(c)	747 N(T)

Note the following:
- A_x drops from 5000 to 2500 as the angle α increases
- the max force in any member (6250 N) decreases with α
- AC and CD go from tension to compression as α increases
- BD goes from tension to compression as α increases

This exercise could be used to discuss design and quasi static analysis in the sense that it is trivial to look at, or even plot, some member forces as α increases in increments of say 1° to illustrate how the member forces react to picking up a load ($\alpha = 0$) and driving away ($\alpha = 1^\circ, 2^\circ ...$).