Precoding for OFDM under Disguised Jamming

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***Abstract*—This paper considers jamming-resistant OFDM system design under full-band disguised jamming, where the jamming symbols are taken from the same constellation as the information symbols over each subcarrier. *First*, we analyze the impact of disguised jamming on OFDM systems. It is shown that due to the symmetricity between the authorized signal and jamming, the BER of OFDM systems without symbol-level precoding or only with repeated symbol-level coding is lower bounded by a modulation specific constant, which cannot be improved by increasing SNR. *Second*, we develop an optimal precoding scheme which minimizes the BER of OFDM systems under full-band disguised jamming. It is shown that the most efficient way to combat full-band disguised jamming in OFDM systems is to concentrate the total available power and distribute it uniformly over a particular number of subcarriers instead of the entire spectrum. The underlying argument is that for a particular subcarrier, when the signal-to-jamming ratio is large enough, then the receiver can distinguish the authorized signal from disguised jamming under the presence of noise. Both theoretical analysis and numerical results demonstrate that the BER performance of OFDM systems under full-band disguised jamming can be improved significantly with the proposed precoding scheme.**

***Index Terms*—OFDM, disguised jamming, precoding, BER minimization.**

I. Introduction

Conventionally, research on communication system design has been focused on capacity improvement under non-intentional interference, such as intersymbol interference, multiuser interference and noise. The jamming resistance of most communication systems today mainly relies on the diversity introduced by error control coding. On the other hand, jamming has widely been modeled as Gaussian noise. Based on the noise jamming model and the Shannon capacity formula, $C = \log_2(1 + SNR)$, an intuitive impression is that jamming is really harmful only when the jamming power is much higher than the signal power. However, this is only partially true. To show it, we need to look at disguised jamming [1]–[3], where the jamming is highly correlated with the signal, and has a power level close or equal to the signal power. Consider the example, $y = s + j + n$, where s is the authorized signal, j is the jamming, n is the noise independent of j and s, and y is the received signal. If j and s are taken randomly and independently from the same constellation, then due to the symmetricity between the jamming and the authorized signal, the receiver is fully confused and cannot really distinguish the authorized signal from jamming. As can be seen, the symbol error rate cannot be easily changed based only on the conventional bit-level channel coding. This observation motivates us to revisit the importance of symbol-level coding, generally known as precoding. In this paper, we first explore the impact of disguised jamming, and then investigate how precoding can be exploited to combat disguised jamming.

As an important multi-carrier transmission system, orthogonal frequency division multiplexing (OFDM) has been identified as a core technique by many recent standards [4], e.g., LTE and WiMAX, mainly due to its high spectral efficiency and robustness under frequency selective channels. For jamming-resistant OFDM system design, a majority of literature [5], [6] primarily focuses on partial-band jamming, which jams only part of all the subcarriers. The basic strategies include: 1) avoiding the jammed bands, but only transmitting on the jamming-free bands; 2) randomizing the jamming effect through carefully designed interleaving, such that the burst errors caused by partial-band jamming can be properly corrected. However, we observed that under the same jamming power constraint, full-band jamming could be more harmful for these systems [7].

In this paper, we consider the jamming-resistant OFDM system design under full-band disguised jamming, where the jamming symbols are taken from the same constellation as the information symbols over each subcarrier. *First*, we analyze the impact of disguised jamming on OFDM systems. It is shown that due to the symmetricity between the authorized signal and jamming, the BER of OFDM systems without symbol-level precoding or only with repeated symbol-level coding is lower bounded by a modulation specific constant, which cannot be improved by simply increasing the SNR. *Second*, we develop an optimal precoding scheme which minimizes the BER of OFDM systems under full-band disguised jamming. It is shown that the most efficient way to combat full-band disguised jamming in OFDM systems is to concentrate the total available power and distribute it uniformly over a particular number of subcarriers instead of the entire spectrum. The underlying argument is that for a particular subcarrier, when the signal-to-jamming ratio is large enough, then the receiver can distinguish the authorized signal from disguised jamming under the presence of noise. Our theoretical analysis and numerical results show that the BER performance of OFDM systems under full-band disguised jamming can be improved significantly with the proposed precoding scheme.

The rest of the paper is organized as follows. In Section II, the system model of precoded OFDM systems is provided. The impact of disguised jamming on OFDM is analyzed in Section III. The optimal precoding scheme as well as the minimum

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BER of OFDM systems under full-band disguised jamming is derived in Section IV. Numerical evaluation is conducted in Section V and we conclude in Section VI.

II. System Model

We consider the OFDM system equipped with a precoder as shown in Fig. 1. In our model, the input data block is first mapped to symbols. Let Ω represent the constellation we use and $\mathbf{x} = [x_0, x_1, ..., x_{K-1}]^T$ the symbol vector after symbol mapping, where $x_i \in \Omega$, K is the length of the symbol vector, and $(\cdot)^T$ denotes the transpose of a vector.

The $N_c \times K$ precoder matrix is denoted by $\mathbf{P}$, where N_c is the number of subcarriers for OFDM transmission. To allow some redundancy, we choose $N_c \geq K$. After symbol mapping, the precoder is applied to the symbol vector $\mathbf{x}$, which results in an $N_c \times 1$ vector $\mathbf{s}$, i.e.,

$$\mathbf{s} = \mathbf{Px}. \tag{1}$$

The entire OFDM symbol can then be generated by performing inverse fast fourier transform (IFFT). This is followed by cyclic prefix (CP) insertion, which adds a guard time to eliminate intersymbol interference caused by multipath signals.

The obtained signal is then transmitted through an additive white Gaussian noise (AWGN) channel, and simultaneously interfered by full-band disguised jamming. The AWGN noise vector $\tilde{\mathbf{n}}$ has zero means and covariance matrix $E(\tilde{\mathbf{n}}^H \tilde{\mathbf{n}}) = \sigma_n^2 \mathbf{I}$, where $(\cdot)^H$ denotes the Hermitian of a matrix. The frequency domain representation of $\tilde{\mathbf{n}}$ is actually a noise vector whose elements correspond to the AWGN noise associating to each OFDM subcarrier. If we denote it by $\mathbf{n} = [n_0, n_1, ..., n_{N_c-1}]^T$, then $\mathbf{n} = \mathbf{F}^H \tilde{\mathbf{n}}$, where $\mathbf{F}$ is the $N_c \times N_c$ IFFT unitary matrix with $[\mathbf{F}]_{n,k} = \frac{1}{\sqrt{N_c}} e^{j2\pi nk/N_c}$. It is noted that, since $\mathbf{F}^H$ is a unitary matrix, $\mathbf{n}$ continues to be a Gaussian random vector with zero means and covariance matrix $E(\mathbf{n}^H \mathbf{n}) = \sigma_n^2 \mathbf{I}$ [5]. Hence, the noise power corresponding to each subcarrier is σ_n^2.

The disguised jamming is typically launched by generating a signal which mimics the legally transmitted signal to confuse the receiver [1]. More specifically, in the OFDM case, the disguised jammer randomly choose one symbol out of the same constellation Ω for each subcarrier and transmit them exactly as the same way in the authorized OFDM transmitter. Namely, if the jamming vector is denoted by $\mathbf{j} = [j_0, j_1, ..., j_{N_c-1}]^T$, where $j_i \in \Omega$ is the disguised symbol associating to the ith subcarrier, then $\tilde{\mathbf{j}} = \mathbf{Fj}$.

At the receiver side, the cyclic prefix is first removed, followed by an FFT operation, which yields

$$\mathbf{y} = \mathbf{s} + \mathbf{j} + \mathbf{n}, \tag{2}$$

in which all the vectors have a dimension of $N_c \times 1$ and their elements correspond to N_c OFDM subcarriers, repectively.

The $K \times N_c$ decoder matrix $\mathbf{D}$ is then applied to $\mathbf{y}$ to recover the transmitted symbols. Hence, the estimated symbol vector, $\hat{\mathbf{x}}$, can be obtained as

$$\hat{\mathbf{x}} = \mathbf{Ds} + \mathbf{Dj} + \mathbf{Dn}. \tag{3}$$

The basic idea of precoding is to optimally exploit the channel information, including that on noise and jamming, at the transmitter to assign symbols, or their linear combination, over different subcarriers. If some redundancy is allowed (i.e., $K < N_c$), an optimal precoder should be able to wisely exploit the introduced redundancy to combat the distortion and interference. In this paper, we aim to find the optimal precoder that can minimize the BER of OFDM systems under full-band disguised jamming, subject to the constraint on transmit power.

III. Conventional OFDM under Disguised Jamming

A. *OFDM without Precoding*

When there is no precoding employed, a channel with disguised jamming can be modeled as an arbitrarily varying channel (AVC) [1]. It has been proven in [1] that, due to the symmetricity between the authorized signal and jamming, the symbol error rate (SER) of a transmission under disguised jamming is lower bounded by

$$\mathcal{P}_s \geq \frac{M-1}{2M}, \tag{4}$$

where M is the constellation size. An intuitive explanation is that under disguised jamming, the receiver has to guess between the truly transmitted symbol and the fake symbol sent by the disguised jammer, if the two symbols are distinct. Note that the error probability of a random guess between two symbols is $\frac{1}{2}$, and the two symbols randomly and independently selected out of Ω by the authorized transmitter and disguised jammer differ in a probability of $\frac{M-1}{M}$. The additional noise would make the error probability even larger.

The lower bounded SER would naturally result in a lower bounded BER, where the relationship is determined by the constellation used. When a binary modulation scheme (e.g., BPSK with M=2) is used, the BER coincides with the SER, and it would be lower bounded by

$$\mathcal{P}_{b,BPSK} \geq \frac{1}{4}, \qquad \textit{without precoding}. \tag{5}$$

The derived lower bounds above point to an important fact that under disguised jamming, bit-level coding can no longer decrease the error probability. The reason is that the bit stream with any bit-level coding ultimately has to be mapped to symbols, which unfortunately have an error probability lower bounded by (4) under disguised jamming.

B. *MC-CDMA: OFDM with Repeated Coding*

Considering that the disguised jamming disenables bit-level coding from improving the error probability performance, it becomes an option to exploit the symbol-level coding, which performs the coding directly on symbols instead of the bit stream. Let us consider MC-CDMA [8], which actually exploits repeated symbol-level coding. In repeated coding, each symbol is transmitted for L times, and it will be estimated at the receiver by averaging all the distorted copies. Take BPSK as an example, with a probability of $\frac{1}{2^L}$, the disguised symbols are all coincidentally opposite to the one sent by the authorized transmitter, in which case the receiver would still

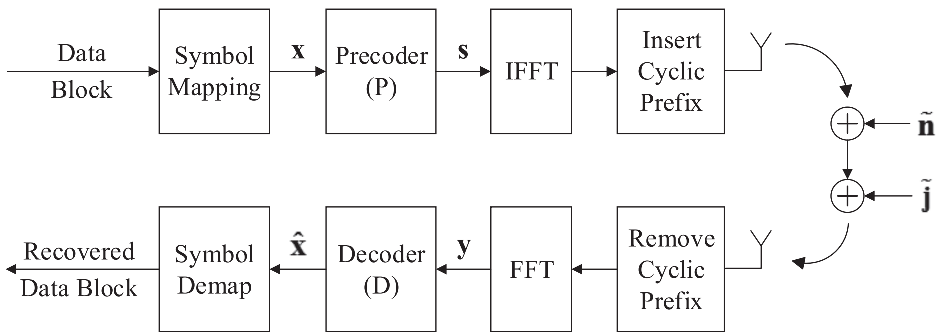

Fig. 1. The system model of OFDM with precoding.

have to randomly guess which one out of the two symbols is transmitted. Hence, considering the noise as an additional impact, the BER under disguised jamming is lowered bounded by

$$\mathcal{P}_{b,BPSK} \geq \frac{1}{2^{L+1}}, \qquad repeated\ coding. \tag{6}$$

It can be observed from (5) and (6) that disguised jamming is a significant threat to OFDM without precoding or only equipped with repeated coding, since the BERs cannot be reduced below the lower bounds no matter how high the SNR is. The significant performance degradation of OFDM under disguised jamming motivates us to design an effective jamming-resistant symbol-level precoding scheme, as will be illustrated in Section IV.

IV. Precoding under Disguised Jamming

In this section, we derive the optimal precoder and decoder matrices using the minimum BER criterion. We start with BPSK modulation and AWGN channels, and then discuss other modulation schemes and frequency selective channels.

Let $\mathcal{P}_{\mathbf{P},\mathbf{D}}$ denote the BER with a precoder matrix $\mathbf{P}$ and decoder matrix $\mathbf{D}$ under full-band disguised jamming, and the problem can then be formulated as

$$\min_{\mathbf{P},\mathbf{D}} \mathcal{P}_{\mathbf{P},\mathbf{D}}; \tag{7a}$$

$$s.t.\ \ tr(\mathbf{P}\mathbf{P}^H) = P_c, \tag{7b}$$

$$\mathbf{D}\mathbf{P} = \mathbf{I}, \tag{7c}$$

where $tr(\cdot)$ is the trace operation. More specifically, (7b) is the constraint on the total available transmit power, and (7c) ensures perfect recovery for the precoding and decoding.

Under the constraint in (7c), the estimated symbol vector in (3) can be further simplified as

$$\hat{\mathbf{x}} = \mathbf{x} + \mathbf{D}\mathbf{j} + \mathbf{D}\mathbf{n}, \tag{8}$$

which is equivalent to

$$\hat{x}_k = x_k + \sum_{i=0}^{N_c-1} d_{k,i} j_i + \sum_{i=0}^{N_c-1} d_{k,i} n_i, \quad k = 0, 1, ..., K-1, \tag{9}$$

where we can see that with respect to $\mathbf{D}$, the estimation of the kth symbol in the transmitted symbol vector only depends on the kth row of $\mathbf{D}$. *This allows us to divide the optimization into two steps: 1) Minimizing the BER independently for each symbol under a parameterized constraint; 2) Minimizing the overall BER including all the symbols by finding the optimal parameters in the constraints.*

1) Independent BER Minimization for Each Symbol: To facilitate the analysis, we present two propositions first.

***Proposition* 1.** *If a BPSK symbol with unit power is distorted by a deviation z and a Gaussian noise with a variance σ^2, the BER can be calculated by*

$$\mathcal{P}_{b,BPSK}(\sigma, z) = \frac{1}{2} Q\left(\frac{1-|z|}{\sigma}\right) + \frac{1}{2} Q\left(\frac{1+|z|}{\sigma}\right), \tag{10}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt.$

Proof: If the symbol $s = -1$ is transmitted, with a fixed deviation z, the received symbol would obey a Gaussian distribution $\hat{s} \sim \mathcal{N}(z-1, \sigma^2)$. The BER with $s = -1$ would be

$$\mathcal{P}(\hat{s} > 0 | s = -1) = Q\left(\frac{1-z}{\sigma}\right). \tag{11}$$

Similarly, the BER with $s = 1$ can be calculated as

$$\mathcal{P}(\hat{s} < 0 | s = 1) = Q\left(\frac{1+z}{\sigma}\right). \tag{12}$$

Assuming that $s = -1$ and $s = 1$ are equally probable, the overall BER can be obtained as shown in (10). ■

***Proposition* 2.** *Under the condition that $\sigma^2 < \beta - 1$, the constrained objective function*

$$J = \sum_{l=0}^{L-1} Q\left(\frac{\beta \pm w_l}{\sigma}\right), \ \ s.t. \ \sum_{l=0}^{L-1} w_l^2 = L \& w_l \geq 0, \ \forall l, \tag{13}$$

achieves its minimum

$$J_{min} = LQ\left(\frac{\beta \pm 1}{\sigma}\right), \tag{14}$$

at $w_l = 1, \ \forall l$. Note that "+" and "−" in (14) correspond to those in (13), respectively.

Proof: We start from the problem with the "−" sign. Using the Lagrange multiplier, we define

$$F = \sum_{l=0}^{L-1} Q\left(\frac{\beta - w_l}{\sigma}\right) + \lambda\left(\sum_{l=0}^{L-1} w_l^2 - L\right). \quad (15)$$

Differentiating (15) with respect to each w_l,

$$\frac{\partial F}{\partial w_l} = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(\beta - w_l)^2}{2\sigma^2}\right\} + 2\lambda w_l, \quad \forall l. \quad (16)$$

By setting $\frac{\partial F}{\partial w_l} = 0$ and considering the constraint in (13), we have $w_l = 1, \ \forall l$. To ensure that this is the minimum point, we calculate the second-order differentiation at this point,

$$\frac{\partial^2 F}{\partial w_l^2} = \frac{1}{\sqrt{2\pi}\sigma}\left(\frac{\beta - 1}{\sigma^2} - 1\right)\exp\left\{-\frac{(\beta-1)^2}{2\sigma^2}\right\}, \ \forall\, l. \quad (17)$$

Let $\frac{\partial^2 F}{\partial w_l^2} > 0$, we obtain the condition for the derived point being the minimum, $\sigma^2 < \beta - 1$. The problem with the "+" sign can be proved similarly. ■

If we define $\sum_{i=0}^{N_c-1} d_{k,i}^2 \triangleq \frac{1}{\beta_k^2}$, the exclusive dependency on the kth row of $\mathbf{D}$ for the kth symbol estimation enables us to find the minimum BER for the kth symbol with respect to β_k, and we have the following theorem.

***Theorem* 1.** *Under the condition that $\sigma_n^2 < \beta_k - 1$, the minimum BER of the kth symbol estimation in (9) can be obtained as*

$$\mathcal{P}_{k,min} = \frac{1}{2}Q\left(\frac{\beta_k - 1}{\sigma_n}\right) + \frac{1}{2}Q\left(\frac{\beta_k + 1}{\sigma_n}\right), \quad (18)$$

where $\beta_k = \sqrt{1/\sum_{i=0}^{N_c-1} d_{k,i}^2}$ and (18) is achieved when

$$d_{k,i} = \begin{cases} \frac{1}{\beta_k}, & i = i_k, \\ 0, & elsewhere, \end{cases} \quad (19)$$

where i_k differs from each other for different k.

Proof: Define $\mathcal{J} \triangleq \{\mathbf{j} = [j_0, j_1, ..., j_{N_c-1}]^T | j_i \in \Omega, i = 0, 1, ..., N_c - 1\}$, and the size of $\mathcal{J}$ would be $|\mathcal{J}| = 2^{N_c}$, since $\Omega = \{-1, +1\}$. According to (9), with a particular jamming vector $\mathbf{j} = [j_0, j_1, ..., j_{N_c-1}]^T$, the kth BPSK symbol x_k is distorted by a deviation $z = \sum_{i=0}^{N_c-1} d_{k,i} j_i$ and a Gaussian noise with a variance $\sigma^2 = \sigma_n^2 \sum_{i=0}^{N_c-1} d_{k,i}^2 = \sigma_n^2/\beta_k^2$. Considering all the 2^{N_c} possible jamming vectors and applying Proposition 1, the BER of the kth symbol can be obtained as

$$\begin{aligned}\mathcal{P}_k &= \frac{1}{2^{N_c}}\sum_{\mathbf{j}\in\mathcal{J}}\left[\frac{1}{2}Q\left(\frac{1 - |\sum_{i=0}^{N_c-1} d_{k,i} j_i|}{\sigma_n/\beta_k}\right)\right. \\ &\quad \left. + \frac{1}{2}Q\left(\frac{1 + |\sum_{i=0}^{N_c-1} d_{k,i} j_i|}{\sigma_n/\beta_k}\right)\right] \\ &= \frac{1}{2^{N_c+1}}\left[\sum_{\mathbf{j}\in\mathcal{J}} Q\left(\frac{\beta_k - \beta_k|\sum_{i=0}^{N_c-1} d_{k,i} j_i|}{\sigma_n}\right)\right. \\ &\quad \left. + \sum_{\mathbf{j}\in\mathcal{J}} Q\left(\frac{\beta_k + \beta_k|\sum_{i=0}^{N_c-1} d_{k,i} j_i|}{\sigma_n}\right)\right].\end{aligned} \quad (20)$$

For any jamming vector $\mathbf{j} = [j_0, j_1, ..., j_{N_c-1}]^T \in \mathcal{J}$, we define $l \triangleq bin2dec([\frac{1-j_0}{2}, \frac{1-j_1}{2}, ..., \frac{1-j_{N_c-1}}{2}])$ and let $w_l \triangleq \beta_k|\sum_{i=0}^{N_c-1} d_{k,i} j_i|$, for $l = 0, 1, ..., 2^{N_c} - 1$. Then (20) can be rewritten as

$$\mathcal{P}_k = \frac{1}{2^{N_c+1}}\left[\sum_{l=0}^{2^{N_c}-1} Q\left(\frac{\beta_k - w_l}{\sigma_n}\right) + \sum_{l=0}^{2^{N_c}-1} Q\left(\frac{\beta_k + w_l}{\sigma_n}\right)\right], \quad (21)$$

with

$$\sum_{l=0}^{2^{N_c}-1} w_l^2 = 2^{N_c}\beta_k^2 \sum_{i=0}^{N_c-1} d_{k,i}^2 = 2^{N_c}. \quad (22)$$

Applying Proposition 2, we can obtain the minimum of (21) as shown in (18), under the condition that $\sigma_n^2 < \beta_k - 1$ and the minimum is achieved at $w_l = 1, \ \forall l$. To achieve this minimum point, only one non-zero element, $\frac{1}{\beta_k}$, can exist among $d_{k,i}, \ \forall i$. There is a further requirement that the only non-zero element in each row needs to be located in different columns, which guarantees that the constraint in (7c) can be satisfied. To sum up, the required decoder matrix $\mathbf{D}$ to achieve the smallest BER for each symbol should be formed in the way as shown in (19). ■

2) Minimization for the Overall BER: With the BER of each symbol minimized, we try to minimize the overall BER including all the symbols, by finding the optimal β_k for each k. Following the pattern in (19), and without loss of generality, we assume the non-zeros of $\mathbf{D}$ locate in the first k columns, i.e.,

$$\mathbf{D} = \begin{bmatrix} \frac{1}{\beta_0} & 0 & \cdot & \cdot & 0 & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \frac{1}{\beta_1} & \cdot & \cdot & 0 & 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \frac{1}{\beta_{K-1}} & 0 & 0 & \cdot & \cdot & 0 \end{bmatrix}_{K\times N_c}.$$

Applying the constraint in (7c), the precoding matrix needs to be

$$\mathbf{P} = \begin{bmatrix} \beta_0 & 0 & \cdot & \cdot & 0 \\ 0 & \beta_1 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \beta_{K-1} \\ 0 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 \end{bmatrix}_{N_c\times K}.$$

Applying the constraint in (7b), the constraint on β_k can be obtained as $\sum_{k=0}^{K-1} \beta_k^2 = P_c$.

Taking into account all the symbols in the transmitted symbol vector and applying Theorem 1, the overall BER can be calculated as

$$\begin{aligned}\mathcal{P}_{\mathbf{P},\mathbf{D}} &= \frac{1}{K}\sum_{k=0}^{K-1}\mathcal{P}_{k,min} \\ &= \frac{1}{2K}\sum_{k=0}^{K-1}\left[Q\left(\frac{\beta_k-1}{\sigma_n}\right)+Q\left(\frac{\beta_k+1}{\sigma_n}\right)\right].\end{aligned} \tag{23}$$

Due to the convexity of the Q function,

$$\mathcal{P}_{\mathbf{P},\mathbf{D}} \geq \frac{1}{2}Q\left(\frac{\frac{1}{K}\sum_{k=0}^{K-1}\beta_k-1}{\sigma_n}\right)+\frac{1}{2}Q\left(\frac{\frac{1}{K}\sum_{k=0}^{K-1}\beta_k+1}{\sigma_n}\right), \tag{24}$$

where the equality holds if and only if each β_k equals each other for all k.

Simultaneously, using the Lagrange multiplier, we have

$$\frac{1}{K}\sum_{k=0}^{K-1}\beta_k \leq \sqrt{\frac{P_c}{K}}, \quad s.t. \sum_{k=0}^{K-1}\beta_k^2 = P_c, \tag{25}$$

in which the equality holds if and only if

$$\beta_k = \sqrt{\frac{P_c}{K}},\ \forall k. \tag{26}$$

Considering the equality of both (24) and (25) holds with the same condition as in (26), the minimum overall BER can be obtained as

$$\mathcal{P}_{\mathbf{P},\mathbf{D}} \geq \frac{1}{2}Q\left(\frac{\sqrt{\frac{P_c}{K}}-1}{\sigma_n}\right)+\frac{1}{2}Q\left(\frac{\sqrt{\frac{P_c}{K}}+1}{\sigma_n}\right), \tag{27}$$

where the equality holds when $\beta_k = \sqrt{\frac{P_c}{K}},\ \forall k$.

The above result is summarized in the following theorem.

Theorem 2. *Under the condition that $\sigma_n^2 < \sqrt{\frac{P_c}{K}}-1$, the BER minimization problem in (7) has a solution,*

$$\min_{\boldsymbol{P},\boldsymbol{D}} \mathcal{P}_{\boldsymbol{P},\boldsymbol{D}} = \frac{1}{2}Q\left(\frac{\sqrt{\frac{P_c}{K}}-1}{\sigma_n}\right)+\frac{1}{2}Q\left(\frac{\sqrt{\frac{P_c}{K}}+1}{\sigma_n}\right), \tag{28}$$

where the minimum is achieved with

$$\boldsymbol{P} = \begin{bmatrix} \sqrt{\frac{N_c}{K}} & 0 & \cdot & \cdot & 0 \\ 0 & \sqrt{\frac{N_c}{K}} & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \sqrt{\frac{N_c}{K}} \\ 0 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 \end{bmatrix}_{N_c\times K},$$

and

$$\boldsymbol{D} = \begin{bmatrix} \sqrt{\frac{K}{N_c}} & 0 & \cdot & \cdot & 0 & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \sqrt{\frac{K}{N_c}} & \cdot & \cdot & 0 & 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \sqrt{\frac{K}{N_c}} & 0 & 0 & \cdot & \cdot & 0 \end{bmatrix}_{K\times N_c}.$$

Theorem 2 indicates that the most efficient way to combat full-band disguised jamming in OFDM systems is to concentrate all the available power and distribute it uniformly on a particular number of subcarriers instead of the entire spectrum. The underlying argument is that for a particular subcarrier, when the signal-to-jamming ratio is large enough, then the receiver can distinguish the authorized signal from disguised jamming under the presence of noise. Further research will be conducted on cognitive disguised jamming.

Discussions: a) *Other Modulation Schemes* Though Theorems 1 and 2 are proved for BPSK at this point, these results shed light on precoder design for other modulations as well. The effectiveness of the proposed precoding scheme on other modulation schemes is demonstrated in Section V-B through simulation results.

b) *Frequency Selective Channels* A typical way to cope with frequency selective channels is to perform a channel equalization after estimation, which is also indispensable in conventional OFDM systems without precoding. Let $\mathbf{h} = [h_0, h_1, ..., h_{N_c-1}]$ denote the frequency domain channel impulse response vector. We perform an equalization on the received symbol vector $\mathbf{y}$ before feeding it to the decoder matrix $\mathbf{D}$. Nothing changes but the symbol estimation in (3) would become $\hat{\mathbf{x}} = \mathbf{D}\tilde{\mathbf{y}}$, where the ith element of $\tilde{\mathbf{y}}$, $\tilde{y}_i$, can be obtained by $\tilde{y}_i = \frac{y_i}{h_i},\ i = 0, 1, ..., N_c-1$. In this case, for best performance, the K subcarriers with the largest SNRs should be selected out of all N_c subcarriers. A numerical evaluation on frequency selective channels is provided in Section V-C.

V. Numerical Results

In this section, the BER performance of the precoded OFDM system under full-band disguised jamming is evaluated and compared with that without precoding and with repeated coding through simulation examples. We consider both AWGN and frequency selective channels. In the following, we assume N_c = 64, R_s = 100, P_c = 64, and the signal-to-jamming ratio is 0dB over the entire spectrum.

A. BPSK under AWGN Channels

In this scenario, under AWGN channels, BPSK is used and we set $K = 16 < N_c$, which provides redundancy for the precoding to combat disguised jamming. In Fig. 2, it is observed that: 1) Uncoded OFDM completely fails under full-band disguised jamming, since the BER maintains at approximately $\frac{1}{4}$ (lower bound in (5)) no matter how high the SNR is; 2) OFDM with repeated coding improves a little on the BER, but is still far away from being satisfactory, since the BER cannot be reduced beneath $\frac{1}{32}$ (lower bound in (6)) no matter how high the SNR is; 3) OFDM with optimal precoding considerably reduces the BER with reasonable SNRs, resulting from the optimal utilization of the introduced redundancy.

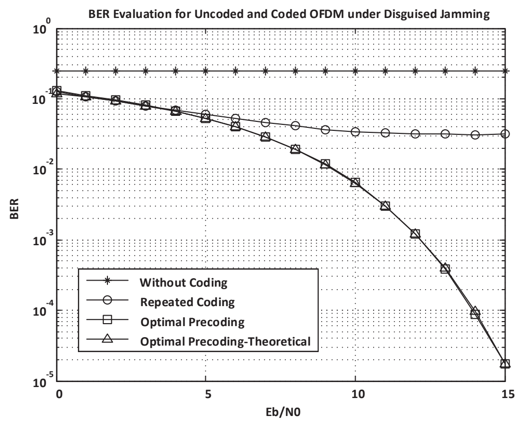

Fig. 2. BER evaluation for BPSK-modulated OFDM with full-band disguised jamming under AWGN channels.

B. 16QAM under AWGN Channels

In this scenario, still under AWGN channels, 16QAM is used to evaluate the performance of the optimal precoding with high-order modulation. K is further reduced to 4 to provide more redundancy. In Fig. 3, we observe the similar results as in Section V-A, which demonstrate that the precoding scheme in Theorem 2 works with high-order modulation as well. However, the increased redundancy and the higher SNR requirement prove that high-order modulation is more fragile to disguised jamming.

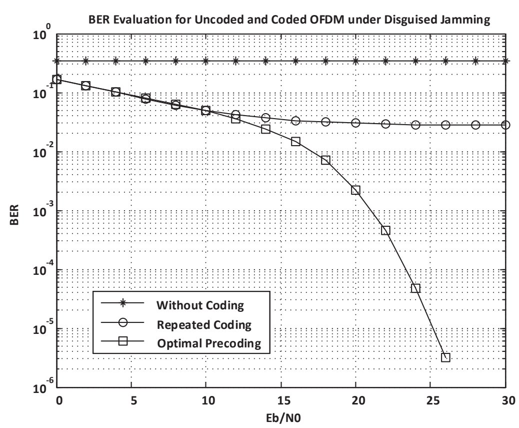

Fig. 3. BER evaluation for 16QAM-modulated OFDM with full-band disguised jamming under AWGN channels.

C. BPSK under Frequency Selective Channels

To evaluate the impact of fading on the proposed precoding scheme, in this scenario, we move the simulation in Section V-A to a typical frequency selective channel. In Fig. 4, it is observed that under frequency selective channels, OFDM with optimal precoding still outperforms the others, which demonstrates the effectiveness of the precoding scheme under frequency selective channels.

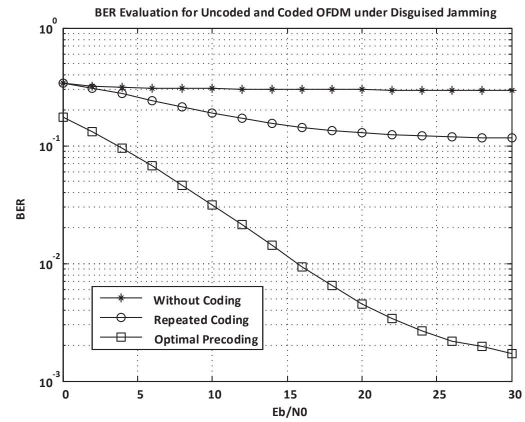

Fig. 4. BER evaluation for BPSK-modulated OFDM with full-band disguised jamming under frequency selective channels.

VI. Conclusions

In this paper, we analyzed the impact of disguised jamming on OFDM systems, and developed an optimal precoding scheme which minimizes the BER of OFDM systems under full-band disguised jamming. It is shown that the most efficient way to combat full-band disguised jamming in OFDM systems is to concentrate all the available power and distribute it uniformly on a particular number of subcarriers instead of the entire spectrum. Both theoretical analysis and numerical results demonstrated that the BER performance of OFDM systems under full-band disguised jamming can be improved significantly with the proposed precoding scheme.

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