Below I discuss some useful tricks.

1.21. The signals are sketched in Figure S1.21.

\[ x(t-1) \]

\[ x(2-t) \]

\[ x(2t+1) \]

\[ x(4-t/2) \]

\[ [x(t)+x(-t)]u(t) \]

\[ x(t+1) \]

\[ x(2t) \]

\[ x(2t+1) \]

\[ x(4-t/2) \]

\[ y(t-1) = x(t-1) \]

\[ y(t-2) = x(t-2) \]

\[ = x(2-t) \]

\[ = x(2-t) \]

\[ \text{which is what we want!} \]

So first flip \( x(t) \), and then shift to the right by 2 \( \text{not left, be careful} \).

\[ y(t) = x(2t) \]

\[ y(t+1) = x(2(t+1)) \]

\[ = x(2t+1) \]

\[ \text{which is what we want.} \]

So compress \( x(t) \) by \( 2 \) \( \text{and then shift to the left by } 1/2 \) \( \text{but 1, be careful!} \)

\[ y(t) = x(4-t/2) \]

\[ y(t) = x(4-t/2) \]

\[ y(t) = x(-t/2) \]

\[ y(1) = x(-4/2) \]

\[ y(1) = x(-4/2) \]

Which is what we want.

So first get a flipped & compressed version of \( x(t) \). Then shift to the right by 1/2.

\[ [x(t)+x(-t)]u(t) \]
Again I discuss some useful tricks.

1.22

Figure S1.22
I will not check system properties for all the systems but a selected few. The left-overs are considerably easier but still if you have difficulties please ask me.
1. \( y(t) = x((t - 1) / 2) \)

(i) The system is not causal because the output at \( t = -1 \) depends on the input at \( t = -2 \) which is in the future.

(ii) Consider a shifted input \( x((t - a) / 2) \).

The output is \( x((t - a) / 2) \neq y((t - b) / 2) = x((t - b) / 2) \).

The system is not time-invariant.

(iii) Let \( a, b \) be arbitrary complex constants.

\[
\begin{align*}
    x(t) &\rightarrow y(t) = x((t - 1) / 2) \\
    x_a(t) &\rightarrow y_a(t) = x_a((t - 1) / 2) \\
    a x(t) + b x_a(t) &\rightarrow a x((t - 1) / 2) + b x_a((t - 1) / 2)
\end{align*}
\]

The system is linear.

2. \[
y(t) = \frac{d}{dt} x(t) \Rightarrow \frac{d}{dt} x(t - t_0) = y(t - t_0)
\]

The system is time invariant.

3. Let \( y(t) = x'(t) \) \( y_2(t) = x'_2(t) \)

The output of the system to \( ax(t) + bx_2(t) \) with \( a, b \in \mathbb{C} \)

\[
\frac{d}{dt} (a x(t) + b x_2(t)) = a x'(t) + b x'_2(t) = ay(t) + by_2(t)
\]

The system is linear.

Assuming the input is differentiable anywhere, it is sufficient to take the limit from the left hand side to calculate the derivative. The system is causal.
Again, I will show the details for few selected examples only.

1.28. (a) Linear, stable.
(b) Time invariant, linear, causal, stable.
(c) Memoryless, linear, causal.
(d) Linear, stable.
(e) Linear, stable.
(f) Memoryless, linear, causal, stable.
(g) Linear, stable.

(i) To know $y[n]$ for $n = 1$ we need $x[0]$ at $n = 1$, which is future. Hence non-causal.

(ii) $x[n] \rightarrow y[n] = x[n]$

$x[n] = x[n-n_0] \rightarrow y[n] = x[n-n_0]$

But $y[n-n_0] = x_1[-(n-n_0)]$

$= x_1[-n-n_0]$

$= y_2[n]$

Hence, not II.

(iii) Linearity is straightforward.

(ii) System is not II because the location of origin is important.

(iii) Linearity is straightforward.

(i) Non-causal, because for $y[n]$, we need $x[0]$.

(ii) $x[n] \rightarrow y[n] = x[n+n_1]$

$x[n] = x[n-n_0] \rightarrow y[n] = x[n-n_0]$

But $y[n-n_0] = x_1[-(n-n_0)] + y_2[n]$

So not II.

(iii) $x[n] \rightarrow y[n] = x[n+n_1]$

$y[n] \rightarrow y[n] = x_0[n+n_1]$

But $y[n+n_1] = a y_1[n] + b y_2[n]$

Hence, Linear.
1.31. (a) Note that \( x_2(t) = x_1(t) - x_1(t - 2) \). Therefore, using linearity we get \( y_2(t) = y_1(t) - y_1(t - 2) \). This is as shown in Figure S1.31.

(b) Note that \( x_3(t) = x_1(t) + x_1(t + 1) \). Therefore, using linearity we get \( y_3(t) = y_1(t) + y_1(t + 1) \). This is as shown in Figure S1.31.

![Figure S1.31](image)

1.32. All statements are true.

1. \( x(t) \) periodic with period \( T \); \( y_1(t) \) periodic, period \( T/2 \).

2. \( y_1(t) \) periodic, period \( T \); \( x(t) \) periodic, period \( 2T \).

3. \( x(t) \) periodic, period \( T \); \( y_2(t) \) periodic, period \( 2T \).

4. \( y_2(t) \) periodic, period \( T \); \( x(t) \) periodic, period \( T/2 \).

1.41. (a) \( y[n] = 2x[n] \). Therefore, the system is time invariant.

(b) \( y[n] = (2n - 1)x[n] \). This is not time-invariant because \( y[n - N_0] \neq (2n - 1)x[n - N_0] \).

(c) \( y[n] = x[n](1 + (-1)^n + 1 + (-1)^{n-1}) = 2x[n] \). Therefore, the system is time invariant.