Learning Objectives

After completing this chapter, you should be able to:
- determine the dimensions and units of physical quantities.
- identify the key fluid properties used in the analysis of fluid behavior.
- calculate common fluid properties given appropriate information.
- explain effects of fluid compressibility.
- use the concepts of viscosity, vapor pressure, and surface tension.

Fluid mechanics is that discipline within the broad field of applied mechanics that is concerned with the behavior of liquids and gases at rest or in motion. It covers a vast array of phenomena that occur in nature (with or without human intervention), in biology, and in numerous engineered, invented, or manufactured situations. There are few aspects of our lives that do not involve fluids, either directly or indirectly.
The immense range of different flow conditions is mind-boggling and strongly dependent on the value of the numerous parameters that describe fluid flow. Among the long list of parameters involved are (1) the physical size of the flow, \( \ell \); (2) the speed of the flow, \( V \); and (3) the pressure, \( p \), as indicated in the figure in the margin for a light aircraft parachute recovery system. These are just three of the important parameters which, along with many others, are discussed in detail in various sections of this book. To get an inkling of the range of some of the parameter values involved and the flow situations generated, consider the following:

**a. Size, \( \ell \)**

Every flow has a characteristic (or typical) length associated with it. For example, for flow of fluid within pipes, the pipe diameter is a characteristic length. Pipe flows include the flow of water in the pipes in our homes, the blood flow in our bodies, and the air flow in our bronchial tree. They also involve pipe sizes that are not within our everyday experiences. Such examples include the flow of oil across Alaska through a four-foot diameter, 799 mile-long pipe, and, at the other end of the size scale, the new area of interest involving flow in nanoscale pipes whose diameters are on the order of \( 10^{-9} \) m. Each of these pipe flows has important characteristics that are not found in the others.

Characteristic lengths of some other flows are shown in Fig. 1.1c.

**b. Speed, \( V \)**

As we note from The Weather Channel, on a given day the wind speed may cover what we think of as a wide range, from a gentle 5 mph breeze to a 100 mph hurricane or a 250 mph tornado. However, this speed range is small compared to that of the almost imperceptible flow of the fluid-like magna below the earth’s surface which drives the motion of the tectonic plates at a speed of about \( 2 \times 10^{-8} \) m/s or the \( 3 \times 10^{-6} \) m/s hyperconvergent air flow past a meteor as it streams through the atmosphere.

Characteristic speeds of some other flows are shown in Fig. 1.1b.

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**F I G U R E 1.1** Characteristic values of some fluid flow parameters for a variety of flows. (a) Object size, (b) fluid speed, (c) fluid pressure.

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**1.1 Some Characteristics of Fluids**

| Property, \( p \) | The pressure within fluids covers an extremely wide range of values. We are accustomed to the 35 psi (2.4 atm.) pressure within our car’s tires, the “120 over 70” typical blood pressure reading, or the standard 14.7 psi atmospheric pressure. However, the large 10,000 psi pressure in the hydraulic ram of an earth mover or the tiny 2 \( \times 10^{-9} \) psi pressure of a sound wave generated at ordinary talking levels are not easy to comprehend.

Characteristic pressures of some other flows are shown in Fig. 1.1c.

The list of fluid mechanics applications goes on and on. But you get the point. Fluid mechanics is a very important, practical subject that encompasses a wide variety of situations. It is very likely that during your career as an engineer you will be involved in the analysis and design of systems that require a good understanding of fluid mechanics. Although it is not possible to adequately cover all the important areas of fluid mechanics within one book, it is hoped that this introductory text will provide a sound foundation of the fundamental aspects of fluid mechanics.

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**1.1 Some Characteristics of Fluids**

One of the first questions we need to explore is, What is a fluid? Or we might ask, What is the difference between a solid and a fluid? We have a general, vague idea of the difference. A solid is "hard" and not easily deformed, whereas a fluid is "soft" and is easily deformed (we can readily move through air). Although quite descriptive, these casual observations of the differences between solids and fluids are not very satisfactory from a scientific or engineering point of view. A closer look at the molecular structure of materials reveals that matter that we commonly think of as a solid (steel, concrete, etc.) has densely spaced molecules with large intermolecular cohesive forces that allow the solid to maintain its shape, and to not be easily deformed. However, for matter that we normally think of as a liquid (water, oil, etc.), the molecules are spaced farther apart, the intermolecular forces are smaller than for solids, and the molecules have more freedom of movement. Thus, liquids can be easily deformed (but not easily compressed) and can be poured into containers or forced through a tube. Gases (air, oxygen, etc.) have even greater molecular spacing and freedom of motion with negligible cohesive intermolecular forces and as a consequence are easily deformed (and compressed) and will completely fill the volume of any container in which they are placed. Both liquids and gases are fluids.

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**Fluids in the News**

Will what works in air work in water? For the past few years a San Francisco company has been working on small, maneuverable submarines designed to travel through water using wings, controls, and thrusters that are similar to those on jet airplanes.

After all, water (for submarines) and air (for airplanes) are both fluids, so it is expected that many of the principles governing the flight of airplanes should carry over to the "flight" of winged submarines.

Of course, there are differences. For example, the submarine must be designed to withstand external pressures of nearly 700 pounds per square inch greater than that inside the vehicle. On the other hand, at high altitude where commercial jet fly, the external pressure is 3.5 psi rather than standard sea level pressure of 14.7 psi, so the vehicle must be pressurized internally for passenger comfort. In both cases, however, the design of the craft for minimal drag, maximum lift, and efficient thrust is governed by the same fluid dynamic concepts.

Although the differences between solids and fluids can be explained qualitatively on the basis of molecular structure, a more specific distinction is based on how they deform under the action of an external load. Specifically, a fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude. A shearing stress (forces per unit area) is created whenever a tangential force acts on a surface as shown in the figure in the margin. When common solids such as steel or other metals are acted on by a shearing stress, they will initially deform (usually a very small deformation), but they will not continuously deform (flow). However, common fluids such as water, oil, and air satisfy the definition of a fluid—that is, they will flow when acted on by a shearing stress. Some materials, such as slurries, tar, putty, toothpaste, and so on, are not easily classified since they will behave as a solid if the applied shearing stress is small, but if the stress exceeds some critical value, the substance will flow. The study of such materials is called rheology.
1.2 Dimensions, Dimensional Homogeneity, and Units

Since in our study of fluid mechanics we will be dealing with a variety of fluid characteristics, it is necessary to develop a system for describing these characteristics both qualitatively and quantitatively. The qualitative aspect serves to identify the nature, or type, of the characteristics (such as length, time, stress, and velocity), whereas the quantitative aspect provides a numerical measure of the characteristics. The quantitative description requires both a number and a standard by which various quantities can be compared. A standard for length might be a meter or foot, for time an hour or second, and for mass a slug or kilogram. Such standards are called units, and several systems of units are in common use as described in the following section. The qualitative description is conveniently given in terms of certain primary quantities, such as length, L, time, T, mass, M, and temperature, θ. These primary quantities can then be used to provide a qualitative description of any other secondary quantity: for example, area, A = L²; velocity, v = LT⁻¹; density, ρ = ML⁻³; and so on, where the symbol = is used to indicate the dimensions of the secondary quantity in terms of the primary quantities. Thus, to describe qualitatively a velocity, v, we would write

\[ v \propto L T^{-1} \]

and say that "the dimensions of a velocity equal length divided by time." The primary quantities are also referred to as basic dimensions.

For a wide variety of problems involving fluid mechanics, only the three basic dimensions, L, T, and M are required. Alternatively, L, T, and F could be used, where F is the basic dimensions of force. Since Newton's law states that force is equal to mass times acceleration, it follows that F = ML²T⁻². Thus, secondary quantities expressed in terms of M can be expressed in terms of F through the relationship above. For example, stress, σ, is a force per unit area, so that \( \sigma = FL^{-1} \), but an equivalent dimensional equation is \( \sigma = ML^{-1}T^{-2} \). Table 1.1 provides a list of dimensions for a number of common physical quantities.

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>FLT System</th>
<th>MLT⁻² System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>( LT^{-1} )</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>Angle</td>
<td>( LT^{-1} )</td>
<td>( ML^{-1}T^{-1} )</td>
</tr>
<tr>
<td>Angular acceleration</td>
<td>( T^{-1} )</td>
<td>( T^{-1} )</td>
</tr>
<tr>
<td>Area</td>
<td>( L^2 )</td>
<td>( L^2 )</td>
</tr>
<tr>
<td>Density</td>
<td>( ML^{-3} )</td>
<td>( ML^{-3} )</td>
</tr>
<tr>
<td>Energy</td>
<td>( FL )</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>Force</td>
<td>( F )</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>Frequency</td>
<td>( T^{-1} )</td>
<td>( T^{-1} )</td>
</tr>
<tr>
<td>Heat</td>
<td>( FL )</td>
<td>( ML^{-1}T^{-3} )</td>
</tr>
<tr>
<td>Mass</td>
<td>( L )</td>
<td>( L )</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>( F )</td>
<td>( ML^{-3} )</td>
</tr>
<tr>
<td>Moment of a force</td>
<td>( F )</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>Moment of inertia (area)</td>
<td>( L^2 )</td>
<td>( L^2 )</td>
</tr>
<tr>
<td>Moment of inertia (mass)</td>
<td>( FLT^{-2} )</td>
<td>( ML^{-2} )</td>
</tr>
<tr>
<td>Momentum</td>
<td>( FT )</td>
<td>( ML^{-2} )</td>
</tr>
</tbody>
</table>

General homogeneous equations are valid in any system of units.

where \( V_0 \) is the initial velocity, \( a \) the acceleration, and \( t \) the time interval. In terms of dimensions the equation is

\[ LT^{-1} = ML^{-1}T^{-1} + LT^{-1} \]

and thus Eq. 1.1 is dimensionally homogeneous.

Some equations that are known to be valid contain constants having dimensions. The equation for the distance, \( d \), traveled by a freely falling body can be written as

\[ d = 16.1 t^2 \]

(1.2)

and a check of the dimensions reveals that the constant must have the dimensions of \( LT^{-2} \) if the equation is to be dimensionally homogeneous. Actually, Eq. 1.2 is a special form of the well-known equation from physics for freely falling bodies,

\[ d = \frac{g t^2}{2} \]

(1.3)

in which \( g \) is the acceleration of gravity. Equation 1.3 is dimensionally homogeneous and valid in any system of units. For \( g = 32.2\ ft/s^2 \), the equation reduces to Eq. 1.2 and thus Eq. 1.2 is valid only for the system of units using feet and seconds. Equations that are restricted to a particular system of units can be denoted as restricted homogeneous equations; as opposed to equations valid in any system of units, which are general homogeneous equations. The preceding discussion indicates one rather elementary, secondary but important, use of the concept of dimensions: the determination of one aspect of the generality of a given equation simply based on a consideration of the dimensions of the various terms in the equation. The concept of dimensions also forms the basis for the powerful tool of dimensional analysis, which is considered in detail in Chapter 7.

The Give and Find are steps that ensure the user understands what is being asked in the problem and explicitly list the items provided to help solve the problems.

The Solution step is where the equations needed to solve the problem are formulated and the problem is actually solved. In this step, there are typically several other tasks that help to set
up the solution and are required to solve the problem. The first is a drawing of the problem; where appropriate, it is always helpful to draw a sketch of the problem. Here the relevant geometry and coordinate system to be used as well as features such as control volumes, forces and pressures, velocities, and mass flow rates are included. This helps in gaining a visual understanding of the problem. Making appropriate assumptions to solve the problem is the second task. In a realistic problem-solving environment, the necessary assumptions are developed as an integral part of the solution process. Assumptions can provide appropriate simplifications or offer useful constraints, both of which can help in solving the problem. Throughout the examples in this text, the necessary assumptions are embedded within the Solution step, as they are in solving a real-world problem. This provides a realistic problem-solving experience.

The final element in the methodology is the Comment. For the examples in the text, this section is used to provide further insight into the problem or the solution. It can also be a point in the analysis at which certain questions are posed. For example: Is the answer reasonable, and does it make physical sense? Are the final units correct? If a certain parameter were changed, how would the answer change? Adopting the above type of methodology will aid in the development of problem-solving skills for fluid mechanics, as well as other engineering disciplines.

**Example 1.1** Restricted and General Homogeneous Equations

**Given** A liquid flows through an orifice located in the side of a tank as shown in Fig. E1.1. A commonly used equation for determining the volume rate of flow, Q, through the orifice is

\[ Q = 0.61 A V g \]

where A is the area of the orifice, g is the acceleration of gravity, and h is the height of the liquid above the orifice.

**Find** Investigate the dimensional homogeneity of this formula.

**Solution**

The dimensions of the various terms in the equation are \( Q = \text{volume/time} \), \( A = L^2 \), \( g = \text{acceleration of gravity} = LT^{-2} \), and \( h = \text{length} = L \).

These terms, when substituted into the equation, yield the dimensional form:

\[
(L^2 T^{-1}) = (0.61)(L^3)(V)(T^{-2})(T^{-2})(g) \]

or

\[
(L^2 T^{-1}) = [(0.61)VT^{-2}](L^{-1}) \]

It is clear from this result that the equation is dimensionally homogeneous [both sides of the formula have the same dimensions of \( L^2 T^{-1} \)], and the numbers (0.61 and \( V \)) are dimensionless.

If we were going to use this relationship repeatedly, we might be tempted to simplify it by replacing \( g \) with its standard value of 32.2 ft/s² and rewriting the formula as

\[
Q = 4.90 A \sqrt{h} \]

A quick check of the dimensions reveals that

\[
LT^{-1} = (4.90)(L^{3/2}) \]

and, therefore, the equation expressed as Eq. 1 can only be dimensionally correct if the number 4.90 has the dimensions of \( L^{3/2} T^{-1} \). Whenever a number appearing in an equation or formula has dimensions, it means that the specific value of the number will depend on the system of units used. Thus, for the case being considered with feet and seconds used as units, the number 4.90 has units of \( ft^{3/2} / s \). Equation 1 will only give the correct value for \( Q \) (in \( ft^3 / s \)) when \( A \) is expressed in square feet and \( h \) in feet. Thus, Eq. 1 is a restricted homogeneous equation, whereas the original equation is a general homogeneous equation that would be valid for any consistent system of units.

**Comment** A quick check of the dimensions of the various terms in an equation is a useful practice and will often be helpful in eliminating errors—that is, as noted previously, all physically meaningful equations must be dimensionally homogeneous. We have briefly alluded to units in this example, and this important topic will be considered in more detail in the next section.

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### 1.2 Dimensions, Dimensional Homogeneity, and Units

#### 1.2.1 Systems of Units

In addition to the qualitative description of the various quantities of interest, it is generally necessary to have a quantitative measure of any given quantity. For example, if we measure the width of this page in the book and say that it is 10 units wide, the statement has no meaning unless the unit of length is defined. If we indicate that the unit of length is a meter, and define the meter as some standard length, a unit system for length has been established (and a numerical value can be given to the page width). In addition to length, a unit must be established for each of the remaining basic quantities (force, mass, time, and temperature). There are several systems of units in use and we shall consider three systems that are commonly used in engineering.

**International System (SI)**: In 1960 the Eleventh General Conference on Weights and Measures, the international organization responsible for maintaining precise uniform standards of measurements, formally adopted the International System of Units as the international standard. This system, commonly termed SI, has been widely adopted worldwide and is widely used (although certainly not exclusively) in the United States. It is expected that the long-term trend will be for all countries to accept SI as the accepted standard and it is imperative that engineering students become familiar with this system. In SI the unit of length is the meter (m), the time unit is the second (s), the mass unit is the kilogram (kg), and the temperature unit is the kelvin (K). Note that there is no degree symbol used when expressing a temperature in kelvin units. The kelvin temperature scale is an absolute scale and is related to the Celsius (centigrade) scale (°C) through the relationship

\[ K = °C + 273.15 \]

Although the Celsius scale is not in itself part of SI, it is common practice to specify temperatures in degrees Celsius when using SI units.

The force unit, called the newton (N), is defined from Newton's second law as

\[ 1 N = (1 kg)(1 m/s^2) \]

Thus, a 1-N force acting on a 1-kg mass will give the mass an acceleration of 1 m/s². Standard gravity \( g \) in SI is 9.807 m/s² (commonly approximated as 9.81 m/s²) so that a 1-kg mass weighs 9.81 N under standard gravity. Note that weight and mass are different, both qualitatively and quantitatively! The unit of work in SI is the joule (J), which is the work done when the point of application of a 1-N force is displaced through a 1-m distance in the direction of a force. Thus,

\[ 1 J = 1 \text{ N} \cdot \text{m} \]

The unit of power is the watt (W) defined as a joule per second. Thus,

\[ 1 W = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s} \]

Prefixes for forming multiples and fractions of SI units are given in Table 1.2. For example, the notation \( km \) would be read as "kilometers" and stands for \( 10^3 \) m. Similarly, mm would be read as "millimeters" and stands for \( 10^{-3} \) m. The centimeter is not an accepted unit of length in
the SI system, so for most problems in fluid mechanics in which SI units are used, lengths will be expressed in millimeters or meters.

**British Gravitational (BG) System.** In the BG system the unit of length is the foot (ft), the time unit is the second (s), the force unit is the pound (lb), and the temperature unit is the degree Fahrenheit (°F) or the absolute temperature unit is the degree Rankine (°R), where

\[ R = °F + 459.67 \]

The mass unit, called the slug, is defined from Newton’s second law (force = mass × acceleration) as

\[ 1 \text{ lb} = (1 \text{ slug})(1 \text{ ft/s}^2) \]

This relationship indicates that a 1-lb force acting on a mass of 1 slug will give the mass an acceleration of 1 ft/s².

The weight, \( W \) (which is the force due to gravity, \( g \)) of a mass, \( m \), is given by the equation

\[ W = mg \]

and in BG units

\[ W(\text{lb}) = m(\text{slugs})g(\text{ft/s}^2) \]

Since the earth’s standard gravity is taken as \( g = 32.174 \text{ ft/s}^2 \) (commonly approximated as 32.2 ft/s²), it follows that a mass of 1 slug weighs 32.2 lb under standard gravity.

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**Fluids in the News**

**How long is a foot?** Today, in the United States, the common length unit is the foot, but throughout antiquity the unit used to measure length has quite a history. The first length units were based on the lengths of various body parts. One of the earliest units was the Egyptian cubit, first used around 3000 B.C. and defined as the length of the arm from elbow to extended fingertips. Other measures followed, with the foot simply taken as the length of a man’s foot. Since this length obviously varies from person to person it was often “standardized” by using the length of the current reigning royalty’s foot. In 1791 a special French commission proposed that a new universal length unit called a meter (mètre) be defined as the distance of one-quarter of the earth’s meridian (north pole to the equator) divided by 10 million. Although controversial, the meter was accepted in 1799 as the standard. With the development of advanced technology, the length of a meter was redefined in 1983 as the distance traveled by light in a vacuum during the time interval of 1/299,792,458 s. The foot is now defined as 0.3048 meters. Our simple rulers and yardsticks indeed have an intriguing history.

**English Engineering (EE) System.** In the EE system, units for force and mass are defined independently; thus special care must be exercised when using this system in conjunction with Newton’s second law. The basic unit of mass in the pound mass (lbm), and the unit of force is the pound (lb). The unit of length is the foot (ft), the unit of time is the second (s), and the absolute temperature scale is the degree Rankine (°R). To make the equation expressing Newton’s second law dimensionally homogeneous we write it as

\[ F = mg \]

where \( g \) is a constant of proportionality which allows us to define units for both force and mass. For the BG system, only the force unit was prescribed and the mass unit defined in a consistent manner such that \( g = 1 \). Similarly, for SI the mass unit was prescribed and the force unit defined in a consistent manner such that \( g = 1 \). For the EE system, a 1-lb force is defined as that force which gives a 1 lbm a standard acceleration of gravity which is taken as 32.174 ft/s². Thus, for Eq. 1.4 to be both numerically and dimensionally correct

\[ 1 \text{ lb} = (1 \text{ lbm})(32.174 \text{ ft/s}^2) \]

It is also common practice to use the notation, \( \text{lb} \), to indicate pound force.

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**FIGURE 1.2** Comparison of SI, BG, and EE units.

With the EE system, weight and mass are related through the equation

\[ W = mg \]

where \( g \) is the local acceleration of gravity. Under conditions of standard gravity (\( g = g_e \)) the weight in pounds and the mass in pound mass are numerically equal. Also, since a 1-lb force gives a mass of 1 lbm an acceleration of 32.174 ft/s² and a mass of 1 slug an acceleration of 1 ft/s², it follows that

\[ 1 \text{ slug} = 32.174 \text{ lbm} \]

In this text we will primarily use the BG system and SI for units. The EE system is used very sparingly, and only in those instances where convention dictates its use, such as for the compressible flow material in Chapter 11. Approximately one-half the problems and examples are given in BG units and one-half in SI units. We cannot overemphasize the importance of paying close attention to units when solving problems. It is very easy to introduce huge errors into problem solutions through the use of incorrect units. Get in the habit of using a consistent system of units throughout a given solution. It really makes no difference which system you use as long as you are consistent; for example, don’t mix slugs and newtons. If problem data are specified in SI units, then use SI units throughout the solution. If the data are specified in BG units, then use BG units throughout the solution. The relative sizes of the SI, BG, and EE units of length, mass, and force are shown in Fig. 1.2.

Tables 1.3 and 1.4 provide conversion factors for some quantities that are commonly encountered in fluid mechanics. For convenient reference these tables are reproduced on the inside of the back cover. Note that in these tables (and others) the numbers are expressed by using computer exponential notation. For example, the number 5.154 × 10^2 is equivalent to 5.154 × 10^2 in scientific notation, and the number 2.832 × 10^2 is equivalent to 2.832 × 10^2. More extensive tables of conversion factors for a large variety of unit systems can be found in Appendix E.

**TABLE 1.3** Conversion Factors from BG and EE Units to SI Units

(See inside of back cover.)

**TABLE 1.4** Conversion Factors from SI Units to BG and EE Units

(See inside of back cover.)
Example 1.2 SI and SI Units

Given: A tank of liquid having a total mass of 36 kg rests on a support in the equipment bay of the Space Shuttle.

Find: Determine the forces (in newtons) that the tank exerts on the support shortly after lift off when the shuttle is accelerating upward as shown in Fig. E1.2a at 15 ft/s².

Solution:
A free-body diagram of the tank is shown in Fig. E1.2b, where \( W \) is the weight of the tank and liquid, and \( F_r \) is the reaction of the floor on the tank. Application of Newton’s second law of motion to this body gives

\[ \sum F = ma \]

or

\[ F_r - W = ma \]

(1)

where we have taken upward as the positive direction. Since \( W = mg \), Eq. 1 can be written as

\[ F_r = m(g + a) \]

(2)

Before substituting any number into Eq. 2, we must decide on a system of units, and then be sure all of the data are expressed in these units. Since we want \( F_r \) in newtons, we will use SI units so that

\[ F_r = 36 \text{ kg} \times (9.81 \text{ m/s}^2 + 15 \text{ ft/s}^2(0.3048 \text{ m/ft})) \]

\[ = 518 \text{ kg m/s}^2 \]

Since \( 1 \text{ N} = 1 \text{ kg m/s}^2 \), it follows that

\[ F_r = 518 \text{ N} \] (downward on floor) (Ans)

\[ \text{FIGURE E1.2a} \] (Photograph courtesy of NASA)

The direction is downward since the force shown on the free-body diagram is the force of the support on the tank so that the force the tank exerts on the support is equal in magnitude but opposite in direction.

Comment: As you work through a large variety of problems in this text, you will find that units play an essential role in arriving at a numerical answer. Be careful! It is easy to mix units and cause large errors. If in the above example the acceleration had been left as 15 ft/s² with \( a \) and \( g \) expressed in SI units, we would have calculated the force as 893 N and the answer would have been 72% too large!

Figures in the News

Units and space travel: A NASA spacecraft, the Mars Climate Orbiter, was launched in December 1998 to study the Martian geography and weather patterns. The spacecraft was slated to begin orbiting Mars on September 23, 1999. However, NASA officials lost communications with the spacecraft early that day and it is believed that the spacecraft broke apart or overheated because it came too close to the surface of Mars. Errors in the maneuvering commands sent from earth caused the Orbiter to sweep within 37 miles of the surface rather than the intended 93 miles. The subsequent investigation revealed that the errors were due to a simple mix-up in units. One team controlling the Orbiter used SI units whereas another team used BG units. This costly experience illustrates the importance of using a consistent system of units.

1.3 Analysis of Fluid Behavior

The study of fluid mechanics involves the same fundamental laws you have encountered in physics and other mechanics courses. These laws include Newton's laws of motion, conservation of mass, and the first and second laws of thermodynamics. Thus, there are strong similarities between the general approach to fluid mechanics and to rigid-body and deformable-body solid mechanics. This is indeed helpful since many of the concepts and techniques of analysis used in fluid mechanics will be ones you have encountered before in other courses.

The broad subject of fluid mechanics can be generally subdivided into fluid statics, in which the fluid is at rest, and fluid dynamics, in which the fluid is moving. In the following chapters we will consider both of these areas in detail. Before we can proceed, however, it will be necessary to define and discuss certain fluid properties that are intimately related to fluid behavior. It is obvious that different fluids can have greatly different characteristics. For example, gases are light and compressible, whereas liquids are heavy (by comparison) and relatively incompressible. A syrup flows slowly from a container, but water flows rapidly when poured from the same container. To quantify these differences, certain fluid properties are used. In the following several sections the properties that play an important role in the analysis of fluid behavior are considered.

1.4 Measures of Fluid Mass and Weight

1.4.1 Density

The density of a fluid, designated by the Greek symbol \( \rho \) (rho), is defined as its mass per unit volume.

Density is typically used to characterize the mass of a fluid system. In the BG system, \( \rho \) has units of slugs/ft³ and SI the units are kg/m³.

The value of density can vary widely between different fluids, but for liquids, variations in pressure and temperature generally have only a small effect on the value of \( \rho \). The small change in the density of water with large variations in temperature is illustrated in Fig. 1.3. Tables 1.5 and 1.6 list values of density for several common liquids. The density of water at 60°F is 1.94 slugs/ft³ or 999 kg/m³. The large difference between these two values illustrates the importance of paying attention to units! Unlike liquids, the density of a gas is strongly influenced by both pressure and temperature, and this difference will be discussed in the next section.

The specific volume, \( \nu \), is the volume per unit mass and is therefore the reciprocal of the density— that is

\[ \nu = \frac{1}{\rho} \]

This property is not commonly used in fluid mechanics but is used in thermodynamics.

\[ \text{FIGURE 1.3} \] Density of water as a function of temperature.
1.4.2 Specific Weight

The specific weight of a fluid, designated by the Greek symbol \( \gamma \) (gamma), is defined as its weight per unit volume. Thus, specific weight is related to density through the equation

\[
\gamma = \rho g
\]

where \( g \) is the local acceleration of gravity. Just as density is used to characterize the mass of a fluid system, the specific weight is used to characterize the weight of the system. In the BG system, \( \gamma \) has units of lb/ft\(^3\) and in SI units are N/m\(^3\). Under conditions of standard gravity \( (g = 32.174 \text{ ft/s}^2 = 9.807 \text{ m/s}^2) \), water at 60 °F has a specific weight of 62.4 lb/ft\(^3\) and 9800 N/m\(^3\).

Tables 1.5 and 1.6 list values of specific weight for several common liquids (based on standard gravity). More complete tables for water can be found in Appendix B (Tables B.1 and B.2).

1.4.3 Specific Gravity

The specific gravity of a fluid, designated as \( SG \), is defined as the ratio of the density of the fluid to the density of water at some specified temperature. Usually the specified temperature is taken as 4 °C (39.2 °F), and at this temperature the density of water is 1.94 slugs/ft\(^3\) or 1000 kg/m\(^3\). In equation form, specific gravity is expressed as

\[
SG = \frac{\rho_f}{\rho_w} = \frac{1}{\gamma_w}
\]

and since it is the ratio of densities, the value of SG does not depend on the system of units used. For example, the specific gravity of mercury at 20 °C is 13.55. This is illustrated by the figure in the margin. Thus, the density of mercury can be readily calculated in either BG or SI units through the use of Eq. 1.7 as

\[
\rho_{\text{Hg}} = (13.55)(1.94 \text{ slugs/ft}^3) = 26.3 \text{ slugs/ft}^3
\]

or

\[
\rho_{\text{Hg}} = (13.55)(1000 \text{ kg/m}^3) = 13.6 \times 10^3 \text{ kg/m}^3
\]

It is clear that density, specific weight, and specific gravity are all interrelated, and from a knowledge of any one of the three the others can be calculated.

1.5 Ideal Gas Law

Gases are highly compressible in comparison to liquids, with changes in gas density directly related to changes in pressure and temperature through the equation

\[
\rho = \frac{p}{RT}
\]

where \( p \) is the absolute pressure, \( \rho \) the density, \( T \) the absolute temperature, and \( R \) is a gas constant. Equation 1.8 is commonly termed the ideal or perfect gas law, or the equation of state for an ideal gas. It is known to closely approximate the behavior of real gases under normal conditions when the gases are not approaching liquefaction.

Pressure in a fluid at rest is defined as the normal force per unit area exerted on a plane surface (real or imaginary) immersed in a fluid and is created by the bombardment of the surface with the fluid molecules. From the definition, pressure has the dimension of FL\(^{-1}\), and in BG units is expressed as lb/ft\(^2\) (psf) or lb/in\(^2\) (psig) and in SI units as N/m\(^2\). In SI, 1 N/m\(^2\) defined as a pascal, abbreviated as Pa, and pressures are commonly specified in pascals. The pressure in the ideal gas law must be expressed as an absolute pressure, denoted abs (psia), which means that it is measured relative to absolute zero pressure (a pressure that would only occur in a perfect vacuum). Standard sea-level atmospheric pressure (by international agreement) is 14.696 psia (abs) or 101.33 kPa (abs). For most calculations these pressures can be rounded to 14.7 psia and 101 kPa, respectively. In engineering it is common practice to measure pressure relative to the local atmospheric pressure, and when measured in this fashion it is called gage pressure. Thus, the absolute pressure can be obtained from the gage pressure by adding the value of the atmospheric pressure. For example, as shown by the figure in the margin on the next page, a pressure of 30 psig (gage) in a tire is equal to 44.7 psia (abs) at standard atmospheric pressure. Pressure is a particularly important fluid characteristic and it will be discussed more fully in the next chapter.
TABLE 1.7
Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure
(Bg Units)

(See inside of front cover.)

TABLE 1.8
Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure
(SI Units)

(See inside of front cover.)

The gas constant, R, which appears in Eq. 1.8, depends on the particular gas and is related to the molecular weight of the gas. Values of the gas constant for several common gases are listed in Tables 1.7 and 1.8. Also in these tables the gas density and specific weight are given for standard atmospheric pressure and gravity and for the temperature listed. More complete tables for air at standard atmospheric pressure can be found in Appendix B (Tables B.3 and B.4).

1.6 Viscosity

The properties of density and specific weight are measures of the "heaviness" of a fluid. It is clear, however, that these properties are not sufficient to uniquely characterize how fluids behave since two fluids (such as water and oil) can have approximately the same value of density but behave quite differently when flowing. There is apparently some additional property that is needed to describe the "fluidity" of the fluid.

To determine this additional property, consider a hypothetical experiment in which a material is placed between two very wide parallel plates as shown in Fig. 1.4a. The bottom plate is rigidly fixed, but the upper plate is free to move. If a solid, such as steel, were placed between the two plates and loaded with the force P as shown, the top plate would be displaced through some small distance, $\delta a$ (assuming the solid was mechanically attached to the plate). The vertical line AB would be rotated through the small angle $\delta \beta$, to the new position $AB'$. We note that to resist the applied force, P, a shearing stress, $\tau$, would be developed at the plate-material interface, and for equilibrium to occur, $P = \tau A$ where A is the effective upper plate area (Fig. 1.4b). It is well known that for elastic solids, such as steel, the small angular displacement, $\delta \beta$ (called the shearing strain), is proportional to the shearing stress, $\tau$, that is developed in the material.

What happens if the solid is replaced with a fluid such as water? We would immediately notice a major difference. When the force P is applied to the upper plate, it will move continuously with a velocity, $U$ (after the initial transient motion has died out) as illustrated in Fig. 1.5. This behavior is consistent with the definition of a fluid—that is, if a shearing stress is applied to a fluid it will deform continuously. A closer inspection of the fluid motion between the two plates would reveal that the fluid in contact with the upper plate moves with the plate velocity, $U$, and the fluid in contact with the bottom fixed plate has a zero velocity. The fluid between the two plates moves with velocity $u = a(y)$ that would be found to vary linearly, $a = ay/b$, as illustrated in Fig. 1.5. Thus, a velocity gradient, $du/dy$, is developed in the fluid between the plates. In this particular case the velocity gradient is a constant since $du/dy = U/b$, but in more complex flow situations, such as that shown by the photograph in the margin, this is not true. The experimental observation that the fluid "sticks" to the solid boundaries is a very important one in fluid mechanics and is usually referred to as the no-slip condition. All fluids, both liquids and gases, satisfy this condition.

In a small time increment, $\delta t$, an imaginary vertical line AB in the fluid would rotate through an angle, $\delta \beta$, so that

$$\tan \delta \beta = \delta \beta = \frac{\delta a}{b}$$

Since $\delta a = U \delta t$ it follows that

$$\delta \beta = \frac{U \delta t}{b}$$

We note that in this case, $\delta \beta$ is a function not only of the force P (which governs $U$) but also of time. Thus, it is not reasonable to attempt to relate the shearing stress, $\tau$, to $\delta \beta$ as is done for solids. Rather, we consider the rate at which $\delta \beta$ is changing and define the rate of shearing strain, $\gamma$, as

$$\gamma = \frac{\delta \beta}{\delta t}$$

which in this instance is equal to

$$\gamma = \frac{U}{b} \frac{du}{dy}$$

A continuation of this experiment would reveal that as the shearing stress, $\tau$, is increased by increasing P (recall that $\tau = P/A$), the rate of shearing strain is increased in direct proportion—that is,

$$\tau \propto \gamma$$

or

$$\tau = \frac{du}{dy}$$

This result indicates that for common fluids such as water, oil, gasoline, and air the shearing stress and rate of shearing strain (velocity gradient) can be related with a relationship of the form

$$\tau = \mu \frac{du}{dy}$$

(1.9)

where the constant of proportionality is designated by the Greek symbol $\mu$ (Ns/m²) and is called the absolute viscosity, dynamic viscosity, or simply the viscosity of the fluid. In accordance with Eq. 1.9, plots of $\tau$ versus $du/dy$ should be linear with the slope equal to the viscosity as illustrated in Fig. 1.6. The actual value of the viscosity depends on the particular fluid, and for a particular fluid the viscosity is also highly dependent on temperature as illustrated in Fig. 1.6 with the two curves for water. Fluids for which the shearing stress is linearly related to the rate of shearing strain (also referred to as rate of angular deformation) are designated as Newtonian fluids after Sir. Newton (1642–1727).

Fortunately most common fluids, both liquids and gases, are Newtonian. A more general formulation of Eq. 1.9 which applies to more complex flows of Newtonian fluids is given in Section 6.8.1.
Fluids in the News

An extremely viscous fluid Pitch is a derivative of tar once used for waterproofing boats. As elevated temperatures it flows quite readily. At room temperature it feels like a solid—it can even be shattered with a blow from a hammer. However, it is a liquid. In 1957 Professor Furness heated some pitch and poured it into a funnel. Since that time it has been allowed to flow freely (or rather, drip slowly) from the funnel. The flow rate is quite small. In fact, to date only seven drops have fallen from the end of the funnel, although the eighth drop is poised ready to fall "soon." While nobody has actually seen a drop fall from the end of the funnel, a苏ker below the funnel holds the previous drops that fall over the years. It is estimated that the pitch is about 100 billion times more viscous than water.

For non-Newtonian fluids, the apparent viscosity is a function of the shear rate.

Fluids for which the shear stress is not linearly related to the rate of shear strain are designated as non-Newtonian fluids. Although there is a variety of types of non-Newtonian fluids, the simplest and most common are shown in Fig. 1.7. The slope of the shear stress versus rate of shear strain graph is denoted as the apparent viscosity, \( \mu_a \). For Newtonian fluids the apparent viscosity is the same as the viscosity and is independent of shear rate.

For shear thinning fluids the apparent viscosity decreases with increasing shear rate—the harder the fluid is sheared, the less viscous it becomes. Many colloidal suspensions and polymer solutions are shear thinning. For example, latex paint does not drip from the brush because the shear rate is small and the apparent viscosity is large. However, it flows smoothly onto the wall because the thin layer of paint between the wall and the brush causes a large shear rate and a small apparent viscosity.

\[ \text{Shear stress} = \eta \times \text{rate of shear strain} \]

\[ \text{Rate of shear strain} \]
the temperature increases, these cohesive forces are reduced with a corresponding reduction in resistance to motion. Since viscosity is an index of this resistance, it follows that the viscosity is reduced by an increase in temperature. In gases, however, the molecules are widely spaced and intermolecular forces negligible. In this case, resistance to relative motion arises due to the exchange of momentum of gas molecules between adjacent layers. As molecules are transported by random motion from a region of low bulk velocity to mix with molecules in a region of higher bulk velocity (and vice versa), there is an effective momentum exchange which resists the relative motion between the layers. As the temperature of the gas increases, the random molecular activity increases with a corresponding increase in viscosity.

The effect of temperature on viscosity can be closely approximated using two empirical formulas. For gases the Sutherland equation can be expressed as

$$\mu = \frac{CT^{\alpha}}{T + S}$$  \hspace{1cm} (1.10)

where $C$ and $S$ are empirical constants, and $T$ is absolute temperature. Thus, if the viscosity is known at two temperatures, $C$ and $S$ can be determined. Or, if more than two viscosities are known, the data can be correlated with Eq. 1.10 by using some type of curve-fitting scheme.

For liquids an empirical equation that has been used is

$$\mu = \frac{D\eta}{T}$$  \hspace{1cm} (1.11)

where $D$ and $B$ are constants and $T$ is absolute temperature. This equation is often referred to as Andrade’s equation. As was the case for gases, the viscosity must be known at least for two temperatures so the two constants can be determined. A more detailed discussion of the effect of temperature on fluids can be found in Ref. 1.

## Example 1.4 Viscosity and Dimensionless Quantities

**GIVEN** A dimensionless combination of variables that is important in the study of viscous flow through pipes is called the Reynolds number $Re$, defined as $VD/\mu$, where, as indicated in Fig. 1.4, $\mu$ is the fluid viscosity, $V$ the mean fluid velocity, $D$ the pipe diameter, and $\mu$ the fluid viscosity. A Newtonian fluid having a viscosity of 0.38 N·s/m² and a specific gravity of 0.91 flows through a 25-mm-diameter pipe with a velocity of 3.6 m/s.

**FIND** Determine the value of the Reynolds number using (a) SI units, and (b) BG units.

**Solution**

(a) The fluid density is calculated from the specific gravity as $\rho = SG \rho_{\text{water}} = 0.91 \times 1000 \text{ kg/m}^3 = 910 \text{ kg/m}^3$ and from the definition of the Reynolds number

$$Re = \frac{\rho V D}{\mu} = \frac{(910 \text{ kg/m}^3)(2.6 \text{ m/s})(0.025 \text{ m})}{0.38 \text{ N·s/m}^2} = 156 \text{ (kg·m/s³)/N}$$

However, since 1 N = 1 kg·m/s² it follows that the Reynolds number is unitsless—that is,

$$Re = 156$$  \hspace{1cm} (Ans)

(b) We first convert all the SI values of the variables appearing in the Reynolds number to BG values by using the conversion factors from Table 1.4. Thus,

$$\rho = (910 \text{ kg/m}^3)(1.940 \times 10^2) = 1.77 \text{ slugs/ft}^3$$

$$V = (2.6 \text{ m/s})(3.281) = 8.53 \text{ ft/s}$$

$$D = (0.025 \text{ m})(3.81) = 0.28 \times 10^{-1} \text{ ft}$$

$$\mu = (0.38 \text{ N·s/m}^2)(2.089 \times 10^{-3}) = 7.94 \times 10^{-3} \text{ lb·s/ft}^2$$

and the value of the Reynolds number is

$$Re = \frac{(1.77 \text{ slugs/ft}^3)(8.53 \text{ ft/s})(0.28 \times 10^{-1} \text{ ft})}{7.94 \times 10^{-3} \text{ lb·s/ft}^2} = 156 \text{ (slag·ft/s)/(lb·s/ft²)} = 156 \text{ (Ans)}$$

**Example 1.5 Newtonian Fluid Shear Stress**

**GIVEN** The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (see Fig. 1.5a) is given by the equation

$$u = \frac{3V}{2} \left( 1 - \left( \frac{y}{h} \right)^2 \right)$$

where $V$ is the mean velocity. The fluid has a viscosity of 0.04 lb·s/ft². Also, $V = 2$ ft/s and $h = 0.2$ in.

**FIND** Determine: (a) the shearing stress acting on the bottom wall, and (b) the shearing stress acting on a plane parallel to the walls and passing through the centerline of the channel.

**Solution**

For this type of parallel flow the shearing stress is obtained from Eq. 1.9,

$$\tau = \frac{d\mu}{dy}$$  \hspace{1cm} (1)

Thus, if the velocity distribution $u = u(y)$ is known, the shearing stress can be determined at all points by evaluating the velocity gradient, $du/dy$. For the distribution given

$$\frac{du}{dy} = \frac{3V}{2h}$$

(a) Along the bottom wall $y = -h$ so that (from Eq. 2)

$$\frac{du}{dy} = \frac{3V}{2h}$$

and therefore the shearing stress is

$$\tau_{\text{wall}} = \mu \frac{3V}{2h} = \frac{(0.04 \text{ lb·s/ft}^2)(3)(2 \text{ ft/s})}{(0.2 \text{ in})(1/12 \text{ in})} = 14.4 \text{ lb/ft}^2$$

This stress creates a drag on the wall. Since the velocity distribution is symmetrical, the shearing stress along the upper wall would have the same magnitude and direction.

(b) Along the midplane where $y = 0$ it follows from Eq. 2 that

$$\frac{du}{dy} = 0$$

and thus the shearing stress is

$$\tau_{\text{midplane}} = 0$$  \hspace{1cm} (Ans)

**Comment** From Eq. 2 we see that the velocity gradient (and therefore the shearing stress) varies linearly with $y$ and in this particular example varies from 0 at the center of the channel to 14.4 lb/ft² at the walls. This is shown in Fig. 1.5b. For the more general case the actual variable will, of course, depend on the nature of the velocity distribution.

Quite often viscosity appears in fluid flow problems combined with the density in the form

$$\rho = \frac{\mu}{\rho}$$
1.7 Compressibility of Fluids

1.7.1 Bulk Modulus

An important question to answer when considering the behavior of a particular fluid is how easily can the volume (and thus the density) of a given mass of the fluid be changed when there is a change in pressure? That is, how compressible is the fluid? A property that is commonly used to characterize compressibility is the bulk modulus, \( E_b \), defined as

\[
E_b = \frac{dp}{dV/V} \quad (1.12)
\]

where \( dp \) is the differential change in pressure needed to create a differential change in volume, \( dV/V \), of a volume \( V \). This is illustrated by the figure in the margin. The negative sign is included since an increase in pressure will cause a decrease in volume. Since a decrease in volume of a given mass, \( m = \rho V \), will result in an increase in density, Eq. 1.12 can also be expressed as

\[
E_b = \frac{\rho}{d\rho/dp} \quad (1.13)
\]

The bulk modulus (also referred to as the bulk modulus of elasticity) has dimensions of pressure, \( FL^{-2} \). In BG units, values for \( E_b \) are usually given as \( lb/in^2 \) (psi) and in SI units as \( N/m^2 \) (Pa). Large values for the bulk modulus indicate that the fluid is relatively incompressible—that is, it takes a large pressure change to create a small change in volume. As expected, values of \( E_b \) for common liquids are large (see Tables 1.5 and 1.6). For example, at atmospheric pressure and a temperature of 60°F it would require a pressure of 5130 psi to compress a unit volume of water 1%. This result is representative of the compressibility of liquids. Since such large pressures are required to effect a change in volume, we conclude that liquids cannot be considered as incompressible for most practical engineering applications. As liquids are compressed, the bulk modulus increases, but the bulk modulus near atmospheric pressure is usually the one of interest. The use of bulk modulus as a property describing compressibility is most prevalent when dealing with liquids, although the bulk modulus can also be determined for gases.

Fluids in the News

This water jet is a blast. Usually liquids can be treated as incompressible fluids. However, in some applications the compressibility of a liquid can play a key role in the operation of a device. For example, a water pulse generator using compressed water has been developed for use in mining operations. It can fracture rock by producing an effect comparable to a conventional explosive such as guncotton. The device uses the energy stored in a water-filled accumulator to generate an ultrahigh-pressure water pulse ejected through a 10- to 25-mm-diameter discharge valve. At the ultrahigh pressures used (300 to 400 MPa, or 3000 to 4000 atmospheres), the water is compressed (i.e., the volume reduced) by about 10 to 15%. When a fast-opening valve within the pressure vessel is opened, the water expands and produces a jet of water that upon impact with the target material produces an effect similar to the explosive forces from conventional explosives. Mining with the water jet can eliminate various hazards that usually arise from conventional chemical explosives, such as those associated with the storage and use of explosives and the generation of toxic gas by-products that require extensive ventilation. (See Problem 1.87.)

1.7.2 Compression and Expansion of Gases

When gases are compressed (or expanded), the relationship between pressure and density depends on the nature of the process. If the compression or expansion takes place under constant temperature conditions (isothermal process), then from Eq. 1.8

\[
P = \text{constant} \quad (1.14)
\]

If the compression or expansion is frictionless and no heat is exchanged with the surroundings (isentropic process), then

\[
P = \rho \text{constant} \quad (1.15)
\]

where \( k \) is the ratio of the specific heats at constant pressure, \( c_p \), to the specific heat at constant volume, \( c_v \) (i.e., \( k = c_p/c_v \)). The two specific heats are related to the gas constant, \( R \), through the equation \( R = c_v - c_p \). As was the case for the ideal gas law, the pressure in both Eqs. 1.14 and 1.15 must be expressed as an absolute pressure. Values of \( k \) for some common gases are given in Tables 1.7 and 1.8, and for air over a range of temperatures, in Appendix B (Tables B.3 and B.4). The pressure-density variations for isothermal and isentropic conditions are illustrated in the margin figure.

With explicit equations relating pressure and density, the bulk modulus for gases can be determined by obtaining the derivative \( dp/d\rho \) from Eq. 1.14 or 1.15 and substituting the results into Eq. 1.13. It follows that for an isothermal process

\[
E_b = \rho \quad (1.16)
\]

and for an isentropic process,

\[
E_b = k \rho \quad (1.17)
\]

Note that in both cases the bulk modulus varies directly with pressure. For air under standard atmospheric conditions with \( \rho = 14.7 \) psi (abs) and \( k = 1.40 \), the isentropic bulk modulus is 20.6 psi. A comparison of this figure with that for water under the same conditions (\( E_b = 512,000 \) psi) shows that air is approximately 15,000 times as compressible as water. It is thus clear that in dealing with gases, greater attention will need to be given to the effect of compressibility on fluid behavior. However, as will be discussed further in later sections, gases can often be treated as incompressible fluids if the changes in pressure are small.

Example 1.6: Isoentropic Compression of a Gas

GIVEN

A cubic foot of air at an absolute pressure of 14.7 psi is compressed isentropically to \( \frac{1}{4} \) ft\(^3\) by the tire pump shown in Fig. E1.6a.

FIND

What is the final pressure?

Solution

For an isentropic compression

\[
P_1 = P_2 \left( \frac{\rho_2}{\rho_1} \right)
\]

where the subscripts \( i \) and \( f \) refer to initial and final states, respectively. Since we are interested in the final pressure, \( P_f \), it follows that

\[
P_f = \left( \frac{\rho_f}{\rho_i} \right) P_i
\]