TRANSIT-TIME CALCULATIONS

Specific Liner Performance Criteria

There are a number of different criteria which can be used to design and/or evaluate an earthen liner. These so-called performance criteria are based on (Goldman et al., 1986; Shackelford, 1992):

1. a specified seepage flux at the liner bottom,
2. a specified contaminant flux for a particular leachate constituent at the liner bottom,
3. the time required to reach a specified seepage flux at the liner bottom,
4. the time taken to reach a specified leachate chemical flux at the liner bottom, and/or
5. the time taken for the concentration of a specific leachate component to reach a specified value at the liner bottom.

Criteria (3), (4), and (5) are referred to as transit-times in that the criteria all involve a time evaluation related to flow or transport. With respect to these five performance criteria, only the transit time criteria based on concentration (No. 5) will be illustrated. The analyses based on the other four criteria are illustrated elsewhere (e.g., Goldman et al., 1986; Shackelford, 1990, 1992a; and Manassero and Shackelford, 1993).

Transport Mechanisms

Three transport mechanisms - advection, mechanical dispersion, and molecular diffusion - generally are recognized as governing the transport of contaminants in porous materials. Advection is the transport of contaminants in response to a gradient in total hydraulic head. Due to advection, nonreactive contaminants are transported at an average rate equal to the seepage velocity of the
flowing liquid waste. Both mechanical dispersion and diffusion are mixing, or dilution, processes which cause the contaminant front to spread out or disperse during transport through the porous material.

Diffusion is a fundamental, irreversible process in which matter is transported spontaneously from one part of a system to another part of the system due to random motions of molecules. On a practical scale, these molecular motions result in the net transport of a chemical species (e.g., ion or molecule) from a region of higher concentration to a region of lower concentration. As a result, diffusion may be thought of as the transport of a chemical species due to a gradient in its concentration.

Mechanical dispersion is a function of the seepage velocity whereas molecular diffusion is not; i.e., the greater the seepage velocity, the greater the effect of mechanical dispersion, and vice versa. Therefore, even though all three mechanisms are active during transport, the effect of mechanical dispersion becomes less significant as the seepage velocity decreases. For the relatively low seepage velocities typically associated with fine-grained soil barriers, the effect of mechanical dispersion often is negligible. As a result, the transport of contaminants through waste containment barriers generally can be described using an appropriate advective-diffusive model.

The appropriate model to be used depends on the susceptibility of the contaminant to geochemical reactions (i.e., nonreactive versus reactive solutes) and the overall flow conditions. In general, there are three possible cases to consider as shown in Fig. 43 (Shackelford, 1989, 1992b; and Manassero and Shackelford, 1993):

1. diffusion without advection (pure diffusion);
2. diffusion with positive advection, and;
3. diffusion with negative advection.
FIGURE 43 - Flow Conditions for Waste Containment Barriers: (a) Diffusion without Advection (Pure Diffusion); (b) Diffusion with Positive Advection; and (c) Diffusion with Negative Advection (after Shackelford, 1989, 1992b, and Manassero and Shackelford, 1993).
In the case of waste containment, diffusion is always present when there is a relatively high concentration of contaminant constituents being contained. Each of these three cases is discussed in the following sections.

**Diffusion Without Advection (Pure Diffusion)**

For the case shown in Fig. 43a, the hydraulic gradient across the clay barrier is zero, so there is no advective flow. However, since the concentration of the contaminants in the leachate is greater than that in the natural soil, a concentration gradient is established across the barrier. As a result of this concentration gradient, diffusion of contaminants will occur through the barrier.

The significance of pure diffusive transport of contaminants across clay barriers can be illustrated with the aid of an example. If the concentration of contaminants in the leachate is assumed to be constant with time, then the concentration of the contaminant reaching the bottom of a clay barrier of thickness L can be described as a function of time, as illustrated in Fig. 44. In Fig. 44, $c_o$ is the concentration of the nonreactive solute in the leachate, $c$ is the corresponding concentration at the bottom of the barrier, and the transit time is the time required for the solute to travel from the top to the bottom of the barrier. A reasonable estimate for the effective diffusion coefficient ($D^*$) of a nonreactive solute, such as chloride (Cl$^-$), is $6.0 \times 10^{-10} \text{ m}^2/\text{s}$ ($6.0 \times 10^{-6} \text{ cm}^2/\text{s}$). Based on this $D^*$ value, the relative concentrations, $c/c_o$, at the bottom of the clay barrier have been plotted in Fig. 44 for a 50-year period as a function of the barrier thickness. Since, in many cases, these values for $c/c_o$ represent concentrations which may exceed allowable standards, it is apparent that diffusion through low-permeability soil barriers can be significant transport process, especially for relatively thin barriers.

**Diffusion With Positive Advection**

For the case depicted in Fig. 43(b), a hydraulic gradient has been established across the clay barrier such that advective flow of contaminants occurs in the same direction as the diffusive transport in the pure diffusive case. The advective flow is termed "positive" since it results in an
FIGURE 44 - Relative Concentration \((c/c_0)\) Versus Transit Time due to Pure Diffusive Transport (after Shackelford, 1989, 1992b).
increase in the contaminant concentration in the natural soil. This case is commonly described in texts on contaminant transport. The transient model for this case, assuming a homogeneous soil, is described by the well known "advective-dispersive" equation.

As previously indicated, the transport of contaminants through clay barriers is a function of the magnitude of the seepage velocity. An example of this dependance is illustrated in Fig. 45 where the transit times for several relative concentrations \((c/c_0)\) have been plotted versus the seepage velocity. The data for Fig. 45 were generated assuming \(D^*\) equals \(6.0 \times 10^{-10} \text{ m}^2/\text{s} \) \( (6.0 \times 10^{-6} \text{ cm}^2/\text{s}) \) and \(x (=L)\) is \(0.914 \text{ m} \) \((3 \text{ ft.})\). The data presented in Fig. 40 indicate that the transit time approaches asymptotically a limiting value which is based on pure diffusive transport as the seepage velocity decreases. In the limit, when the seepage velocity approaches zero, the transit time for the advective-diffusive transport case approaches the transit time for the pure diffusive transport case. Therefore, the transit times based on pure diffusive transport given in Fig. 39 represent the limiting values for the case of diffusion with positive advection. For example, the transit time based on pure diffusive transport for a barrier thickness of \(0.914 \text{ m} \) \((3 \text{ ft})\) and a relative concentration of \(0.5\) is approximately 49 years (see Fig. 39). This value is represented in Fig. 40 by a vertical line which indicates the independence of diffusion on the seepage velocity. As a result, the best containment barrier that can be built for the case of positive advection is one in which diffusion is the controlling mechanism of contaminant transport. However, relatively short transit times may still result even though diffusion is the controlling mechanism of transport, especially for relatively thin barriers and/or low values of \(c/c_0\) (see Figs. 44 and 45).

**Diffusion With Negative Advection**

In some instances, the direction of advective transport may be opposite to that of diffusive transport, as shown in Fig. 43(c). This situation may occur in practice, for example, when the barrier (either natural or man-made) is located over a confined aquifer under artesian pressure, or
when vertical barriers, such as slurry walls, are used to isolate a contaminated area (see Fig. 46). In this case, the advective flow is termed "negative" since it works to prevent the escape of pollutants from the contaminated area. Although the seepage velocity works against the migration of contaminants from the containment system,

significant diffusive transport of contaminants may still result if relatively thin barriers are built to contain relatively high concentrations of priority pollutants over extensive periods.

Example Design Problem

An example design problem for the case of diffusion with positive advection (Fig. 43b) is presented to illustrate (1) the procedure for estimating the required thickness of a compacted soil liner, and (2) the effect of diffusion on calculated results. The design will be based on an analytical solution to the one-dimensional advection-dispersion (diffusion) equation for solute transport in saturated soil in the form of Fig. 47. (e.g., see Ogata, 1970; and Ogata and Banks, 1961). In Fig. 47, $T_R$ is a dimensionless time factor, or

$$T_R = \frac{v_s t}{R_d L}$$

(11)

where $v_s$ is the seepage velocity, $t$ is the transit time, $R_d$ is the retardation factor, and $L$ is the thickness of the soil liner, and $P_L$ is a dimensionless factor known as the Peclet number, or

$$P_L = \frac{v_s L}{D^*}$$

(12)
FIGURE 47 - Relative Concentration of Leachate Constituent \( \left( \frac{c}{c_o} \right) \) as a Function of Dimensionless Parameters \( T_R \) and \( P_L \) (after Shackelford, 1990).
An iterative procedure can be used with Fig. 47 to estimate the liner thickness required to provide the necessary transit time for a specified leachate concentration, c. The iterative procedure involves the following steps:

1. Assume a liner thickness, L;
2. Calculate $P_L$ (Eq. 16);
3. Determine $T_R$ from Fig. 42 for the desired value of $c/c_0$ and the calculated value of $P_L$;
4. Use Eq. 15 to determine the transit time, $t$; and
5. Repeat step (1) through (4) until $t$ is greater than or equal to the design life of the containment barrier.

For the purpose of illustration, a somewhat low seepage velocity of $1.8 \times 10^{-8}$ cm/s ($1.8 \times 10^{-10}$ m/s) will be assumed for the soil in the field, the minimum thickness of the barrier is 3 feet, and the design life of the containment system is 40 years. Also, the analysis will be performed only for chloride (Cl\(^-\)), a non-reactive solute. The appropriate chemical properties for chloride are:

$c_0 = 3484$ mg/L

$c = 250$ mg/L (Drinking Water Standard)

$D^* = 4.7 \times 10^{-6}$ cm\(^2\)/s

$R_d = 1.0$ (non-reactive)

Therefore, the performance criteria is a relative concentration, $c/c_0$, of 0.072 ($= 250/3484$) at the bottom of the liner after 40 years. The value of $P_L$ is:

$$P_L = \frac{(1.8 \times 10^{-8} \text{ cm/s})(0.9144 \text{ m})(100 \text{ cm/m})}{4.7 \times 10^{-6} \text{ cm}^2/\text{s}} = 0.35$$

With $c/c_0 = 0.072$, $T_R$ is approximately 0.05 (Fig. 47). Therefore, the transit time (Eq. 11) is:

$$t = \frac{T_R R_d L}{V_s} = \frac{(0.05)(1.0)(0.9144 \text{ m})(100 \text{ cm/m})}{(1.8 \times 10^{-8} \text{ cm/s})(31.536 \times 10^6 \text{ s/yr})} = 8.1 \text{ years}$$
Since a transit time of 8.1 years is much less than the desired design life of 40 years, a 0.9144-m (3-ft.) barrier is too thin to contain the chloride within the specified concentration limit. For the second iteration, a barrier thickness of 1.829 m (6 ft.) will be assumed. The corresponding calculations result in the following values for the required parameters:

\[ P_L = 2 \times 0.35 = 0.70 \]

\[ T_R \sim 0.09 \text{ (Fig. 42)} \]

\[ t \sim 30 \text{ years} \]

Since \( t = 30 \text{ years} \) is still less than the desired time, try \( L = 2.134 \text{ m (7.0 ft.)} \), or:

\[ P_L = \frac{2.134 \text{ m}}{0.9144 \text{ m}} \times 0.35 \approx 0.82 \]

\[ T_R \sim 0.105 \text{ (Fig. 42)} \]

\[ t \sim 39.5 \text{ years} \]

Therefore, a barrier thickness of about 2.134 m (7.0 ft.) will be required to satisfy the specified leachate concentration criterion for chloride.

A 0.9144-m (3-ft.)-thick barrier could be used successfully if the concentration of the chloride in the leachate could be reduced by some type of chemical treatment process. Again, Fig. 42 can be used to estimate the required reduction in the chloride concentration. For example, the Peclet number for chloride for a 0.9144-m (3-ft.) barrier was 0.35. The value for \( T_R \) for a 40-year operational life is:

\[ T_R = \frac{v_s t}{R_d L} = \frac{(1.8 \times 10^{-8} \text{ cm/s})(40 \text{ yr})(31.536 \times 10^6 \text{ s/yr})}{(1.0)(0.9144 \text{ m})(100 \text{ cm /m})} = 0.248 \]

For these values of \( P_L \) and \( T_R \), the relative concentration, \( c/c_0 \), from Fig. 47 is about 0.473, so the required chloride concentration in the leachate is:

\[ c_0 = \frac{c}{0.473} = \frac{250}{0.473} = 529 \text{ mg/L} \]
Therefore, a 0.9144-m (3-ft.)-thick barrier can be used if the chloride concentration is reduced from 3484 mg/l to 529 mg/l.

Finally, the transit time for chloride based solely on the seepage velocity for a 0.9144-m (3-ft.)-thick barrier is:

\[
t = \frac{L}{v_s} = \frac{(0.9144 \text{ m})(100 \text{ cm/m})}{(1.8 \times 10^{-8} \text{ cm/s})(31.536 \times 10^6 \text{ s/yr})} = 161 \text{ yr}
\]

This value of 161 years is significantly greater than the 8.1 years for chloride using Fig. 47. The difference in the two values is due to the combination of a relatively low seepage velocity and the diffusive transport of the chloride, which is not evaluated when using only the seepage velocity.

Additional details on transit time calculations, including consideration of solute flux as well as solute concentration and analyses for reactive as well as non-reactive solutes, is provided by Shackelford (1990, 1992a, 1993) and Manassero and Shackelford (1993).

**Summary and Conclusions**

Diffusion of contaminants through fine-grained soil barriers is a significant, if not dominant, transport mechanism. The significance of diffusion to the overall migration of contaminants increases as the seepage velocity decreases. Even if the seepage velocity is zero, release rates of contaminants by diffusion can be high, especially when relatively high concentrations of contaminants are being contained by relatively thin barriers. As a result, severe consequences can result if diffusive transport of contaminants is not considered in the design of waste containment barriers.
REFERENCES


