Energy and Power Spectral Density and Autocorrelation.

Lecture Outline

- **Energy Spectral Density**
- **Autocorrelation of Energy Signals**
- **Power Spectral Density (PSD)**
- **Properties of PSD**
- **Filtering Power Signals**
- **Modulating Power Signals**

1. **Energy and Power Signals**
   - An energy signal $x(t)$ has $0 < E < \infty$ for average energy
   $$E = \int_{-\infty}^{\infty} |x(t)|^2 dt.$$
   - A power signal $x(t)$ has $0 < P < \infty$ for average power
   $$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt.$$
   - Can think of average power as average energy/time.
   - An energy signal has zero average power. A power signal has infinite average energy. Power signals are generally not integrable so don't necessarily have a Fourier transform.
   - We use power spectral density to characterize power signals that don't have a Fourier transform.

2. **Energy Spectral Density (ESD)**
   - Defined as $\Psi_x(f) = |X(f)|^2$.
   - Measures the distribution of signal energy $E = \int |x(t)|^2 dt = \int \Psi_x(f) df$ over frequency.
   - Properties of ESD include $\Psi_x(f) \geq 0$, $\Psi_x(-f) = \Psi_x(f)$ for $x(t)$ real, and for $x(t)$ input to a filter with frequency response $H(f)$, the filter output $y(t)$ has ESD $\Psi_y(f) = |H(f)|^2 \Psi_x(f)$.

3. **Autocorrelation of Energy Signals**
   - Defined for real signals as $R_x(\tau) = \int x(t)x(t-\tau)dt = x(\tau) * x(-\tau)$.
   - Measures the similarity of a signal with a delayed version of itself.
   - Autocorrelation defines signal energy: $E = R_x(0)$.
   - Since $|R_x(\tau)| \leq R_x(0)$, can use autocorrelation for signal synchronization.
   - The autocorrelation is symmetric: $R_x(\tau) = R_x(-\tau)$.
   - The autocorrelation and ESD are Fourier Transform pairs: $R_x(\tau) \Leftrightarrow \Psi_x(f)$.

4. **Power Spectral Density (PSD)**
   - Power signals have infinite energy: Fourier transform and ESD may not exist.
   - Power signals need alternate spectral density definition with similar properties as ESD.
   - Can obtain ESD for a power signal $x(t)$ that is time windowed with window size $2T$.
   - PSD defined as the normalized limit of the ESD for the windowed signal $x_T(t)$: $S_x(f) = \lim_{T\to\infty} \frac{1}{2T}|X_T(f)|^2$.
   - PSD measures the distribution of signal power $P = \lim_{T\to\infty} \frac{1}{2T}\int |x_T(t)|^2 dt = \int S_x(f)df$ over frequency domain.

5. **Properties of PSD**
   - $S_x(f) \geq 0$
   - $S_x(-f) = S_x(f)$

6. **Filtering and Modulation of Power Signals:**
   - Let $x(t)$ be a power signal with PSD $S_x(f)$.
   - If $x(t)$ is input to a filter with frequency response $H(f)$, then the filter output $y(t)$ has PSD $S_y(f) = |H(f)|^2 S_x(f)$.
   - If $S_x(f)$ is bandlimited with bandwidth $B << f_c$, then for $z(t) = x(t)\cos(2\pi f_c t)$, $S_z(f) = .25[S_x(f-f_c) + S_x(f+f_c)]$.

Main Points:

- Energy spectral density measures signal energy distribution across frequency.
- Autocorrelation function of an energy signal measures signal self-similarity versus delay: can be used for synchronization.
- A signal's autocorrelation and ESD are Fourier transform pairs.
- Power signals often do not have Fourier transforms: instead we characterize them using PSD.
- Can determine the impact of filtering and modulation of power signals based on PSD.