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Physical Layer Security of Multiband Communications Under Hostile Jamming

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***Abstract*—This paper considers a game between a power-limited authorized user and a power-limited jammer, who operate independently over the same AWGN channel consisting of multiple bands. We explore the possibility for the authorized user or the jammer to randomly utilize part (or all) of the available spectrum and/or apply nonuniform power allocation. It is found that: either for the authorized user to maximize its capacity, or for the jammer to minimize the capacity of the authorized user, the best strategy for both of them is to distribute the transmission power or jamming power uniformly over all the available spectrum. The minimax capacity can be calculated based on the channel bandwidth and the signal to jamming and noise ratio, and it matches with the Shannon channel capacity formula. Numerical results are provided to illustrate the theoretical analysis.**

***Index Terms*—Multiband communications, game theory, jamming, capacity analysis.**

I. Introduction

Hostile jamming, in which the authorized user's signal is deliberately interfered by the adversary, is one of the most commonly used techniques for limiting the effectiveness of an opponent's communication. In traditional anti-jamming techniques [1], they either assume specific jamming models or try to estimate the jamming pattern before selecting an anti-jamming scheme. The underlying assumption is that the jamming is varying slowly such that the authorized user has sufficient time to track and react to the jamming. However, if the jammer is intelligent and can switch its patterns fast enough, then it would be impossible for the authorized user to detect and react in real time. To the best of our knowledge, existing work on this topic has been focused on single band communications [2] or just power allocation for multiband/multicarrier communications [3]. An interesting question is: can the authorized user or the jammer benefit from randomly utilizing part instead of all of the available spectrum and/or applying nonuniform power allocation in the presence of an intractable opponent?

In this paper, we try to address this question by investigating a game between a power-limited authorized user and a power-limited jammer, who operate independently over the same AWGN channel consisting of multiple bands. From a game theoretical perspective, we find that: either for the authorized user to maximize its capacity, or for the jammer to minimize the capacity of the authorized user, the best strategy for both of them is to distribute the transmission power or jamming power uniformly over all the available spectrum. The minimax capacity of the authorized user is given by $C = B\log_2(1+P_s/(P_J+P_N))$, where B is the bandwidth of the overall spectrum, P_N the noise power, P_s and P_J the total power for the authorized user and the jammer, respectively. In other words, the minimax capacity above is the smallest capacity that can be achieved by the authorized user if it utilizes all the available spectrum and applies uniform power allocation, no matter what strategy is applied by the jammer; meanwhile, it is also the largest capacity that can be achieved by the authorized user if the jammer jams all the available spectrum and applies uniform power allocation, no matter what strategy is applied by the authorized user.

II. System Model and Problem Formulation

We consider a multiband communication scenario, where *an authorized user* and *a jammer* operate over N_c frequency bands or subchannels (not necessarily being consecutive), and each has a bandwidth of B/N_c Hz. That is, the bandwidth of the total available spectrum is B Hz. Each subchannel is modeled as an AWGN channel. More specifically, we assume the total noise power over the entire spectrum is P_N, and the noise power corresponding to each subchannel is P_N/N_c. Recall that Gaussian jamming is optimal when the jammer has no knowledge of its target [4], hence we assume the jamming is Gaussian for each jammed subchannel. In the following, P_s denotes the total available power for the authorized user, and P_J the total jamming power.

The authorized user is always trying to maximize its capacity under jamming by applying an optimal strategy, that is, to transmit information over all or part of the available subchannels with optimal subchannel selection and power allocation. Accordingly, the jammer would like to find an optimal strategy that can minimize the capacity of the authorized user. It is assumed that both the authorized user and the jammer have no knowledge of the selected subchannels and power levels applied by their opponent.

Each strategy applied by the authorized user is determined by the number of activated subchannels, the subchannel selection process and the power allocation process. More specifically: (1) The authorized user activates K_s $(1 \leq K_s \leq N_c)$ out of N_c subchannels each time for information transmission. (2) The subchannel selection process is characterized using a binary indicator vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_{N_c}]$, where the

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random variable $\alpha_m = 1$ or 0 indicates whether the mth subchannel is selected or not, and $\sum_{m=1}^{N_c} \alpha_m = K_s$. Let $\boldsymbol{\omega}_s = [\omega_{s,1}, \omega_{s,2}, ..., \omega_{s,N_c}]$ be the corresponding probability vector, where $\omega_{s,m}$ denotes the probability that the mth subchannel is selected each time (i.e., $\omega_{s,m} = Pr\{\alpha_m = 1\}$), and $\sum_{m=1}^{N_c} \omega_{s,m} = K_s$. (3) The authorized user numbers the selected K_s subchannels from 1 through K_s following the order as they appear in the original spectrum, and performs power allocation over them. The power allocation process is characterized using a vector $\mathbf{P}_s = [P_{s,1}, P_{s,2}, ..., P_{s,K_s}]$, in which $P_{s,n}$ denotes the power allocated to the nth selected subchannel, and $\sum_{n=1}^{K_s} P_{s,n} = P_s$ is the power constraint. Let $\mathcal{W}_{s,K_s} = \{\boldsymbol{\omega}_s = [\omega_{s,1}, \omega_{s,2}, ..., \omega_{s,N_c}] | 0 \leq \omega_{s,m} \leq 1, \sum_{m=1}^{N_c} \omega_{s,m} = K_s\}$, and $\mathcal{P}_{s,K_s} = \{\mathbf{P}_s = [P_{s,1}, P_{s,2}, ..., P_{s,K_s}] | 0 < P_{s,n} \leq P_s, \sum_{n=1}^{K_s} P_{s,n} = P_s\}$. The strategy space for the authorized user can thus be defined as

$$\mathcal{X} = \{(K_s, \boldsymbol{\omega}_s, \mathbf{P}_s) | 1 \leq K_s \leq N_c, \boldsymbol{\omega}_s \in \mathcal{W}_{s,K_s}, \mathbf{P}_s \in \mathcal{P}_{s,K_s}\}. \tag{1}$$

The strategy space $\mathcal{X}$ covers all the possible subchannel utilization strategies as K_s varies from 1 to N_c.

Similarly, the jammer jams K_J ($1 \leq K_J \leq N_c$) out of N_c subchannels each time following a binary indicator vector $\boldsymbol{\beta} = [\beta_1, \beta_2, ..., \beta_{N_c}]$ with $\sum_{m=1}^{N_c} \beta_m = K_J$. The subchannel selection process is characterized using a probability vector $\boldsymbol{\omega}_J = [\omega_{J,1}, \omega_{J,2}, ..., \omega_{J,N_c}]$, where $\omega_{J,m} = Pr\{\beta_m = 1\}$ and $\sum_{m=1}^{N_c} \omega_{J,m} = K_J$. Then the jammer numbers the K_J selected subchannels from 1 through K_J in the same manner as the authorized user, and performs power allocation over them using a power-allocation vector $\mathbf{P}_J = [P_{J,1}, P_{J,2}, ..., P_{J,K_J}]$ with $\sum_{n=1}^{K_J} P_{J,n} = P_J$. Let $\mathcal{W}_{J,K_J} = \{\boldsymbol{\omega}_J = [\omega_{J,1}, \omega_{J,2}, ..., \omega_{J,N_c}] | 0 \leq \omega_{J,m} \leq 1, \sum_{m=1}^{N_c} \omega_{J,m} = K_J\}$ and $\mathcal{P}_{J,K_J} = \{\mathbf{P}_J = [P_{J,1}, P_{J,2}, ..., P_{J,K_J}] | 0 < P_{J,n} \leq P_J, \sum_{n=1}^{K_J} P_{J,n} = P_J\}$, the strategy space for the jammer can thus be defined as

$$\mathcal{Y} = \{(K_J, \boldsymbol{\omega}_J, \mathbf{P}_J) | 1 \leq K_J \leq N_c, \boldsymbol{\omega}_J \in \mathcal{W}_{J,K_J}, \mathbf{P}_J \in \mathcal{P}_{J,K_J}\}. \tag{2}$$

From a game theoretical perspective, the strategic decision-making of the authorized user and the jammer can be modeled as a two-person zero-sum game [5], which is characterized by a triplet $(\mathcal{X}, \mathcal{Y}, C)$, where

1) $\mathcal{X}$ is the strategy space of the authorized user;
2) $\mathcal{Y}$ is the strategy space of the jammer;
3) C is a real-valued payoff function defined on $\mathcal{X} \times \mathcal{Y}$.

The interpretation is as follows. Let (x, y) denote the strategy pair, in which $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ are the strategies applied by the authorized user and the jammer, respectively. The payoff function $C(x, y)$ is defined as the expected or average capacity of the authorized user choosing a strategy $x \in \mathcal{X}$ in the presence of the jammer choosing a strategy $y \in \mathcal{Y}$. In other words, $C(x, y)$ is the amount that the authorized user wins and simultaneously the jammer loses in the game with a strategy pair (x, y). Define $\mathcal{A} = \{\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_{N_c}] | \alpha_m \in \{0, 1\}, \sum_{m=1}^{N_c} \alpha_m = K_s\}$, and $\mathcal{B} = \{\boldsymbol{\beta} = [\beta_1, \beta_2, ..., \beta_{N_c}] | \beta_m \in \{0, 1\}, \sum_{m=1}^{N_c} \beta_m = K_J\}$. Let $p(\boldsymbol{\alpha})$ and $p(\boldsymbol{\beta})$ denote the probabilities that the authorized user selects subchannels using $\boldsymbol{\alpha}$ and the jammer selects subchannels using $\boldsymbol{\beta}$, respectively. Let $T_{s,m}$ and $T_{J,m}$ be the power allocated to the mth subchannel by the authorized user and the jammer, respectively, which are determined by

$$T_{s,m} = \begin{cases} P_{s,g_m}, & \alpha_m = 1, \\ 0, & \alpha_m = 0, \end{cases} \quad T_{J,m} = \begin{cases} P_{J,q_m}, & \beta_m = 1, \\ 0, & \beta_m = 0, \end{cases} \tag{3}$$

where $g_m = \sum_{i=1}^{m} \alpha_i$ is the new index of subchannel m in the K_s selected subchannels if it is activated by the authorized user, and $q_m = \sum_{i=1}^{m} \beta_i$ is the new index of subchannel m in the K_J selected subchannels if it is jammed by the jammer. Apparently, we have $1 \leq g_m \leq K_s$ and $1 \leq q_m \leq K_J$, for all $1 \leq m \leq N_c$. Note that the subchannel selection processes used by the authorized user and the jammer are independent of each other. Then, the *average* capacity of the authorized user in the game with a strategy pair (x, y) can be calculated as

$$C(x, y) = \sum_{\boldsymbol{\alpha} \in \mathcal{A}} \sum_{\boldsymbol{\beta} \in \mathcal{B}} p(\boldsymbol{\alpha}) p(\boldsymbol{\beta}) \times \sum_{m=1}^{N_c} \frac{B}{N_c} \log_2 \left(1 + \frac{T_{s,m}}{T_{J,m} + P_N/N_c}\right). \tag{4}$$

Based on the definitions above, the minimax capacity of the authorized user can be obtained by [6]

$$C(x^*, y^*) = \max_{x \in \mathcal{X}} \min_{y \in \mathcal{Y}} C(x, y) = \min_{y \in \mathcal{Y}} \max_{x \in \mathcal{X}} C(x, y). \tag{5}$$

It can be seen from (5) that the authorized user tries to choose an optimal strategy $x^* \in \mathcal{X}$ to maximize its capacity, while the jammer tries to minimize it by choosing an optimal strategy $y^* \in \mathcal{Y}$. The capacity $C(x^*, y^*)$ in (5) can be achieved when a saddle point strategy pair (x^*, y^*) is chosen, which is characterized by the following inequalities [7]

$$C(x, y^*) \leq C(x^*, y^*) \leq C(x^*, y), \quad \forall x \in \mathcal{X}, y \in \mathcal{Y}. \tag{6}$$

This implies that: with strategy x^*, the smallest capacity that can be achieved by the authorized user is $C(x^*, y^*)$, no matter which strategy is applied by the jammer; on the other hand, if the jammer applies strategy y^*, the largest capacity that can be achieved by the authorized user is also $C(x^*, y^*)$, no matter which strategy is applied by the authorized user. As a result, to find the optimal transmission strategy and jamming strategy, we need to find the saddle point strategy pair (x^*, y^*).

III. Optimal Strategy for Multiband Communications under Jamming

Recall that K_s denotes the number of subchannels activated by the authorized user, and K_J the number of subchannels jammed by the jammer. In this section, we derive the saddle point strategy pair (x^*, y^*) in two steps: (1) For any fixed K_s and K_J with $1 \leq K_s, K_J \leq N_c$, calculate the corresponding minimax capacity and denote it by $\widetilde{C}(K_s, K_J)$. Let $K_s = 1, 2, ..., N_c$ and $K_J = 1, 2, ..., N_c$, we can obtain an $N_c \times N_c$ payoff matrix $\widetilde{\mathbf{C}}$. (2) For the derived payoff matrix $\widetilde{\mathbf{C}}$, locate its saddle point, and then the minimax capacity of the authorized user in (5) can be calculated accordingly.

A. The Minimax Problem for Fixed K_s and K_J

With fixed K_s and K_J, the strategy space for the authorized user becomes $\widetilde{\mathcal{X}} = \{(K_s, \boldsymbol{\omega}_s, \mathbf{P}_s)|\text{Fixed } K_s, \boldsymbol{\omega}_s \in \mathcal{W}_{s,K_s}, \mathbf{P}_s \in \mathcal{P}_{s,K_s}\} \subset \mathcal{X}$, and similarly the strategy space for the jammer becomes $\widetilde{\mathcal{Y}} = \{(K_J, \boldsymbol{\omega}_J, \mathbf{P}_J)|\text{Fixed } K_J, \boldsymbol{\omega}_J \in \mathcal{W}_{J,K_J}, \mathbf{P}_J \in \mathcal{P}_{J,K_J}\} \subset \mathcal{Y}$. It should be noted that the user-activated subchannels and the jammed subchannels vary from one time to another, although the total number of the user-activated or jammed subchannels is fixed. In the following, we will find the saddle point of $C(x, y)$ for $x \in \widetilde{\mathcal{X}}$ and $y \in \widetilde{\mathcal{Y}}$.

***Lemma* 1.** *For any $z \geq 0$ and $a > 0$, the real-valued function, $f(z) = \log_2(1 + z/a)$, is concave.*

Proof: $f''(z) = -1/\ln 2/(z + a)^2 < 0$, for any $z \geq 0$ and $a > 0$. ∎

***Lemma* 2.** *For any $z \geq 0$, $a > 0$ and $b > 0$, the real-valued function, $f(z) = \log_2(1 + a/(z + b))$, is convex.*

Proof: $f''(z) = \frac{a}{\ln 2} \frac{(2z+a+2b)}{(z+a)^2(z+a+b)^2} > 0$, for any $z \geq 0$, $a > 0$ and $b > 0$. ∎

The solution to the minimax problem for fixed K_s and K_J is given in the proposition below.

***Proposition* 1.** *Let K_s be the number of subchannels activated by the authorized user, and K_J the number of subchannels jammed by the jammer. For any fixed (K_s, K_J) pair, the saddle point of $C(x, y)$ under the power constraints P_s and P_J for $x \in \widetilde{\mathcal{X}}$ and $y \in \widetilde{\mathcal{Y}}$ is reached when both authorized user and the jammer choose to apply uniform subchannel selection and uniform power allocation strategy. That is, for fixed K_s and K_J, the saddle point strategy pair $(\widetilde{x}^*, \widetilde{y}^*)$ that satisfies*

$$C(\widetilde{x}, \widetilde{y}^*) \leq C(\widetilde{x}^*, \widetilde{y}^*) \leq C(\widetilde{x}^*, \widetilde{y}), \quad \forall \widetilde{x} \in \widetilde{\mathcal{X}}, \widetilde{y} \in \widetilde{\mathcal{Y}}, \tag{7}$$

is given by $\widetilde{x}^* = (K_s, \boldsymbol{\omega}_s^*, \boldsymbol{P}_s^*)$ with

$$\begin{cases} \omega_{s,m}^* = K_s/N_c, & m = 1, 2, ..., N_c, \\ P_{s,n}^* = P_s/K_s, & n = 1, 2, ..., K_s, \end{cases} \tag{8}$$

and $\widetilde{y}^* = (K_J, \boldsymbol{\omega}_J^*, \boldsymbol{P}_J^*)$ with

$$\begin{cases} \omega_{J,m}^* = K_J/N_c, & m = 1, 2, ..., N_c, \\ P_{J,n}^* = P_J/K_J, & n = 1, 2, ..., K_J. \end{cases} \tag{9}$$

In this case, the minimax capacity of the authorized user can be obtained as

$$\begin{aligned} \widetilde{C}(K_s, K_J) =& K_s \frac{K_J}{N_c} \frac{B}{N_c} \log_2 \left(1 + \frac{P_s/K_s}{P_J/K_J + P_N/N_c}\right) \\ &+ K_s \left(1 - \frac{K_J}{N_c}\right) \frac{B}{N_c} \log_2 \left(1 + \frac{P_s/K_s}{P_N/N_c}\right). \end{aligned} \tag{10}$$

Proof: (1) We first prove that $(\widetilde{x}^*, \widetilde{y}^*)$ defined in (8) and (9) satisfy the left part of (7), $C(\widetilde{x}, \widetilde{y}^*) \leq C(\widetilde{x}^*, \widetilde{y}^*)$. Assuming the jammer applies the strategy $\widetilde{y}^*$ with uniform subchannel selection and uniform power allocation as indicated in (9). For the authorized user who applies an arbitrary strategy $\widetilde{x} \in \widetilde{\mathcal{X}}$, we numbered the activated K_s subchannels as $n = 1, 2, ..., K_s$. For each subchannel activated by the authorized user, the probability that it is jammed is K_J/N_c, since the jammer jams each subchannel with a uniform probability $\omega_{J,m}^* = K_J/N_c$. Accordingly, the probability that each subchannel is not jammed is $1 - K_J/N_c$.

Considering all the subchannels activated by the authorized user, when the authorized user applies an arbitrary strategy $\widetilde{x} \in \widetilde{\mathcal{X}}$, and the jammer applies strategy $\widetilde{y}^*$, the average capacity can be calculated as

$$\begin{aligned} C(\widetilde{x}, \widetilde{y}^*) =& \sum_{n=1}^{K_s} \left[\frac{K_J}{N_c} \frac{B}{N_c} \log_2 \left(1 + \frac{P_{s,n}}{P_J/K_J + P_N/N_c}\right)\right. \\ &\left. + \left(1 - \frac{K_J}{N_c}\right) \frac{B}{N_c} \log_2 \left(1 + \frac{P_{s,n}}{P_N/N_c}\right)\right] \\ =& \frac{K_J}{N_c} \frac{B}{N_c} \sum_{n=1}^{K_s} \log_2 \left(1 + \frac{P_{s,n}}{P_J/K_J + P_N/N_c}\right) \\ &+ \left(1 - \frac{K_J}{N_c}\right) \frac{B}{N_c} \sum_{n=1}^{K_s} \log_2 \left(1 + \frac{P_{s,n}}{P_N/N_c}\right). \end{aligned} \tag{11}$$

Note that $\sum_{n=1}^{K_s} P_{s,n} = P_s$, and applying Lemma 1, we have

$$\begin{aligned} C(\widetilde{x}, \widetilde{y}^*) \leq& K_s \frac{K_J}{N_c} \frac{B}{N_c} \log_2 \left(1 + \frac{P_s/K_s}{P_J/K_J + P_N/N_c}\right) \\ &+ K_s \left(1 - \frac{K_J}{N_c}\right) \frac{B}{N_c} \log_2 \left(1 + \frac{P_s/K_s}{P_N/N_c}\right) \\ =& C(\widetilde{x}^*, \widetilde{y}^*), \end{aligned} \tag{12}$$

where the equality holds if and only if $P_{s,n} = P_s/K_s, \forall n$.

(2) Proof of the right part of (7), $C(\widetilde{x}^*, \widetilde{y}^*) \leq C(\widetilde{x}^*, \widetilde{y})$. Assuming the authorized user applies the strategy $\widetilde{x}^*$ with uniform subchannel selection and uniform power allocation as indicated in (8). For the jammer who applies an arbitrary strategy $\widetilde{y} \in \widetilde{\mathcal{Y}}$, we numbered the jammed K_J subchannels as $n = 1, 2, ..., K_J$. For each jammed subchannel, the probability that it also serves as a subchannel activated by the authorized user is $\omega_{s,m}^* = K_s/N_c$. Hence, the average number of jammed subchannels which are also activated by the authorized user is $K_J K_s/N_c$, and the average number of subchannels activated by the authorized user that are jamming-free would be $K_s - K_J K_s/N_c = K_s(1 - K_J/N_c)$.

Considering both the jammed and jamming-free subchannels, when the jammer applies an arbitrary strategy $\widetilde{y} \in \widetilde{\mathcal{Y}}$, and the authorized user applies strategy $\widetilde{x}^*$, the average capacity can be calculated as

$$\begin{aligned} C(\widetilde{x}^*, \widetilde{y}) =& \sum_{n=1}^{K_J} \frac{K_s}{N_c} \frac{B}{N_c} \log_2 \left(1 + \frac{P_s/K_s}{P_{J,n} + P_N/N_c}\right) \\ &+ K_s \left(1 - \frac{K_J}{N_c}\right) \frac{B}{N_c} \log_2 \left(1 + \frac{P_s/K_s}{P_N/N_c}\right). \end{aligned} \tag{13}$$

Note that $\sum_{n=1}^{K_J} P_{J,n} = P_J$, and applying Lemma 2, we have

$$\begin{aligned}C(\widetilde{x}^*, \widetilde{y}) \geq & K_s \frac{K_J}{N_c}\frac{B}{N_c}\log_2\left(1+\frac{P_s/K_s}{P_J/K_J+P_N/N_c}\right) \\ & + K_s\left(1-\frac{K_J}{N_c}\right)\frac{B}{N_c}\log_2\left(1+\frac{P_s/K_s}{P_N/N_c}\right) \\ = & C(\widetilde{x}^*, \widetilde{y}^*),\end{aligned} \tag{14}$$

where the equality holds if and only if $P_{J,n} = P_J/K_J, \forall n$. ■

Proposition 1 shows that: when the number of user-activated subchannels and the number of jammed subchannels are both fixed, uniform subchannel selection and uniform power allocation would serve as the best strategies, not only for the authorized user to maximize its capacity but also for the jammer to minimize the capacity of the authorized user.

B. Capacity Optimization over K_s and K_J

In Section III-A, we derived the closed-form minimax capacity of the authorized user for fixed K_s and K_J. Considering all possible K_s and K_J, we would have an $N_c \times N_c$ matrix $\widetilde{\mathbf{C}}$, in which $\widetilde{C}(K_s, K_J)$ is the minimax capacity of the authorized user for fixed K_s and K_J, as indicated in (10). Now finding the minimax capacity in (5) can be reduced to finding the saddle point of the matrix $\widetilde{\mathbf{C}}$. Note that the saddle point of a matrix $\mathbf{A}$ is an entry $a_{i,j}$, which is simultaneously the minimum of the ith row and the maximum of the jth column.

***Lemma* 3.** *For a real-valued function $f(z) = \ln(1+z) - z/(1+z)$, $f(z) > 0$, when $z > 0$.*

Proof: When $z > 0$, $f'(z) = z/(1+z)^2 > 0$. Thus, $f(z) > f(0) = 0$. ■

***Lemma* 4.** *For the capacity function*

$$\begin{aligned}\widetilde{C}(K_s, K_J) = & K_s \frac{K_J}{N_c}\frac{B}{N_c}\log_2\left(1+\frac{P_s/K_s}{P_J/K_J+P_N/N_c}\right) \\ & + K_s\left(1-\frac{K_J}{N_c}\right)\frac{B}{N_c}\log_2\left(1+\frac{P_s/K_s}{P_N/N_c}\right),\end{aligned} \tag{15}$$

we have

$$\frac{\partial \widetilde{C}}{\partial K_s} > 0 \;\; \& \;\; \frac{\partial \widetilde{C}}{\partial K_J} < 0. \tag{16}$$

Proof: (1) The first-order derivative of $\widetilde{C}$ over K_s,

$$\begin{aligned}\frac{\partial \widetilde{C}}{\partial K_s} = & \frac{K_J}{N_c}\frac{B}{N_c}\frac{1}{\ln 2}\left[\ln\left(1+\frac{\frac{P_s}{K_s}}{\frac{P_J}{K_J}+\frac{P_N}{N_c}}\right) - \frac{\frac{P_s}{K_s}}{\frac{P_s}{K_s}+\frac{P_J}{K_J}+\frac{P_N}{N_c}}\right] \\ & + \left(1-\frac{K_J}{N_c}\right)\frac{B}{N_c}\frac{1}{\ln 2}\left[\ln\left(1+\frac{\frac{P_s}{K_s}}{\frac{P_N}{N_c}}\right) - \frac{\frac{P_s}{K_s}}{\frac{P_s}{K_s}+\frac{P_N}{N_c}}\right].\end{aligned} \tag{17}$$

Let $z_1 = \frac{P_s/K_s}{P_J/K_J+P_N/N_c}$, then $\frac{z_1}{1+z_1} = \frac{P_s/K_s}{P_s/K_s+P_J/K_J+P_N/N_c}$. Similarly, let $z_2 = \frac{P_s/K_s}{P_N/N_c}$, then $\frac{z_2}{1+z_2} = \frac{P_s/K_s}{P_s/K_s+P_N/N_c}$. Applying Lemma 3 to (17), we have

$$\frac{\partial \widetilde{C}}{\partial K_s} > 0. \tag{18}$$

(2) The first-order derivative of $\widetilde{C}$ over K_J,

$$\begin{aligned}\frac{\partial \widetilde{C}}{\partial K_J} = & \frac{K_s}{N_c}\frac{B}{N_c}\frac{1}{\ln 2}\Bigg[\ln\left(1+\frac{\frac{P_s}{K_s}}{\frac{P_J}{K_J}+\frac{P_N}{N_c}}\right) - \ln\left(1+\frac{\frac{P_s}{K_s}}{\frac{P_N}{N_c}}\right) \\ & + \frac{\frac{P_s}{K_s}\frac{P_J}{K_J}}{\left(\frac{P_s}{K_s}+\frac{P_J}{K_J}+\frac{P_N}{N_c}\right)\left(\frac{P_J}{K_J}+\frac{P_N}{N_c}\right)}\Bigg] \\ < & \frac{K_s}{N_c}\frac{B}{N_c}\frac{1}{\ln 2}\Bigg[\frac{\frac{P_s}{K_s}\frac{P_J}{K_J}}{\frac{P_N}{N_c}\left(\frac{P_s}{K_s}+\frac{P_J}{K_J}+\frac{P_N}{N_c}\right)+\frac{P_s}{K_s}\frac{P_J}{K_J}} \\ & - \ln\left(1+\frac{\frac{P_s}{K_s}\frac{P_J}{K_J}}{\frac{P_N}{N_c}\left(\frac{P_s}{K_s}+\frac{P_J}{K_J}+\frac{P_N}{N_c}\right)}\right)\Bigg].\end{aligned} \tag{19}$$

Let $z_0 = \frac{P_s/K_s P_J/K_J}{P_N/N_c(P_s/K_s+P_J/K_J+P_N/N_c)}$, then $\frac{z_0}{1+z_0} = \frac{P_s/K_s P_J/K_J}{P_N/N_c(P_s/K_s+P_J/K_J+P_N/N_c)+P_s/K_s P_J/K_J}$. Applying Lemma 3 to (19), we have

$$\frac{\partial \widetilde{C}}{\partial K_J} < 0. \tag{20}$$

■

Following Lemma 4, we have the proposition below.

***Proposition* 2.** *The saddle point of matrix $\widetilde{\mathbf{C}}$ is indexed by $(K_s^*, K_J^*) = (N_c, N_c)$. Equivalently, for all $1 \leq K_s, K_J \leq N_c$, we have*

$$\widetilde{C}(K_s, N_c) \leq \widetilde{C}(N_c, N_c) \leq \widetilde{C}(N_c, K_J). \tag{21}$$

C. The Minimax Capacity

In Section III-A, we derived the saddle point strategies and the corresponding minimax capacity for fixed K_s and K_J; in Section III-B, the gaming relationship over different K_s and K_J was investigated, and we found the saddle point strategies indicating how to choose K_s and K_J for the authorized user and the jammer, respectively. Consequently, exploring all possible strategies in the defined strategy spaces $\mathcal{X}$ and $\mathcal{Y}$, we manage to find the saddle point strategies to the original minimax problem in (5). The result is summarized in the theorem below.

***Theorem* 1.** *Assuming that an authorized user and a jammer are operating independently over the same AWGN channel consisting of N_c subchannels. Either for the authorized user to maximize its capacity, or for the jammer to minimize the capacity of the authorized user, the best strategy for both of them is to distribute the power uniformly over all the N_c subchannels. In this case, the minimax capacity is given by*

$$C = B\log_2\left(1+\frac{P_s}{P_J+P_N}\right), \tag{22}$$

where B is the bandwidth of the overall spectrum, P_N the noise power, P_s and P_J the total power for the authorized user and the jammer, respectively.

Proof: The proof follows directly from Proposition 1 and 2. The minimax capacity in (22) can be derived simply by substituting $K_s = K_J = N_c$ into (10). ■

IV. Numerical Results

In this section, we evaluate the impact of different strategies applied by the authorized user and the jammer on the capacity of the authorized user. We assume $N_c = 64$, $B = 1$ MHz, $P_s = P_J = 16$ W, and the overall SNR is 10dB. In light of Proposition 1, we assume that both the authorized user and the jammer apply uniform subchannel selection.

A. Capacity v.s. Power Allocation with Fixed K_s and K_J

We evaluate the capacity of the authorized user under different transmission and jamming power allocation schemes. Performing power allocation, we set the power allocation vector as one whose elements, if sorted, would form an arithmetic sequence, and we use the maximum power difference among all the selected subchannels as the metric of uniformity. Hence, the maximum power difference indicates how far the power allocation is away from being uniform, and a zero difference means uniform power allocation. Here we evaluate the capacity in two cases: (1) uniform jamming power allocation, while the power allocation for the authorized user is nonuniform; (2) the case which is opposite to (1).

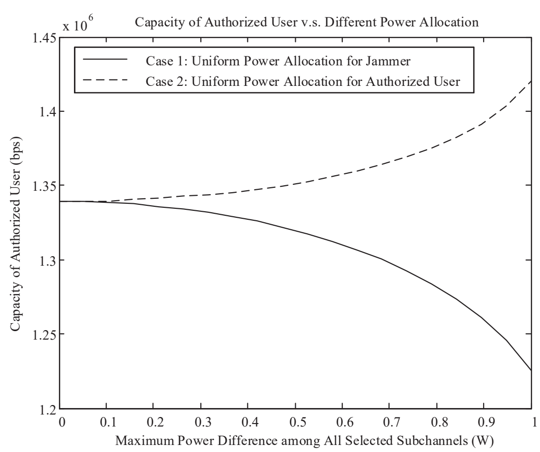

Fig. 1. Capacity v.s. different power allocation.

Fig. 1 shows the results when both the authorized user and the jammer select half of all the available subchannels. It can be seen that: (1) if the jammer applies uniform power allocation, the authorized user maximizes its capacity when it applies uniform power allocation as well; (2) if the authorized user applies uniform power allocation, the jammer minimizes the capacity of the authorized user when it applies uniform power allocation as well; (3) the minimax capacity (the intersection in the figure) serves as a lower bound when the authorized user applies uniform power allocation under all possible jamming power allocation schemes, and meanwhile an upper bound when the jammer applies uniform power allocation under all possible transmission power allocation schemes. The results above match well with Proposition 1.

B. Capacity v.s. Number of Selected Subchannels

We evaluate the capacity of the authorized user with different number of selected subchannels by the authorized user or the jammer. For each (K_s, K_J) pair, both the authorized user and the jammer apply uniform power allocation. It is observed in Fig. 2 that the best strategy is to utilize all the N_c subchannels, either for the authorized user to maximize its capacity, or for the jammer to minimize the capacity of the authorized user. This result matches well with Proposition 2.

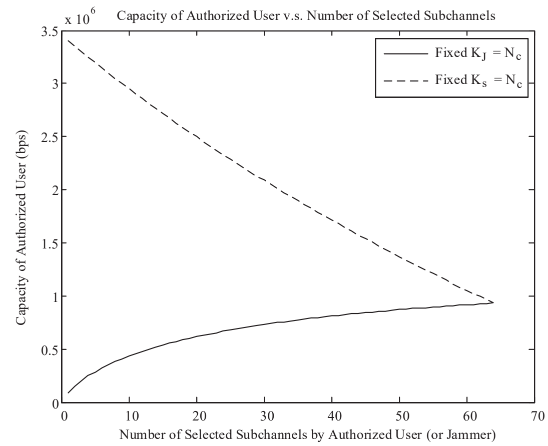

Fig. 2. Capacity v.s. number of selected subchannels.

V. Conclusions

In this paper, we investigated a game between a power-limited authorized user and a power-limited jammer, who operate independently over the same AWGN channel consisting of multiple bands. Both theoretical analysis and numerical results demonstrated that: either for the authorized user to maximize its capacity, or for the jammer to minimize the capacity of the authorized user, the best strategy for both of them is to distribute the transmission power or jamming power uniformly over all the available spectrum.

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