

Energy Harvesting for Two-Way OFDM Communications under Hostile Jamming

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Abstract—Hostile jamming can cause significant performance degradation in wireless communications, but it also provides an unexplored source of additional signal power. In this letter, we propose an energy-harvesting receiver for two-way orthogonal frequency division multiplexing (OFDM) systems under hostile jamming. More specifically, in the downlink, the receiver is designed to simultaneously process information and harvest energy from the received desired signal as well as the jamming interference through a power splitter. The harvested energy can then be used as an additional source of power to enhance the uplink transmission. We investigate the optimal power allocation and power splitting ratio to maximize the sum-rate of the uplink and downlink transmissions. To reduce the complexity, a suboptimal energy harvesting scheme with closed-form solution is proposed. We also obtain a lower bound on the sum-rate of the proposed scheme under strong full-band jamming.

Index Terms—Energy harvesting, hostile jamming, power splitting, sum-rate maximization.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is a spectrally efficient multi-carrier transmission technique in wireless communications. Due to its high efficiency and simple transceiver design, OFDM has been widely adopted in many civilian wireless standards, e.g., LTE and WiMAX [1]. However, as most of the other wireless techniques, OFDM is fragile under hostile jamming [2].

To enhance the jamming resistance of OFDM systems, several encoding and decoding schemes have been proposed in [3], [4]. In [3], linear precoder and decoder were proposed to minimize the mean-square error (MSE) for OFDM systems under multi-tone jamming. For pilot tone based jamming attacks, the authors in [4] proposed approaches to mitigate the attack effect by randomizing the pilot tone locations.

Though jamming presents a threat to the reliability of data transmissions, it can potentially provide an unexplored source

of additional signal power if the receiver is able to harvest energy from the radio-frequency (RF) signals. Recently, simultaneous wireless information and power transmission (SWIPT) technology has gained considerable attention in both industrial and academic fields [5]–[10]. In a SWIPT system, the receiver can process information and harvest energy from RF signals simultaneously. The SWIPT technique has been extended to multi-antenna systems [6], OFDM systems [7], [8], and cooperative networks [9], [10]. For downlink SWIPT OFDM with power-splitting receiver, the authors in [7] investigated the optimal power allocation to maximize the energy efficiency. In [8], a reconfigurable receiver architecture was proposed to realize SWIPT in two-way multi-antenna OFDM communication systems. However, the uplink and downlink transmissions were treated separately, and there was no consideration on jamming or hostile interference.

In this letter, we consider energy-harvesting transceiver design for two-way OFDM communications under hostile jamming. The basic idea of the proposed scheme is that the receiver harvests energy from the received mixed signals with a power-splitting architecture in the downlink, and the harvested energy can then be used as an additional source of power to enhance the uplink transmission. We investigate the optimal power allocation and power splitting ratio to maximize the sum-rate of the downlink and uplink transmissions. The resultant non-convex optimization problem is solved using a simple one-dimensional search in combination with convex optimization. To reduce the computational complexity, we propose a suboptimal scheme with closed-form solution for energy harvesting in SWIPT OFDM. We also obtain a lower bound on the sum-rate for the proposed scheme under strong full-band jamming.

II. SYSTEM MODEL

We consider a half-duplex OFDM system where two nodes \mathcal{A} and \mathcal{B} exchange information under hostile jamming. Both nodes are equipped with a single antenna. Node \mathcal{A} is a powerful infrastructure device (e.g., a base station) with a fixed power supply P_A , while node \mathcal{B} is a mobile device with a limited power supply P_B , and is able to decode information and harvest energy from the received radio signals (including both the desired signal and jamming interference). The total bandwidth of the system is divided into N subcarriers for data transmission.

Let $x_{A,n}$ and $x_{B,n}$ denote the unit-power information symbol to be transmitted on the n -th subcarrier at nodes \mathcal{A} and \mathcal{B} , respectively. In the downlink transmission (from node \mathcal{A} to \mathcal{B}), the received signal on subcarrier n at node \mathcal{B} is

$$y_{B,n} = \sqrt{\alpha_D P_{A,n}} h_n x_{A,n} + w_{B,J,n} + v_{B,a,n}, \quad (1)$$

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where α_D , $P_{A,n}$ and $h_n \sim \mathcal{CN}(0, 1)$ denote the path loss, the allocated transmit power and the small-scale Rayleigh fading coefficient of the n -th subcarrier in the downlink, respectively. $w_{B,J,n} \sim \mathcal{CN}(0, \sigma_{B,J,n}^2)$ is the Gaussian distributed jamming interference, and $v_{B,a,n} \sim \mathcal{CN}(0, \sigma_{B,a}^2)$ is the additive white Gaussian noise (AWGN). Here $x \sim \mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian variable with mean μ and variance σ^2 .

At node \mathcal{B} , a power splitting receiver is employed for simultaneous information receiving and energy harvesting [5]. Specifically, the received signal is splitted into two parts: one for energy harvesting and the other for information recovering. Let $\rho \in [0, 1]$ denote the power splitting ratio, that is, $\sqrt{\rho}y_{B,n}$ is used for energy harvesting. Then the total harvested power at node \mathcal{B} is

$$P_{EH} = \eta\rho \left[\sum_{n=1}^N (\alpha_D P_{A,n} |h_n|^2 + \sigma_{B,J,n}^2) + N\sigma_{B,a}^2 \right], \quad (2)$$

where $0 < \eta < 1$ denotes the energy conversion efficiency.

The other part of the received signal, $\sqrt{1-\rho}y_{B,n}$, is then used for information detection. The baseband signal can be expressed as

$$\tilde{y}_{B,n} = \sqrt{1-\rho} \sqrt{\alpha_D P_{A,n}} h_n x_{A,n} + \sqrt{1-\rho} (w_{B,J,n} + v_{B,a,n}) + v_{B,b,n}, \quad (3)$$

where $v_{B,b,n} \sim \mathcal{CN}(0, \sigma_{B,b}^2)$ is the equivalent AWGN noise in the baseband [5]. From (3), the achievable rate for the downlink transmission is given by

$$R_{DL} = \sum_{n=1}^N \log_2 \left(1 + \frac{(1-\rho)\alpha_D P_{A,n} |h_n|^2}{(1-\rho)(\sigma_{B,J,n}^2 + \sigma_{B,a}^2) + \sigma_{B,b}^2} \right). \quad (4)$$

For the uplink transmission, node \mathcal{B} transmits the signal to node \mathcal{A} with its inherent power supply P_B as well as the harvested power P_{EH} from the downlink phase. The received signal on the n -th subcarrier at node \mathcal{A} can be written as

$$y_{A,n} = \sqrt{\alpha_U P_{B,n}} g_n x_{B,n} + w_{A,J,n} + v_{A,n}, \quad (5)$$

where α_U , $P_{B,n}$ and $g_n \sim \mathcal{CN}(0, 1)$ denote the path loss, the transmit power and the small-scale Rayleigh fading coefficient of the n -th subcarrier, respectively. $w_{A,J,n} \sim \mathcal{CN}(0, \sigma_{A,J,n}^2)$ and $v_{A,n} \sim \mathcal{CN}(0, \sigma_A^2)$ denote the jamming interference and the overall AWGN at node \mathcal{A} , respectively.

As mentioned earlier, node \mathcal{A} is a powerful infrastructure device and all the received signals are used for information detection, i.e., no energy harvesting is involved. The achievable rate of the uplink is obtained as

$$R_{UL} = \sum_{n=1}^N \log_2 \left(1 + \frac{\alpha_U P_{B,n} |g_n|^2}{\sigma_{A,J,n}^2 + \sigma_A^2} \right). \quad (6)$$

III. PERFORMANCE OPTIMIZATION

In this section, we investigate the optimal power splitting ratio and the optimal power allocation at nodes \mathcal{A} and \mathcal{B} to maximize the sum-rate.

A. Sum-Rate Maximization

From (4) and (6), the sum-rate maximization problem of the considered SWIPT OFDM system can be formulated as

$$\begin{aligned} \max_{\rho, \tilde{\mathbf{P}}_A, \tilde{\mathbf{P}}_B} \quad & R_{sum} = R_{DL} + R_{UL} \\ \text{s.t.} \quad & 0 \leq \rho \leq 1, \quad \sum_{n=1}^N P_{A,n} \leq P_A, \\ & \sum_{n=1}^N P_{B,n} \leq P_B + P_{EH}. \end{aligned} \quad (7)$$

where $\tilde{\mathbf{P}}_{\mathcal{X}} = [P_{\mathcal{X},1}, \dots, P_{\mathcal{X},N}]^T$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{B}\}$.

For conventional non-energy-harvesting OFDM systems, $\rho = 0$, and it is well known that water-filling is optimal in terms of rate maximization. When jamming is present, the system performance generally degrades. In the proposed scheme, on the other hand, the jamming power can be utilized to improve the overall system performance. The main point here is that the power allocation for the downlink and uplink should be jointly optimized to achieve maximum sum-rate.

Note that the optimization problem (7) is non-convex due to the power splitting operation at node \mathcal{B} . However, when ρ is fixed, the harvested power P_{EH} in (2) is a linear function of $\tilde{\mathbf{P}}_A$. Hence, the constraints in problem (7) become linear constraints. Based on these observations, it can be seen that problem (7) turns out to be convex when ρ is fixed, and can be efficiently solved using standard algorithms for convex optimization. Hence, the sum-rate maximization problem (7) for the proposed scheme can be solved by a one-dimensional exhaustive search over $\rho \in [0, 1]$.

B. A Suboptimal Scheme

To reduce the computational complexity, we propose a suboptimal scheme for information transfer and energy harvesting below. For downlink transmission, we propose to use the classical water-filling power allocation:

$$P_{A,n}^* = \left[\lambda - \frac{\sigma_{B,J,n}^2 + \sigma_{B,a}^2 + \sigma_{B,b}^2}{\alpha_D |h_n|^2} \right]^+, \quad \forall n, \quad (8)$$

where $[x]^+ = \max(x, 0)$, and λ is a constant determined by the transmit power constraint. Then, the harvested power at node \mathcal{B} can be expressed as

$$P_{EH} = c_0 \rho, \quad (9)$$

where

$$c_0 = \eta \left[\sum_{n=1}^N (\alpha_D P_{A,n}^* |h_n|^2 + \sigma_{B,J,n}^2) + N\sigma_{B,a}^2 \right]. \quad (10)$$

Assuming the transmission power allocation at node \mathcal{A} is $P_{A,n}^*$, as given in (8), the downlink rate in (4) can be calculated as

$$\begin{aligned} R_{DL} &= \sum_{n=1}^N \log_2 \left(\frac{(1-\rho)(\alpha_D P_{A,n}^* |h_n|^2 + \sigma_{B,J,n}^2 + \sigma_{B,a}^2) + \sigma_{B,b}^2}{(1-\rho)(\sigma_{B,J,n}^2 + \sigma_{B,a}^2) + \sigma_{B,b}^2} \right) \\ &\geq \sum_{n=1}^N \log_2 \left(\frac{(1-\rho)(\alpha_D P_{A,n}^* |h_n|^2 + \sigma_{B,J,n}^2 + \sigma_{B,a}^2)}{(1-\rho)(\sigma_{B,J,n}^2 + \sigma_{B,a}^2) + \sigma_{B,b}^2} \right), \end{aligned} \quad (11)$$

where $\bar{\sigma}_{B,J}^2 = \frac{1}{N} \sum_{n=1}^N \sigma_{B,J,n}^2$. In the inequality above, we have used the fact that $\sigma_{B,b}^2 \geq 0$, and $\prod_{n=1}^N x_n \leq (\frac{1}{N} \sum_{n=1}^N x_n)^N$.

For the uplink transmission, uniform power allocation is used among the subcarriers at node \mathcal{B} , i.e., $P_{B,n} = (P_B + P_{EH})/N, \forall n$. Similarly, the uplink rate is lower bounded by

$$R_{UL} \geq \sum_{n=1}^N \log_2 \left(\frac{\alpha_U (P_B + c_0 \rho) |g_n|^2}{N(\bar{\sigma}_{A,J}^2 + \sigma_A^2)} \right), \quad (12)$$

where $\bar{\sigma}_{A,J}^2 = \frac{1}{N} \sum_{n=1}^N \sigma_{A,J,n}^2$.

As a result, the sum-rate of the downlink and uplink transmissions is lower bounded by

$$R_{sum} \geq \sum_{n=1}^N \log_2 \left(\frac{(1-\rho)(\alpha_D P_{A,n}^* |h_n|^2 + \sigma_{B,J,n}^2 + \sigma_{B,a}^2)}{(1-\rho)(\bar{\sigma}_{B,J}^2 + \sigma_{B,a}^2) + \sigma_{B,b}^2} \right) + \sum_{n=1}^N \log_2 \left(\frac{\alpha_U (P_B + c_0 \rho) |g_n|^2}{N(\bar{\sigma}_{A,J}^2 + \sigma_A^2)} \right). \quad (13)$$

Next, we derive a closed-form solution of ρ to maximize the sum-rate lower bound above. From (13), it can be seen that maximizing the sum-rate lower bound is equivalent to maximize the following objective function:

$$\begin{aligned} \max_{\rho} \quad & \frac{(1-\rho)(P_B + c_0 \rho)}{(1-\rho)(\bar{\sigma}_{B,J}^2 + \sigma_{B,a}^2) + \sigma_{B,b}^2} \\ \text{s.t.} \quad & 0 \leq \rho \leq 1. \end{aligned} \quad (14)$$

By using Lagrange's method, the optimal ρ is given by (15),

$$\rho^* = \left[\frac{\bar{\sigma}_{B,a}^2 + \sigma_{B,b}^2 - \sigma_{B,b} \sqrt{\sigma_{B,b}^2 + (1 + P_B/c_0) \bar{\sigma}_{B,a}^2}}{\bar{\sigma}_{B,a}^2} \right]^+. \quad (15)$$

where $\bar{\sigma}_{B,a}^2 = \bar{\sigma}_{B,J}^2 + \sigma_{B,a}^2$.

Discussions: For OFDM systems under full-band strong jamming, we have $\bar{\sigma}_{B,a}^2 \gg \sigma_{B,b}^2$. From (15), we can see that $\rho^* \rightarrow 1$, which implies that almost all the received signal power is harvested to enhance the uplink transmission.

C. Performance Lower Bound

For OFDM systems under strong full-band Gaussian jamming, we have the following result for the proposed scheme.

Theorem 1: For OFDM systems under Rayleigh fading and strong full-band jamming with $\sigma_{B,J,n}^2 \gg \alpha_D P_{A,n} |h_n|^2$, and $\sigma_{A,J,n}^2 \gg \alpha_U P_{B,n} |g_n|, \forall n$, the average sum-rate of the proposed scheme is lower bounded by

$$R_{sum}^{avg} \geq N e^{c_1} E_1(c_1) \log_2(e), \quad (16)$$

where $c_1 = (\bar{\sigma}_{A,J}^2 + \sigma_A^2)/\eta \alpha_U \bar{\sigma}_{B,J}^2$, and $E_1(x)$ is the exponential integral function [11].

Proof: Consider the downlink first. From (4), the achievable rate $R_{DL} \rightarrow 0$ under strong full-band jamming ($\sigma_{B,J,n}^2 \gg \alpha_D P_{A,n} |h_n|^2$). Based on the discussions in the previous subsection, the splitting ratio $\rho \rightarrow 1$ under strong full-band jamming.

Hence, we can choose $\rho = 1$, i.e., all the received signals in the downlink are used for energy harvesting. From (9), the harvested power is

$$P_{EH} = c_0 > \eta \sum_{n=1}^N \sigma_{B,J,n}^2. \quad (17)$$

Now consider the uplink transmission with equal power allocation among all the subcarriers, i.e., $P_{B,n} = (P_B + P_{EH})/N, \forall n$. Following (6) and (17), the achievable rate of the uplink is lower bounded by

$$R_{UL} \geq \sum_{n=1}^N \log_2 \left(1 + \frac{\eta \alpha_U \bar{\sigma}_{B,J}^2 |g_n|^2}{\bar{\sigma}_{A,J}^2 + \sigma_A^2} \right). \quad (18)$$

Then the average sum-rate of the proposed scheme is lower bounded by

$$R_{sum}^{avg} \geq \sum_{n=1}^N \mathcal{E} \left[\log_2 \left(1 + \frac{\eta \alpha_U \bar{\sigma}_{B,J}^2 |g_n|^2}{\bar{\sigma}_{A,J}^2 + \sigma_A^2} \right) \right]. \quad (19)$$

where $\mathcal{E}[\cdot]$ denotes the expectation over $|g_n|$. Using the equality $\int_0^{+\infty} e^{-\mu x} \ln(1 + \beta x) dx = \frac{1}{\mu} e^{\frac{\mu}{\beta}} E_1\left(\frac{\mu}{\beta}\right)$ ([11, eq. 4.337.1]), we can obtain the result in (16). ■

Discussions: Consider the downlink transmission under strong jamming and without energy harvesting, i.e., $\sigma_{B,J,n}^2 \gg \alpha_D P_{A,n} |h_n|^2$ and $\rho = 0$. From (4), we have

$$R_{DL} \simeq R'_{DL} = \log_2(e) \sum_{n=1}^N \frac{\alpha_D P_{A,n} |h_n|^2}{\sigma_{B,J,n}^2}. \quad (20)$$

where we have used the approximation that $\log(1+x) \simeq x$ for small x . The optimal jamming power can be determined by solving the following problem:

$$\min_{\{\sigma_{B,J,n}^2\}_{n=1}^N} R'_{DL} \quad \text{s.t.} \quad \sum_{n=1}^N \sigma_{B,J,n}^2 = \sigma_{B,J}^2, \quad (21)$$

which is given by $\sigma_{B,J,n}^2 = |h_n| \sqrt{P_{A,n} \sigma_{B,J}^2} / (\sum_{k=1}^N |h_k| \sqrt{P_{A,k}}), \forall n$. Hence, for the jammer, the optimal jamming strategy is to adjust the jamming power level on each subcarrier based on the knowledge of the channel, namely channel state information (CSI) assisted adaptive jamming.

Moreover, we would like to point out that the proposed scheme can readily be extended to the multiuser case, where each user can perform energy harvesting separately with its own power splitting ratio. For best performance, the power-splitting ratios and power allocation schemes need to be jointly optimized for maximum sum-rate.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed scheme for a SWIPT OFDM system with $N = 16$ subcarriers. The path loss is given by $\alpha_D = d^{-\alpha}$, where $d = 10$ m is the distance between the two nodes and $\alpha = 3$ is the path loss exponent. The channels are reciprocal with $\alpha_U = \alpha_D$ and $g_n = h_n, \forall n$. The jammer is close to node \mathcal{B} with a total jamming

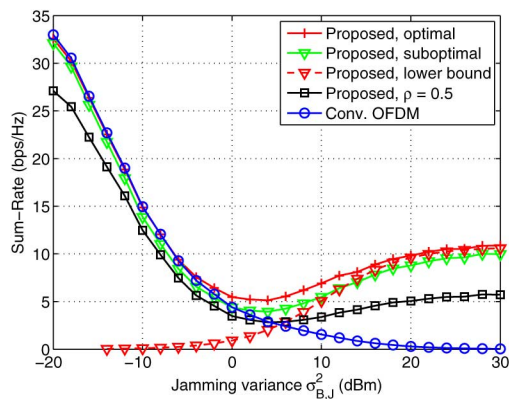


Fig. 1. Example 1: Performance comparison under full-band jamming.

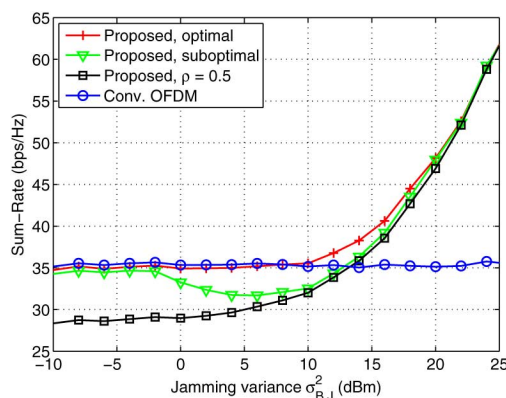


Fig. 2. Example 2: Performance comparison under partial-band jamming.

power 30 dB stronger than that at node \mathcal{A} . We consider CSI-assisted adaptive jamming. The energy conversion efficiency is set to be $\eta = 0.7$. The maximum transmit power of node \mathcal{A} is $P_A = 20$ dBm, and that of node \mathcal{B} is $P_B = 0$ dBm. The noise variances are given by $\sigma_A^2 = -30$ dBm, and $\sigma_{B,a}^2 = \sigma_{B,b}^2 = -33$ dBm, respectively.

Example 1: Full-band jamming Fig. 1 shows the sum-rate of the proposed scheme under CSI-assisted full-band jamming. The overall jamming power at node \mathcal{B} is $\sigma_{B,J}^2 = \sum_{n=1}^N \sigma_{B,J,n}^2$. It is observed that for weak jamming ($\sigma_{B,J}^2 \leq -5$ dBm), there is no benefit to harvest energy from the received signals. As the jamming becomes stronger, the sum-rate of the conventional OFDM system decreases rapidly. In contrast, for the proposed scheme, the harvested energy in the downlink can improve the performance of the uplink transmission, leading to a much higher sum-rate than that of the conventional OFDM system. It can also be seen that the performance of the suboptimal scheme is very close to that of the optimal one.

Example 2: Partial-band jamming Fig. 2 shows the sum-rate of SWIPT OFDM systems under partial-band jamming where 4 out of 16 subcarriers are jammed. The other settings are the same as in Example 1. We also assume that the jammed subcarriers can be detected by the two nodes. It can be seen that for the proposed scheme, the achievable sum-rate improves under strong jamming through energy harvesting from the received signals at node \mathcal{B} .

Power splitting ratios versus the jamming power The optimal and suboptimal power splitting ratios for the above two

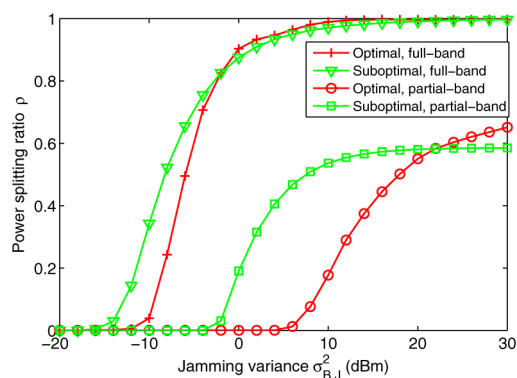


Fig. 3. The power splitting ratios versus the jamming power.

examples are shown in Fig. 3. Clearly, for weak jamming, there is no benefit to harvest energy from the received signals and $\rho \simeq 0$. For full-band strong jamming ($\sigma_{B,J}^2 \geq 10$ dBm), $\rho \simeq 1$, which implies that almost all the received signals are used for energy harvesting to enhance the uplink transmission.

V. CONCLUSION

In this letter, we considered power-splitting energy-harvesting receiver design for OFDM systems under hostile jamming. We investigated the optimal power allocation and power splitting ratio for sum-rate maximization. The achievable performance lower bound under strong full-band jamming was derived. Both analytical and simulation results showed that energy harvesting can improve the sum-rate performance of OFDM systems under strong jamming.

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