Integrated Lecture and Laboratory Modules for Contaminant Transport Studies in Undergraduate Geotechnical Engineering

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Abstract: The engineering concepts necessary for fully describing contaminant transport processes in soil and groundwater have been largely neglected by traditional civil engineering curricula in the United States. New lecture and laboratory modules have been developed and integrated into the undergraduate soil mechanics course for civil engineering students at the Colorado School of Mines. The lecture module includes theoretical development of the advection-dispersion equation and a set of practical example problems designed to clarify the relative importance of advection, dispersion, and molecular diffusion in realistic contaminant transport scenarios. The laboratory experiment is developed using relatively inexpensive soil permeability testing equipment common to most academic and industrial geotechnical engineering laboratories. Unlike conventional chemical transport testing, which requires relatively complicated, time-consuming, and expensive analytical instrumentation, the new laboratory exercise relies on the use of simple colored dye for simulating the movement of a contaminant through a one-dimensional soil column. The key soil parameters governing the transport of the dye, including the diffusion coefficient, hydraulic conductivity, dispersion coefficient, and retardation coefficient, are quantified by observing the location and color intensity of the dye as it moves through the column. Emphasis is placed on maximizing the simplicity and visual impact of the experimental exercise. The effectiveness of the lecture and laboratory modules is assessed through a student survey.

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Introduction

Contaminant transport processes in soils and groundwater are becoming increasingly important as national policy with regard to the health of our natural environment is ever more regulated. For the modern civil engineer, direct encounters with subsurface chemical transport phenomena are often unavoidable.

Sources for subsurface contaminants are diverse and widespread. Common examples include hazardous chemical spills at factories, industrial storage sites, military installations or nuclear facilities, commercial and residential use of agricultural pesticides, release of grossly acidic chemical species from mining operations such as tailings ponds or waste-rock piles, and migration of unchecked leachate from waste containment facilities such as landfills or hazardous material storage facilities. Upon release into the environment, the types of materials through which these contaminants may migrate are equally as diverse. Transport may occur, for example, in the unsaturated groundwater zone as a result of a near-surface chemical spill, or in the saturated groundwater zone as the contaminant infiltrates beyond the water table. The soils under consideration may be either natural deposits, which are typically complicated by inherent nonhomogeneity or anisotropy, or man-made soil systems such as compacted clay liners, slurry cut-off walls, and permeable reactive barriers designed to stop or slow down the contaminant migration process. In all cases, however, contaminant transport is governed by three fundamental engineering concepts: diffusion, advection, and dispersion.

To date, these basic engineering concepts have been largely neglected by traditional undergraduate civil engineering curricula in the United States. To address the growing disparity between civil engineering education and professional practice, a new lecture module and laboratory experiment for demonstrating chemical transport phenomena in soils has been developed for undergraduate civil engineering students at the Colorado School of Mines (CSM). The lecture and laboratory modules have been integrated into the courses “Soil Mechanics” and “Soil Mechanics and Environmental Engineering,” respectively, which are required for all CSM civil engineering students (approximately 50 students per semester) and are typically taken during the third or fourth year of study. This paper describes the new chemical transport modules and summarizes an assessment of their effectiveness based on the results of a student survey.

Lecture Module: Theoretical Background

The lecture module is designed to provide a basic familiarity with three fundamental engineering concepts governing transport phe-
Figure 1 on page 20 should be as follows:

Fig. 1. Conceptual illustration of contaminant transport in a 1D soil aquifer due to chemical diffusion
nomina in soil: chemical diffusion, chemical advection, and mechanical dispersion. The transient advection-dispersion equation for describing the dynamic contaminant transport process is derived and solved in one dimension. Practical example problems are supplied throughout the lecture module to clarify the basic theoretical concepts and demonstrate the relative importance of the three major transport mechanisms in realistic contamination transport scenarios.

**Chemical Diffusion**

Chemical diffusion is a process by which a chemical species moves from an area of higher concentration to an area of lower concentration. A simple example is the dilution of sugar in a cup of water. Without disturbing or stirring the water in the cup, the sugar will dissolve and spread throughout the entire cup until its concentration reaches a constant value. This spreading process is called diffusion. The rate of the diffusion process depends mainly on the property of the medium and the distribution of chemical species. In this case, the sugar. Diffusion is generally much faster in pure liquids than in solids or porous materials like soil. In soils, diffusion through the pore water is much faster when the soil is saturated.

To describe the diffusion process in a typical geoenvironmental engineering scenario, consider the 1D soil layer or aquifer shown in Fig. 1. The aquifer is bounded by two reservoirs. The reservoir on the left represents a constant source of chemical solution in this case, the cup of water) and type of chemical species (in this case, the sugar). Diffusion is generally much faster in pure liquids than in solids or porous materials like soil. In soils, diffusion through the pore water is much faster when the soil is saturated.

Initially at \( t=t_0 \), the soil is saturated with free water. Since there is no head loss across the aquifer (i.e., the elevation of the water table in both reservoirs is the same), there will be no fluid flow in the soil. The hydraulic head is the same throughout the system and the distribution in concentration of chemical species is initially zero everywhere. At any time \( t > t_0 \), chemical diffusion initiates due to the difference in chemical concentration between the left boundary and the adjacent pore water in the soil and right reservoir. Over time, the chemical solution will gradually diffuse into the soil and its concentration distribution changes with time (i.e., \( t_1 < t_2 < t_3 \)) creating a "smeared" concentration front. Eventually, the concentration front will reach the right reservoir. The rate of chemical migration at any point within the soil layer is proportional to the gradient of chemical concentration, and can be described quantitatively by Fick’s first law:

\[
q_D = -D \frac{dC}{dx}
\]  

where \( q_D \) = mass flux of chemical species per unit area per unit time due to diffusion; \( D \) = diffusion coefficient in \( \text{m}^2/\text{s} \); and \( dC/dx \) = gradient in chemical concentration through the soil. The negative sign in Eq. (1) reflects the fact that chemical transport occurs from areas of higher concentration to areas of lower concentration.

The diffusion coefficient is a material constant and is dependent on the type of chemical species and the medium through which diffusion occurs. Whenever free water is the medium, \( D \) typically ranges from \( 10^{-9} \) to \( 10^{-8} \) \( \text{m}^2/\text{s} \). In a porous medium such as saturated soil, the diffusion coefficient \( D \) could be 1 to 3 orders of magnitude smaller due to the tortuous diffusion path caused by the complex pore structure.

**Chemical Advection**

The second principal contaminant transport mechanism is chemical advection. Continuing our simple analogy of sugar in a cup of water, we can accelerate the mixing process by mechanically disturbing or stirring the water, as we typically prefer to do. The process of chemical transport under the resultant fluid flow is called advection or convection.

A conceptual model for chemical advection in soil is illustrated in Fig. 2. Assume that there is no chemical diffusion and that steady state flow in the soil layer has been established in response to the constant head difference \( h \). If the water in the left reservoir is suddenly replaced with a solution of constant chemical concentration \( C_0 \), we will start to observe a sharp, or piston-like, concentration front migrating along the direction of the superimposed fluid flow. This process proceeds until the concentration front reaches the right reservoir. The rate of chemical migration at any time and at any cross section located behind the concentration front is

\[
q_A = v C
\]  

where \( q_A \) = mass flux of chemical species per unit area per unit time due to advection; and \( v \) = fluid seepage velocity.
Mechanical Dispersion

As chemical species in soils are transported by advection, several mechanisms cause the fluid flow velocity to vary locally from place to place. These mechanisms include (1) differences in pore path dimensions (caused by varying grain sizes), (2) differences in transport lengths among the different paths (caused by the tortuosity of the flow path), and (3) local differences in flow velocity within any one path (caused by friction effects occurring along the boundaries of the flow path).

These nonuniform velocities cause the diffusion coefficient to be dependent on fluid velocity $v$. Mixing, which results from this dependency, is called mechanical dispersion. A mathematical description of mechanical dispersion is often achieved by modifying the diffusive mass flux from Eq. (1) using an effective diffusion coefficient $D^*$:

$$q_D = -D^* \frac{\partial C}{\partial x} = -(\alpha v + D) \frac{\partial C}{\partial x}$$

where $\alpha$ = dynamic dispersivity, a material property with units of length. The dynamic dispersivity is a function of the chemical travel distance $x$, and can be empirically expressed as

$$\alpha = 0.0175x^{1.46}$$

The total steady mass flux due to chemical diffusion, advection, and mechanical dispersion is the summation of Eqs. (2) and (3), known as the advection-dispersion solute flux equation:

$$q_T = -(\alpha v + D) \frac{\partial C}{\partial x} + v C$$

Transient Advection-Dispersion

The dynamic process of chemical transport is a time-dependent or transient process (e.g., from the time we start to induce a constant concentration in the waste pond to the time a steady chemical concentration distribution in the aquifer is established). This transient process is important in understanding or solving many problems of chemical transport in soils.

If we consider an infinitesimal element in the soil layer shown in Figs. 1 and 2, neglect irreversible decay of the chemical species, and apply the mass conservation principle, then the net chemical mass flux through the element is equal to the change in the chemical concentration in that element

$$\nabla \cdot q_T = -R \frac{\partial C}{\partial t}$$

where $R$ is called the distribution or retardation factor, which reflects the capacity for soil particles to adsorb chemical species on their surface. The value of the retardation factor is typically between 1 and 5. For a nonretarded system, the retardation factor is equal to 1.

Substituting Eq. (5) into Eq. (6) and considering transport in only 1D leads to the governing equation for transient chemical advection-dispersion in soils:

$$(\alpha v + D) \frac{\partial^2 C}{\partial x^2} - \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial t} = R \frac{\partial C}{\partial t}$$

The solution of Eq. (7), together with appropriate initial and boundary conditions, provides the complete description of chemical concentration in space and time. Given the following typical boundary conditions for transient advection-dispersion in one dimension, the analytical solution of Eq. (7) at some distance $x$ from the left source $C_o$, at time $t$, is (Ogata 1970; Javandel et al. 1984)

$$C(x,t=0) = 0; \quad C(x=0,t) = C_o; \quad \frac{\partial C(x=\infty,t)}{\partial x} = 0$$

$$C(x,t) = \frac{1}{2} \left\{ \text{erfc} \left[ \frac{Rx-vt}{2\sqrt{(\alpha v+D)Rt}} \right] + \exp \left( \frac{vx}{\alpha v+D} \right) \text{erfc} \left[ \frac{Rx+vt}{2\sqrt{(\alpha v+D)Rt}} \right] \right\}$$

where $C(x,t)/C_o$ = concentration of the chemical species at point $x$ and time $t$ relative to the source concentration; and erfc$(z)$ = complementary error function. The complementary error function can be obtained using a mathematical lookup table or estimated graphically by using Fig. 3. Given the graphical solution for the complementary error function, Eq. (9) is relatively simple for students at the undergraduate level to solve.

Qualitative solutions to Eq. (7) are shown on Fig. 4. Referring to Fig. 4(a), at time $t=t_1$, a short time after imposing a constant concentration in the left reservoir, the concentration distribution in the soil layer depends on the relative importance of the principal transport mechanisms. Similar patterns can be observed in a later time $t=t_2$. However, one can observe a wider spreading in space for both diffusion and dispersion mechanisms. Similarly, Fig. 4(b) shows a qualitative solution to the transient advection-dispersion equation for a nonretarded system in two dimensions. Although the total mass of the chemical species remains the same at all times, the contaminant plume moves in the direction of the groundwater flow by advection while simultaneously decreasing in concentration (or increasing in size) as the time and travel distance increase under the combined diffusion-dispersion mechanisms.

Fig. 3. Graphical representation of the complementary error function erfc$(z)$
Laboratory Module: Column Testing Exercise

Column tests for experimentally determining the relevant advection-dispersion parameters have been well established in the field of geotechnical and geoenvironmental engineering (e.g., Shackelford 1995). The basic premise of these tests is to measure or back-calculate the relevant advection-dispersion parameters, which include seepage velocity \( v \), the molecular diffusion coefficient \( D \), the mechanical dispersion coefficient \( a \), and the retardation coefficient \( R \), by determining the rate of transport of a specific chemical species under superimposed water flow in a 1D column of soil.

In general, however, the established column testing procedures do not readily lend themselves to the undergraduate laboratory environment. Notable limitations include the requirement for excessively long testing times, particularly for relatively fine-grained materials, which can require days or weeks of testing, and the requirement for expensive and complex analytical equipment for quantifying the rate of chemical transport through the soil column. For teaching purposes, where cost effectiveness, time effectiveness, simplicity, and interactive student involvement are preferred, the established column testing methods are undesirable.

A new laboratory module was developed at CSM to provide students with intimate experience in the fundamental aspects of contaminant transport processes in soil. The laboratory module is conducted following the previously described lecture module, ideally within the same week. The module allows the students to make direct measurements of each controlling parameter required as input into Eq. (9) for predicting the spatial and temporal distribution of migrating chemical species in a 1D column of soil. Emphasis is placed on the simplicity, cost effectiveness, and visual impact of the laboratory exercise.

Column Apparatus

Five soil columns were constructed for use by groups of students numbering from three to five. Fig. 5 shows a schematic drawing of a typical column, comprised of a cast acrylic tube filled with a compacted sand/clay mixture. The inner cross-sectional area of the tube is 4.45 cm² and the length of the soil-filled portion typically ranges from 10 to 13 cm. A symmetric system of endcaps, tie rods, rubber stoppers, and springs is used to rigidly hold the soil in place, forming two small fluid reservoirs on either end of the column. The left reservoir (or head-water reservoir) is the upstream side representing a constant contaminant source while the right reservoir (or tail-water reservoir) represents the downstream discharge site or sink. Head and tail water pressures during testing are measured and controlled using a conventional soil permeability testing panel consisting of two air/water interface columns controlled by regulated air pressure.

Simulated Contaminant

For advection-dispersion testing, colored dye (FDA Blue #1) is used to simulate a contaminant introduced into the head-water reservoir. The influent source concentration \( C_0 \) is prepared by adding 1 g of dye to 400 g of distilled water. The dye solution is...
forced to flow through the saturated sand/clay mixture by increasing the hydraulic gradient using the pressure panel. The transport of the dye solution under the combined processes of advection, diffusion, dispersion, and retardation is observed by monitoring the location and color intensity of the dye as it migrates through the transparent column. The material and flow properties controlling these processes (i.e., $v$, $D$, $\alpha$, and $R$) are directly quantified by the students using simple and visually oriented analysis procedures described in the following sections.

During the final phase of testing, the color intensity of the dye solution in the effluent reservoir is monitored and compared to a calibrated “color wheel,” which allows the students to determine the dye’s relative concentration $C/C_0$ as it begins to breakthrough the soil column. Fig. 6 shows a photograph of the calibrated color wheel, which shows vials of dye solution ranging in intensity from $C/C_0=0$ (distilled water) to $C/C_0=1$ (1 g dye to 400 g water). Students are asked to determine $C/C_0$ in the effluent reservoir as a function of time, generating a concentration “breakthrough” curve that can then be compared with a theoretical breakthrough curve obtained using the measured soil parameters and the analytical solution for transient advection-dispersion [i.e., Eq. (9)].

### Soil Properties and Preparation

The type of soil for column testing was carefully selected to obtain desirable hydraulic conductivity and retardation properties. Hydraulic conductivity is required to be low enough to cause observable dispersion, yet high enough to permit the entire testing procedure to be completed within a 3-h session. In order for the soil to exhibit significant retardation, a small amount of clay was required. It was found that a mixture of poorly-graded fine sand (0.075 mm $< D < 0.15$ mm) and kaolinite clay ($D < 0.002$ mm) resulted in ideal conductivity ($k = 1 \times 10^{-6}$ m/s) and retardation properties ($R = 2 - 3$). Mixtures are prepared at a ratio by mass of 95% sand to 5% clay. Prior to the student’s arrival in the laboratory session, each column is prepared and assembled by compacting the sand/clay mixture into the acrylic tubes at a porosity of approximately 40%. Each is then saturated overnight by forcing distilled water to flow through the column using the pressure control panel.

### Diffusion Coefficient

The student’s first task is to determine the diffusion coefficient for the transport of FDA Blue #1 dye through the saturated sand/clay mixture. Based on diffusion theory, the diffusion coefficient $D$ can be estimated by the following simple equation:

$$D \approx \frac{x^2}{t}$$  \hspace{1cm} (10)

where $x =$ average diffusion distance over a period of time $t$. Ideally, the students are asked to estimate $D$ by injecting colored dye into the upstream reservoir, maintaining no-flow conditions (i.e., zero hydraulic gradient), and observing the location of the spreading front of the dye as a function of time. Because the diffusion process is extremely slow, however, the dye is typically injected by the instructor several days prior to the laboratory session using an identically prepared soil column.

### Hydraulic Conductivity and Seepage Velocity

The student’s next task is to determine the saturated hydraulic conductivity of the sand/clay mixture and the associated seepage velocity. Hydraulic conductivity is determined by measuring the effluent fluid flow rate at steady state under a controlled head gradient and applying Darcy’s Law (i.e., constant head technique). The measured discharge velocity $v$ is related to the seepage velocity $v$, required for the solution of Eq. (9), by the known soil porosity $n$ as follows:

$$v = \frac{v_d}{n}$$  \hspace{1cm} (11)

### Mechanical Dispersion Coefficient

As the flow of the dye solution from the head-water reservoir proceeds through the column, an empirical equation may be used to estimate the magnitude of the mechanical dispersion coefficient $\alpha$

$$\alpha \approx \frac{\Delta x^2}{t} = \frac{\Delta x^2}{\Delta x} = \Delta x$$  \hspace{1cm} (12)

where $x =$ dispersion distance (cm) over a period of time $t$. The dispersion distance is measured as the length between the location of the sharp concentration front and the furthest point along the column where any trace of dye may be observed. Fig. 7 shows examples of this measurement for a column test at two times.

### Retardation Coefficient

Retardation is responsible for a restriction in the velocity of the dye solution relative to the overall fluid velocity. Generally,
chemical species travel slower than the water in which they are dissolved due to the surface adsorption capacity of soil particles, particularly clay particles, which possess surface charge and relatively large surface area. The retardation factor $R$ can be defined as the ratio of the pore-water velocity to the chemical travel velocity:

$$ R = \frac{v}{v_c} $$

where $v$ = seepage velocity (cm/s); and $v_c$ = chemical travel velocity (cm/s). Students are asked to estimate $R$ using Eq. (13) by observing the velocity of the advancing sharp concentration front relative to the previously determined seepage velocity. This can be accomplished simply by noting the downstream advance of the dye using a ruler. For the sand/clay mixture, $R$ values ranging from 2 to 3 were found to be typical.

An interesting side experiment can be conducted by the students by injecting FDA green dye into the soil column under flowing conditions. It can be observed that the blue component of the green dye is retarded by the sand/clay mixture while the yellow component is not. Continued injection of green dye results in a distinctive and thought-provoking separation phenomenon.

### Breakthrough Testing

As the dye solution begins to break through the soil column and into the tail-water reservoir, the students are asked to monitor the color intensity of the effluent dye as a function of time by comparison to the calibrated color wheel (Fig. 6). The effluent is periodically sampled allowing it to flow into a small glass vial. Each vial is held up to the calibrated color wheel so that its relative dye concentration $C/C_0$ may be estimated. The relative dye concentration is then plotted as a function of time elapsed since the introduction of the dye into the head-water reservoir, i.e., the “breakthrough” curve. The students are then asked to calculate the theoretical breakthrough curve for the system using the parameters measured in the previous exercises and Eq. (9). Because the students are monitoring the effluent dye concentration, the value of $x$ in Eq. (9) is set equal to the length of the soil column. Fig. 8 shows an idealized comparison between experimental and theoretical breakthrough data. The theoretical curve is based on the soil and flow parameters shown.

### Student Assessment

The effectiveness of the chemical transport lecture and laboratory modules was assessed through a student questionnaire. Students enrolled in the CSM “Soil Mechanics” course were asked to answer several questions regarding the structure, relevance, difficulty, and effectiveness of the new modules. Table 1 summarizes these questions and the student responses. The majority of students felt that the new modules were useful and relevant to their future careers. All agreed that the laboratory module was helpful in further developing their understanding of contaminant transport phenomena. A common criticism of the lecture module was the requirement for “too much math.” Many also felt that the amount of time required for complete testing, which approached 3 h, was too excessive.

### Summary and Conclusions

The engineering concepts necessary for describing subsurface contaminant transport have been largely neglected by traditional undergraduate civil engineering curricula in the United States, creating a growing disparity between education and professional practice. To address this problem, a new lecture module and laboratory experiment for demonstrating chemical transport phenomena in soils has been developed for undergraduate civil engineering students at the Colorado School of Mines. This paper has described these new modules.

Emphasis in the development of the laboratory session was placed on maximizing cost and time effectiveness, simplicity, and visual impact. Excluding a pressure control panel, which is already in the possession of most typical geotechnical laboratories, the total cost of developing the laboratory exercise was on the order of $500. The exercise can be conducted within a 3-h laboratory session, although a significant amount of prelaboratory setup time (about 8 h) is required by the instructor or teaching assistant. Simple colored dye is used to simulate a contaminant in a flowing soil-water system, thus precluding the requirement for expensive and complex analytical equipment involved in traditional chemical transport testing and creating a highly visually oriented learning environment. Student assessment of the new modules, obtained through an informal questionnaire, was generally positive.
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Appendix. Example Problems

Example 1: Diffusion Example

If a 2M solution of trichloroethylene (TCE) is maintained in the left reservoir (or waste pond) of Fig. 1 and the right reservoir (or flowing stream) is maintained with free water, the length of the aquifer is 100 m, and the diffusion coefficient is $10^{-9}$ m$^2$/s, what is the discharge rate of TCE into the stream at steady state?

Solution: The gradient of TCE across the aquifer at steady state is

$$\frac{\partial C}{\partial x} = \left[ \frac{2 \text{ mol}}{L \cdot 0} \right] = -0.02 \left[ \frac{\text{mol}}{L \cdot \text{m}} \right]$$

The steady state flux of TCE into the stream, according to Fick's Law (1) is

$$q_D = -\left(10^{-9}\right) \left[ \frac{\text{m}^2}{\text{s}} \right] (-0.02) \left[\frac{\text{mol}}{L \cdot \text{m}}\right]$$

$$= (10^{-9}) \left[ \frac{\text{m}^2}{\text{s}} \right] (0.02) \left[ \frac{\text{mol}}{\text{m}^3} \right] (10^3)$$

$$= 2 \times 10^{-8} \left[ \frac{\text{mol}}{\text{s} \cdot \text{m}^2} \right]$$

$$= 0.63 \left[ \frac{\text{mol}}{\text{year} \cdot \text{m}^2} \right]$$

One mole of TCE is equivalent to 131.4 g. Therefore, each square meter of the stream adjacent to the aquifer will leach about 83 g of TCE per year due purely to the chemical diffusion process.

Example 2: Advection Example

If the difference in water level for the left waste pond and right stream shown on Fig. 2 is $h = 1$ m, the chemical concentrations in the pond and stream are the same as the previous example, and the hydraulic conductivity of the aquifer is $k = 10^{-8}$ m/s, what is the steady state advective flux of TCE into the stream?

Solution: The seepage velocity is described in terms of hydraulic conductivity $k$ and hydraulic gradient $i$ according to Darcy's Law as

$$v = ki = (10^{-8}) \left[ \frac{\text{m}}{\text{s}} \right] \left[ \frac{1 \text{ m}}{100 \text{ m}} \right] = 1 \times 10^{-10} \left[ \frac{\text{m}}{\text{s}} \right]$$

According to Eq. (2), the steady state advective flux of TCE is

$$q_A = (1 \times 10^{-10}) \left[ \frac{\text{mol}}{\text{s}} \right] \left[ \frac{\text{mol}}{\text{m}^3} \right] (10^3) = 2 \times 10^{-7} \left[ \frac{\text{mol}}{\text{s} \cdot \text{m}^2} \right] = 6.31 \left[ \frac{\text{mol}}{\text{year} \cdot \text{m}^2} \right]$$

The advective flux is about 10 times the diffusive flux from the previous example. Under most circumstances, contaminant transport in soils by advection is faster than by diffusion.

Example 3: Dispersion Example

What is the steady-state diffusive TCE discharge into the stream for Example 1 if mechanical dispersion is also considered?

Solution: The dynamic dispersivity at the right stream 100 m from the source pond, according to Eq. (4), is

$$\alpha = 0.0175(100)^{1.46} [\text{m}] = 14.56 [\text{m}]$$

The effective diffusion coefficient $D^*$ is

$$D^* = \alpha v + D$$

$$= (14.56) \left[ \frac{\text{m}}{10^{-10}} \right] \left[ \frac{\text{m}}{\text{s}} \right] + (10^{-9}) \left[ \frac{\text{m}^2}{\text{s}} \right]$$

$$= 2.456 \times 10^{-9} \left[ \frac{\text{m}^2}{\text{s}} \right]$$

The steady-state diffusive flux including mechanical dispersion, according to Eq. (3), is

$$q_D = (2.456)(10^{-9}) \left[ \frac{\text{m}^2}{\text{s}} \right] (2)(10^{-2}) \left[ \frac{\text{mol}}{\text{m}^3} \right] (10^3)$$

$$= 4.912 \times 10^{-8} \left[ \frac{\text{mol}}{\text{s} \cdot \text{m}^2} \right]$$

$$= 1.54 \left[ \frac{\text{mol}}{\text{year} \cdot \text{m}^2} \right]$$

indicating that the mechanical dispersion increases the process of diffusion by a factor of 1.456.

Example 4: Steady State Advection-Dispersion Example

What is the total annual discharge of TCE into the stream due to diffusion, advection, and dispersion for the problem defined in Examples 1–3?

Solution: Applying Eq. (5) leads to

$$q_T = - \left( \alpha v + D \right) \frac{\partial C}{\partial x} + v C$$

$$= 1.54 \left[ \frac{\text{mol}}{\text{year} \cdot \text{m}^2} \right] + 6.31 \left[ \frac{\text{mol}}{\text{year} \cdot \text{m}^2} \right] = 7.85 \left[ \frac{\text{mol}}{\text{year} \cdot \text{m}^2} \right]$$

indicating that for every year and square meter of the stream adjacent to the aquifer, approximately 1,032 g of TCE will be leached from the aquifer.

References


