8.1 Rainwater runoff from a parking lot flows through a 3-ft-diameter pipe, completely filling it. Whether flow in a pipe is laminar or turbulent depends on the value of the Reynolds number. (See Video V8.2.) Would you expect the flow to be laminar or turbulent? Support your answer with appropriate calculations.

\[
Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu} \quad \text{If } Re > 4000 \text{ the flow is turbulent. The corresponding velocity is}
\]

\[
V = \frac{Re \nu}{D} = \frac{(4000)(1.21 \times 10^{-5} \text{ft/s})}{3 \text{ ft}} = 0.0161 \text{ ft/s}
\]

Most likely the velocity will be greater than this, i.e., turbulent flow.

8.2 (See Fluids in the News article titled “Nanoscale flows,” Section 8.1.1.) (a) Water flows in a tube that has a diameter of \( D = 0.1 \text{ m} \). Determine the Reynolds number if the average velocity is 10 diameters per second. (b) Repeat the calculations if the tube is a nanoscale tube with a diameter of \( D = 100 \text{ nm} \).

(a) \( Re = \frac{V D}{\nu} \), where \( D = 0.1 \text{ m} \), \( V = 10 \text{ (0.1 m)/s} = 1 \text{ m/s} \), and \( \nu = 1.12 \times 10^{-6} \text{ m}^2 / \text{s} \)

Thus,

\[
Re = \frac{\left(1 \text{ m/s}\right)(0.1 \text{ m})}{1.12 \times 10^{-6} \text{ m}^2 / \text{s}} = 89300
\]

(b) \( Re = \frac{V D}{\nu} \), where \( D = 100 \text{ nm} \left(\frac{1 \text{ m}}{10^9 \text{ nm}}\right) = 10^{-7} \text{ m} \), \( V = 10 \text{ (10}^{-7} \text{ m)/s} = 10^{-6} \text{ m/s} \), and \( \nu = 1.12 \times 10^{-6} \text{ m}^2 / \text{s} \)

Thus,

\[
Re = \frac{\left(10^{-6} \text{ m/s}\right)(10^{-7} \text{ m})}{1.12 \times 10^{-6} \text{ m}^2 / \text{s}} = 8.93 \times 10^{-8}
\]
8.6 Carbon dioxide at 20 °C and a pressure of 550 kPa (abs) flows in a pipe at a rate of 0.04 N/s. Determine the maximum diameter allowed if the flow is to be turbulent.

For turbulent flow, \( Re = \frac{\rho V D}{\mu} > 4000 \), where \( Q = VA = \frac{\pi}{4} D^2 V \)
or
\[ Re = \frac{4 \rho Q D}{\pi \mu D^2} = \frac{4 \rho Q}{\pi \mu D} = 4000 \]

Thus, \( D = \frac{4 \rho Q}{4000 \pi \mu} \), where \( \rho Q = 0.04 \frac{N}{s} \) and \( \mu = 1.4 \times 10^{-5} \frac{Ns}{m^2} \) (Table 1.8)

Hence, \( D = \frac{4 \left( 0.04 \frac{N}{s} \right) \left( 9.81 \frac{m}{s^2} \right)}{4000 \pi \left( 1.47 \times 10^{-5} \frac{Ns}{m^2} \right)} = 0.0883 \ m \)
8.12 Water flows in a constant diameter pipe with the following conditions measured: At section (a) \( p_a = 32.4 \) psi and \( z_a = 56.8 \) ft; at section (b) \( p_b = 29.7 \) psi and \( z_b = 68.2 \) ft. Is the flow from (a) to (b) or from (b) to (a)? Explain.

Assume the flow is uphill. Thus,

\[
\frac{P_a}{\gamma} + \frac{V_a^2}{2g} + Z_a = \frac{P_b}{\gamma} + \frac{V_b^2}{2g} + Z_b + h_2
\]

or with \( V_a = V_b \),

\[
h_2 = \frac{P_a}{\gamma} + Z_a - \frac{P_b}{\gamma} - Z_b = \frac{(32.4 \text{ psi} - 29.7 \text{ psi})(144 \text{ lb/ft}^3)}{62.4 \text{ lb/ft}^3} + 56.8 \text{ ft} - 68.2 \text{ ft}
\]

or \( h_2 = -5.17 \text{ ft} < 0 \), which is impossible. Thus, the flow is downhill, from (b) to (a).

8.13 A fluid of specific gravity 0.96 flows steadily in a long, vertical 1-in.-diameter pipe with an average velocity of 0.50 ft/s. If the pressure is constant throughout the fluid, what is the viscosity of the fluid? Determine the shear stress on the pipe wall.

Assume laminar, calculate \( \mu \), then check to determine if \( Re < 2100 \). For laminar flow down the pipe (\( \Theta = -90^\circ \)) with constant pressure (\( \Delta \rho = 0 \))

\[
\frac{V}{(\Delta \rho - 5\ell \sin \theta)D^2}{32 \mu} \quad \text{gives} \quad V = \frac{\dot{m}D^2}{32 \mu} \quad \text{or} \quad \mu = \frac{\dot{m}D^2}{32 V}
\]

\[
\mu = \frac{0.96(62.4 \text{ lb/ft}^3)(1 \text{ ft})^2}{32(0.5 \text{ ft}^2)} = 0.0260 \frac{\text{lb-s}}{\text{ft}^2}
\]

Note:

\[
Re = \frac{\dot{m}VD}{\mu} = \frac{0.96(1.94 \text{ lb}) (0.5 \text{ ft}^2)(1 \text{ ft})}{0.0260 \frac{\text{lb-s}}{\text{ft}^2}} = 2.98 < 2100
\]

Thus, the flow is laminar as assumed and \( \mu = 0.0260 \frac{\text{lb-s}}{\text{ft}^2} \)

Also, with \( \Delta \rho = 0, \quad \Sigma F_x = 0 \) gives \( \pi D\ell \tau_w = \frac{\dot{m}D^2}{2} \), or

\[
\tau_w = \frac{\dot{m}D}{4} = 0.96 \frac{62.4 \text{ lb}}{4} \left( \frac{1 \text{ ft}}{2} \right) = 1.25 \frac{\text{lb}}{\text{ft}^2}
\]
8.14 A fluid flows through a horizontal 0.1-in.-diameter pipe. When the Reynolds number is 1500, the head loss over a 20-ft length of the pipe is 6.4 ft. Determine the fluid velocity.

\[ h_L = f \frac{L}{D} \frac{V^2}{2g} \], where since \( Re = 1500 < 2100 \) the flow is laminar.

Thus, \( f = \frac{64}{Re} = \frac{64}{1500} = 0.0427 \) so that

\[ \frac{6.4 \text{ ft}}{0.0427} = \frac{20 \text{ ft}}{(0.1/12 \text{ ft})} \frac{V^2}{2(32.2 \text{ ft/s}^2)} \]

or

\[ V = 2.01 \text{ ft/s} \]

8.15 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

\[ \rho = 1260 \frac{\text{kg}}{\text{m}^3}, \quad \mu = 1.50 \frac{\text{N} \cdot \text{s}}{\text{m}^2} \]

For laminar flow in a pipe,

\[ V = \text{average velocity} = \frac{1}{2}V_{\text{max}} = \frac{1}{2}(1 \text{ m/s}) = 0.5 \text{ m/s} \]

Thus,

\[ Re = \frac{\rho V D}{\mu} = \frac{(1260 \text{ kg/m}^3)(0.5 \text{ m/s})(0.075 \text{ m})}{1.50 \frac{\text{N} \cdot \text{s}}{\text{m}^2}} = 31.5 < 2100 \]

The flow is laminar, so that

\[ V = \frac{(\Delta \rho - 32 \mu \sin \theta)D^2}{32 \mu L} \], where \( \theta = 90^\circ \)

Thus,

\[ \Delta \rho = \frac{32 \mu L V}{D^2} + 32 = \frac{32(1.50 \frac{\text{N} \cdot \text{s}}{\text{m}^2})(10 \text{ m})(0.5 \text{ m/s})}{(0.075 \text{ m})^2} + (9.81 \frac{\text{m}}{\text{s}^2})(1260 \frac{\text{kg}}{\text{m}^3})(10 \text{ m}) \]

\[ = 1.66 \times 10^5 \frac{\text{N}}{\text{m}^2}, \text{ or } \Delta \rho = 166 \text{ kPa} \]

Also,

\[ \phi_1 + z_1 + \frac{V_1^2}{2g} = \phi_2 + z_2 + \frac{V_2^2}{2g} + h_L \], or with \( V_1 = V_2 \), \( z_2 - z_1 = l \), and

\[ \phi_1 = \rho_2 + \Delta \rho \] this gives

\[ h_L = \frac{\Delta \rho}{g} - l = \frac{1.66 \times 10^5 \frac{\text{N}}{\text{m}^2}}{(9.81 \frac{\text{m}}{\text{s}^2})(1260 \frac{\text{kg}}{\text{m}^3})} - 10 \text{ m} = 3.43 \text{ m} \]
8.21 A fluid flows through a horizontal 0.1-in.-diameter pipe with an average velocity of 2 ft/s and a Reynolds number of 1300. Determine the head loss over a 20-ft length of pipe.

\[ \text{Re} = \frac{\nu D}{\nu} = 1500 \quad \text{or} \quad \nu = \frac{\frac{\nu}{1500}}{\frac{\nu}{1500}} = \frac{(2 \text{ ft/s})(0.1 \text{ ft})}{1500} = 1.11 \times 10^{-5} \text{ ft}^2/\text{s} \]  
(1)

Also, from Eq. 5.57, \( \frac{\rho_2}{\rho_1} + \frac{V_2^2}{2g} + z_2 + \frac{\nu_1^2}{2g} + z_1 + h_s - h_a \)

which with \( V_1 = V_2, \quad z_1 = z_2, \quad \text{and} \quad h_s = 0 \) gives

\[ \rho_1 - \rho_2 = \delta h_a \]  
(2)

Since \( \text{Re} = 1500 < 2100 \) the flow is laminar so that from Eq. 8.8:

\[ V = \frac{\Delta \rho D^2}{32 \mu L} \quad \text{where} \quad \Delta \rho = \rho_1 - \rho_2 = \delta h_a \]

Thus,

\[ V = \frac{\delta h_a D^2}{32 \mu L} = \frac{\rho g h_a D^2}{32 \mu L} = \frac{g h_a D^2}{32 \nu L} \quad \text{since} \quad \nu = \mu/\rho \text{ and } \delta = \rho g \]

Hence, using \( \nu \) from Eq. 1:

\[ h_a = \frac{32 \nu L V}{g D^2} = \frac{32 \left(1.11 \times 10^{-5} \text{ ft}^2/\text{s}\right)(20 \text{ ft})(2 \text{ ft})}{(32.2 \text{ ft}^2/\text{s})(0.1 \text{ ft})^2} = 6.36 \text{ ft} \]

8.22 A viscous fluid flows in a 0.10-m-diameter pipe such that its velocity measured 0.012 m away from the pipe wall is 0.8 m/s. If the flow is laminar, determine the centerline velocity and the flowrate.

For laminar flow in a pipe

\[ u(r) = V_c \left[ 1 - \left( \frac{r}{D} \right)^2 \right], \quad \text{where} \quad D = 0.1 \text{ m} \quad \text{and} \quad u = 0.8 \text{ m/s at} \quad r = \frac{0.1 \text{ m}}{2} = 0.05 \text{ m} \]

Thus,

\[ 0.8 \frac{\text{m}}{\text{s}} = V_c \left[ 1 - \left( \frac{2(0.03 \text{ m})}{0.1 \text{ m}} \right)^2 \right] \quad \text{or} \quad V_c = 1.89 \frac{\text{m}}{\text{s}} \]

so that

\[ Q = \pi D^2 V = \pi D^2 (0.5 V_c) = \pi (0.1 \text{ m})^2 (0.5)(1.89 \frac{\text{m}}{\text{s}}) = 7.42 \times 10^{-3} \frac{\text{m}^3}{\text{s}} \]
For laminar flow \[ V = \frac{(\Delta \rho - \gamma \ell \sin \theta) D^2}{32 \mu} \tag{1} \]

where for this case \( \Delta \rho = 0 \), \( \ell = 5280 \text{ ft} \), \( D = 0.25 \text{ ft} \) and for maximum \( \Delta Z \), \( Re = 2100 \).

Thus,

\[ \frac{\rho V D}{\mu} = 2100 \quad \text{or} \quad V = \frac{2100 \mu}{\rho D} = \frac{2100 \left(2.34 \times 10^{-5} \frac{lb - ft}{in^2} \right)}{1.94 \times 10^{-5} \frac{lb - ft}{in^2} \left(0.25 \text{ ft}\right)} = 0.10 \text{ ft} \]

Hence, from Eq. (1):

\[ \sin \theta = -\frac{32 \mu V}{\gamma D^2} = -\frac{32 \left(2.34 \times 10^{-5} \frac{lb - ft}{in^2} \right)(0.10 \text{ ft})}{62.4 \frac{lb}{in^2} \left(0.25 \text{ ft}\right)^2} = -1.94 \times 10^{-5} \]

or

\[ \Delta Z = \ell \sin \theta = -(5280 \text{ ft}) \times (-1.94 \times 10^{-5}) = 0.102 \text{ ft} \]

Note: Could use the energy equation

\[ Z_1 - f \frac{pV^2}{2g} = Z_2 \]

with \( Re = 2100 \) so that \( V = 0.10 \text{ ft} \) and

\[ f = \frac{64}{Re} = \frac{64}{2100} = 0.0305 \]

and obtain the same result: \( Z_1 - Z_2 = 0.102 \text{ ft} \)