

APPLICATIONS OF RELIABILITY ANALYSIS TO POWER ELECTRONICS SYSTEMS

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ABSTRACT

Reliability and cost considerations play an important role in the choice of various alternatives. These could be alternate designs to meet an objective or could be alternate planning scenarios. To be able to make trade-offs between cost and reliability, reliability needs to be quantified and there need to be methods for predicting the reliability. This paper describes three techniques of reliability analysis, a technique based on Markov processes, cut set method and Monte Carlo simulation. These techniques are illustrated using two examples. The Markov process is illustrated by application to study fault tolerant features in PEBB based converter configurations. Similarly the Monte Carlo simulation and cut set techniques are illustrated by application to standby power systems including uninterruptible power supply systems.

BACKGROUND

This section gives a brief introduction of the reliability evaluation methods. There are many methods available in the literature; here three methods will be reviewed. The two analytical methods are the Markov processes and cut sets and one form of Monte Carlo Simulation is also described.

ANALYTICAL METHOD

The analytical method described here uses the concepts of Markov chains and Markov cut-sets. A detailed treatment of these techniques is available in [1]; a summary of the methods is presented here.

Markov Chains

A Markov chain is a sequence of events consisting of the transition of a component or system of components amongst a set of states such that the state to which the component or system transits in the future depends only on the current state of the component or system and not on the states it has transited

through in the past. If a system conforms to (or, more realistically, approximates) this behavior, then the theory of Markov chains may be applied to it. In this kind of analysis, information of interstate transition rates is used to determine the rate of transiting to failure states.

The transition rate from state i to state j is the mean rate of the system passing from state i to state j . In this paper, the durations of component states are assumed exponentially distributed, which means that the interstate transition rates are constant. This is explained in the next section.

The state probabilities can be obtained by solving

$$BP = C \quad (1)$$

where B matrix obtained from A by replacing the elements of an arbitrarily selected row k by 1;

A matrix of transition rates such that the element $a_{ij} = \lambda_{ji}$ and $a_{ii} = -\sum_j a_{ji}$;

λ_{ij} constant transition rate from state i to j ;

P column vector whose i th term p_i is the steady-state probability of the system being in state i ;

C column vector with k th element equal to one and other elements set to zero.

Once the steady-state probabilities have been calculated, the reliability indices [1] can be computed using the following relationships.

Frequency of system failure:

The relationship for the frequency of system failure [1] is given by Eq. (2) or Eq. (3)

$$f_f = \sum_{i \in (S-F)} p_i \times \sum_{j \in F} \lambda_{ij} \quad (2)$$

$$= \sum_{i \in F} p_i \times \sum_{j \in (S-F)} \lambda_{ij} \quad (3)$$

where f_f is the frequency of the system failure, S is the system state space and F is the subset of failed states.

Mean Down Time:

The expected time of stay in F in one cycle is

$$r_f = P_f / f_f \quad (4)$$

where

$$P_f = \sum_{i \in F} p_i \quad (5)$$

Cut Sets

A cut set is a set of components that, if removed from the system, results in loss of power to the load point. If this set does not contain any cut set as a subset, then it is called a minimal cut set. In this definition, component is used in a wider sense of hardware or a particular system condition. In general cut sets containing m components are called m -order cut sets. Generally up to second order cut sets are considered and the contribution from higher order cut sets is considered negligible. The following relationships are important in cut set calculations.

1) Frequency and duration of a cut set

(a) First order cut set [6] :

$$f_{cs_i} = \lambda_i \quad (6)$$

$$r_{cs_i} = r_i \quad (7)$$

where f_{cs_i}, r_{cs_i} = frequency and mean duration of cut set i

λ_i, r_i = failure rate and mean duration of component comprising cut set i

(b) Second order cut set [6]

$$f_{cs_i} = \lambda_i \lambda_j (r_i + r_j) \quad (8)$$

$$r_{cs_i} = r_i r_j / (r_i + r_j) \quad (9)$$

where λ_i, λ_j = failure rates of components i and j comprising cut set i

r_i, r_j = mean failure durations of components i and j comprising cut set i

(c) Conditional second order cut set [6]:

Assuming that component i fails given component j has failed,

$$f_{cs_i} = \lambda_i \lambda_j (r_j) \quad (10)$$

$$r_{cs_i} = r_i r_j / (r_i + r_j) \quad (11)$$

The terms *failure frequency* and *failure rate* are often used interchangeably. Strictly speaking failure rate is the mean number of failures per unit of the up time and failure frequency is the mean number of failures per unit of the total time. If the probability of system being up is close to unity, then these two quantities are very close. In general, failure frequency is less than failure rate.

2) Frequency and duration of interruption [6]:

$$f_s = \text{frequency of load interruption} = \sum_{\forall i} f_{cs_i} \quad (12)$$

$$r_s = \text{mean duration of load interruption} = \sum f_{cs_i} r_{cs_i} / f_s \quad (13)$$

The theory concerning Markov chains and minimal cut-sets is dealt with in greater detail in (4). These concepts are used as follows to analyze the sample system.

Monte Carlo simulation

This is another method of determining reliability indices. This method is described in detail in [1], while an application of this method to the sample system is given in [9]. A summary of the method is presented here.

The reliability indices of an actual physical system can be estimated by collecting data on the occurrence of failures and the durations of repair. The Monte Carlo method mimics the failure and repair history of the components and the system by using the probability distributions of the component state durations. Statistics are then collected and indices estimated using statistical inference.

Though there are different ways of implementing Monte Carlo simulations, the technique described here is the *next event* method and is capable of simulating dependent events. This is a sequential simulation method which proceeds by generating a sequence of events using random numbers and probability distributions of random variables representing component state durations. A flowchart is given in Fig. 1.

The input data consists of the failure rate (λ) and mean down time (r) of every component. The failure rate is the reciprocal of the mean up time. The mean down time is the reciprocal of the repair rate (μ). The failure and repair rates, λ and μ , of a component will be used to determine how long the component will remain in the 'UP' state and the 'DOWN' state.

Simulation could be started from any system state, but it is customary to begin simulation with all the components in the 'UP' state.

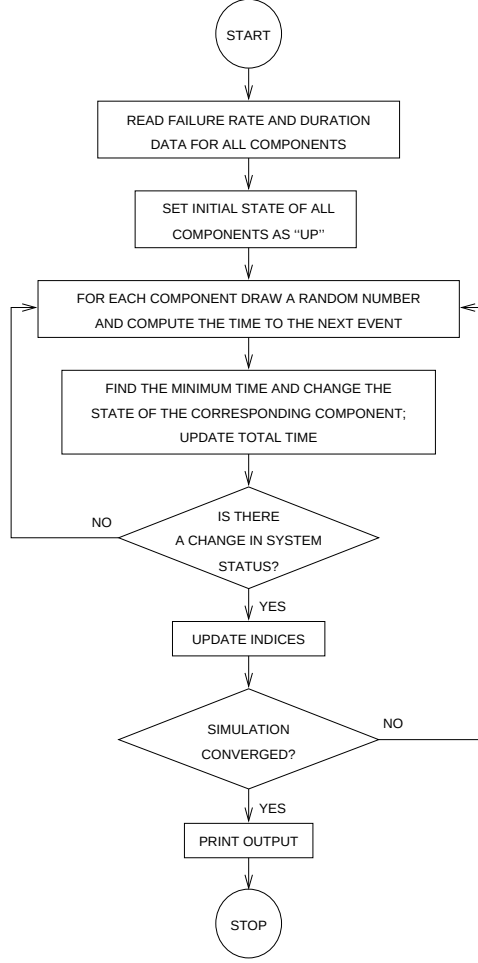


Fig. 1. Flowchart for next-event simulation.

The time to the next event is generated by using the inverse of probability distribution method. This is explained as follows.

If the transition times of the components are assumed to be exponentially distributed:

$$f(t) = \rho e^{-\rho t} \quad (14)$$

where ρ is the transition rate. The mean transition time is, therefore,

$$\int_0^{\infty} f(t) dt = \frac{1}{\rho} \quad (15)$$

This means that if, for instance, a component is UP, then, regardless of how long it has been in the UP state, the expected time to the next failure is $1/\lambda$, i.e., the mean up time.

The *probability distribution function* of the transition time T would be

$$F(t) = P(t \leq T) = \int_0^t \rho e^{-\rho\tau} d\tau = 1 - e^{-\rho t} \quad (16)$$

Now $F(t)$ can be regarded as a random variable, uniformly distributed between 0 and 1. This means that the survivor function

$$S(T) = 1 - F(T) = e^{-\rho T} \quad (17)$$

is also uniformly distributed between 0 and 1. So if a random number R_n is generated, $0 \leq R_n \leq 1$, it can be associated with the event that the next transition occurs after time t_r , given by

$$R_n = e^{-\rho t_r}, \text{ that is, } t_r = -\frac{\ln(R_n)}{\rho} \quad (18)$$

This method is used to determine the time to the next transition for every component, using λ or μ for ρ , depending on whether the component is UP or DOWN.

At the end of any simulated time interval $[0, t]$, where t = total up time in $[0, t]$ + total down time in $[0, t]$, the estimates of the reliability indices are given as follows.

failure rate:
$$\lambda_t = \frac{\text{number of failures in } [0, t]}{\text{total up time in } [0, t]} \quad (19)$$

mean down time:
$$r_t = \frac{\text{total down time in } [0, t]}{\text{number of failures in } [0, t]} \quad (20)$$

The values of λ and r at the instant the simulation converges are the reliability indices for the system as obtained from the Monte Carlo method. The simulation is said to have converged when the indices attain stable values. This ‘stabilization’ of the value of an index i is measured by its standard error, defined as:

$$\eta = \frac{\sigma_i}{\sqrt{n_c}} \quad (21)$$

where σ_i = standard deviation of the index i

n_c = number of cycles simulated

Convergence is said to occur when the standard error drops below a prespecified fraction, ε , of the index i , i.e., when

$$\eta \leq \varepsilon i \quad (22)$$

If, for instance, the mean down time r is chosen as the index to converge upon, then, after every system restoration simulated, the following relation is tested for validity:

$$\frac{\sigma_r}{\sqrt{n_c}} \leq \varepsilon r \quad (23)$$

If this criterion is satisfied, the simulation is said to have converged.

Simulation is advantageous in that it not only allows the computation of indices at various points in the system, but also permits the accumulation of data pertaining to the distribution of these indices, thereby affording a better understanding of the system behavior.

APPLICATIONS

The techniques described in the paper are illustrated by two applications. The first application is to model fault tolerant features in the PEBB- based converters proposed in ref. [3]. The second application is the study of reliability improvement by using UPS as standby power

MODELING FAULT TOLERANT FEATURES IN THE PEBB-BASED CONVERTERS [3]

In this section a modeling approach is presented to analyze the fault tolerant behavior of PEBB based converters proposed in [3]. Fig. 2 (a) shows a detailed connection diagram of combining three, 3-phase inverter blocks with 18 IGBT switches. The interconnection inductances between the modules limit the high frequency circulating currents. Fig. 2 (b) shows the vector diagram of the fundamental voltages generated by IGBT switch pairs.

1. Fault Tolerant Features & Reliability Analysis:

From Fig. 2 (a) and (b) it is clear that failure (open circuit) of IGBT switches 3,6 and 7,10 or 2,5 and 13,16 or 15,18 and 8,11 results in an open delta situation and the modular inverters are still capable of supplying the load at a reduced power level. However, the failure of IGBT switch pairs 1,4 or 9,12 or 14,17 will cause a critical failure and result in a system shut down.

The trade off between reliability and cost can be achieved by the knowledge of cost of interruptions and constructing reliability models and computing the reliability indices. The two indices that are useful in this situation are:

1. Frequency of failure: This is the mean number of failures per year.
2. Mean duration of failure: This is the mean duration of a failure event.

The cost of interruption in some applications depends on the total down time whereas in other applications the frequency of interruptions is also important.

A central problem in the optimization of the overall cost is developing a reliability model that reflects the factors that effect the reliability. Some approaches for constructing this model are discussed in [8-10]. A suitable approach would be to use of Markov Processes to model these configurations. In this approach, various states resulting from the failure and repair or replacement of components are identified. Then the interstate transition rates are assigned that are functions of the failure rates and repair times of the components. The state probabilities and other indices are then calculated using the mathematical techniques described in [1-2].

2. State Transition Diagram:

The state transition diagram for the topology shown in Fig. 2 (a) is shown in Fig. 3. The various parameters are:

λ = Failure rate of one leg of the inverter.

μ_1 = Repair rate when only one leg is failed.

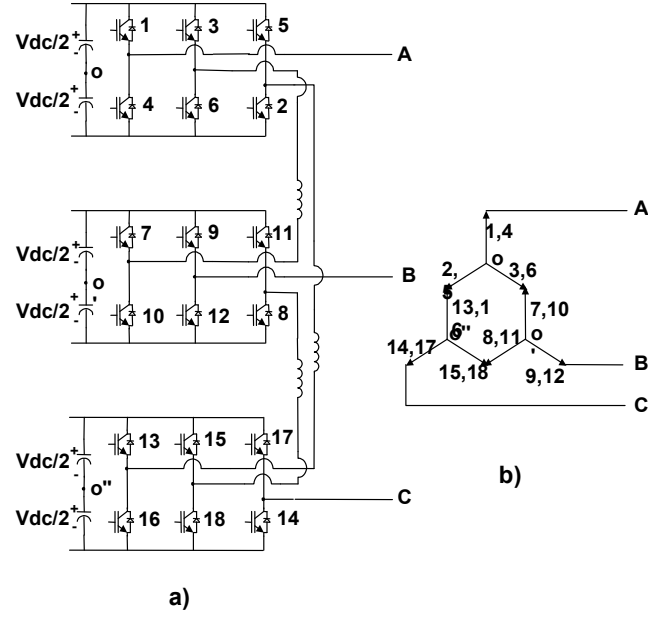
μ_2 = Repair rate when two legs are failed.

μ_3 = Repair rate when three legs are failed.

μ_I = Repair rate when failure is detected as a result of inspection before the configuration failure.

In developing this state transition model, following assumptions are made:

1. Once the configuration is failed, there are no more component failures as the configuration will not be powered.
2. There is some process of inspection or detection of failures even before the configuration is completely down.
3. The configuration is either up or down. It is, however, possible to model degraded modes of failure.



$$\text{Frequency of system failure} = 3\lambda P_1 + 7\lambda P_3 + 7\lambda P_4$$

$$\text{Mean Down Time} = \text{System unavailability} / \text{Frequency of system failure}.$$

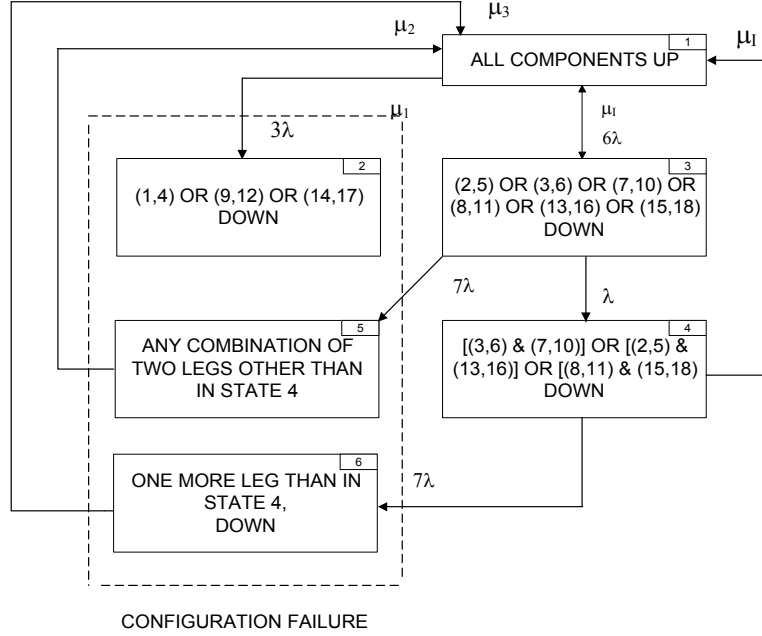


Fig. 3. State transition diagram of configuration described in Fig. 2 (a)

STANDBY POWER SYSTEM INCLUDING UPS

System Description

The test system is shown in Fig. 4. This system is the same as the one analyzed in [8, 9]. The power is normally supplied from utility input power through the UPS. A synchronized bypass and static switch protect the critical load in the event of an inverter failure. If voltage is lost to the critical load, the STS reestablishes voltage in less than one quarter of a cycle. This is considered continuous power for most loads.

When the generators are in standby mode, their failures remain undetected except during periodic inspections. Therefore, while starting, there is probability p_s that a generator may fail to start.

Only one generating unit is taken out for planned maintenance. If a generator fails while the other is on planned maintenance, it is possible to accelerate the maintenance on the second generator by a factor of α .

If power fails at bus A, the battery can sustain the load for upto 4 hours.

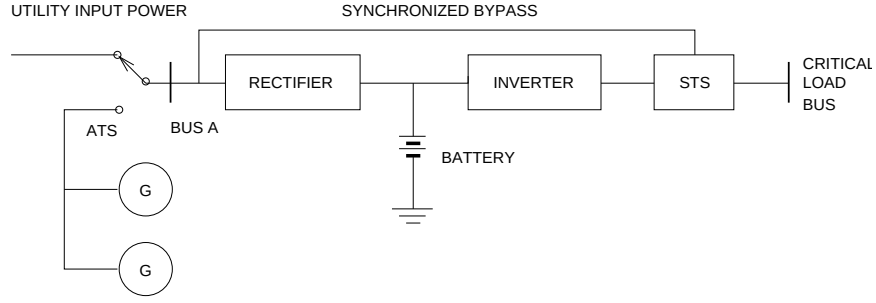


Fig. 4. Parallel supplied non-redundant uninterruptible power supply

TABLE I lists the data pertaining to the system.

Simulation Model

The flowchart shown in Fig. 1 was implemented. In computing the time to the next event, the method described earlier was used, but the following dependencies were also included:

1. A generator cannot fail while the utility supply is up
2. A generator cannot be taken out for maintenance if the utility is down or if the other generator is down or under maintenance
3. The UPS cannot be taken out for maintenance unless power is available at bus A

TABLE I: SYSTEM DATA

Equipment/Supply	λ (f/y)	r (h/f)
Utility Supply, single circuit	0.53700	5.66
Generator (per hour of use)	0.00536	478.00
Inverter	1.25400	107.00
Rectifier	0.03800	39.00
ATS	0.00600	5.00
STS	0.08760	24.00
Battery	0.03130	24.00
Equipment Maintenance	$freq$ (/y)	dur (h)
Generator	1.00	10.00
UPS	1.00	4.00
Other data:		
Battery can supply load for		4.0 h
Common mode failure of generators	(λ_{cm})	0.0
Acceleration factor for planned maintenance of generators	(α)	2.0
Probability of failure to start a generator	(p_s)	0.015

The failure of a generator to start was modeled as follows. A random number z was generated, $0 \leq z \leq 1$. If $z \leq p_s$, the generator was assumed to fail.

Results

In this section the indices obtained from simulation are compared with those obtained analytically, using cut set analysis, in [8].

Cut Set Analysis

The reliability indices of the test system have been analytically evaluated, using the cut set approach, in [5]. A summary of the solution steps and the results will be presented here, to provide a basis for validation of the simulation technique.

First, the combination of the utility supply and the two generators is analyzed for failure modes. Fig. 5 shows the possible states this combination can assume, and the transition rates between these states. Based on these transition rates, the probabilities and frequencies of occurrence of the failed states are determined. This enables computation of the failure rate and duration of the utility-generator subsystem, which are determined to be

$$\lambda_p = 0.001576/y \quad \text{and} \quad r_p = 5.443h$$

The next step involves computation of the rate and duration of power failure at bus A:

$$\lambda_A = \lambda_p + \lambda_{ATS} = 0.007576/y$$

$$r_A = \frac{\lambda_p r_p + \lambda_{ATS} r_{ATS}}{\lambda_p + \lambda_{ATS}} = 5.092h$$

The remainder of the cut set analysis is performed as shown in TABLE II. The system indices, i.e., rate and duration of power loss at the Critical Load Bus (CLB) are determined to be

$$\lambda_{CLB} = \sum_{\text{cut-sets}} \lambda = 0.005225/y$$

$$r_{CLB} = \frac{\sum \lambda r}{\sum \lambda} = 9.648h$$

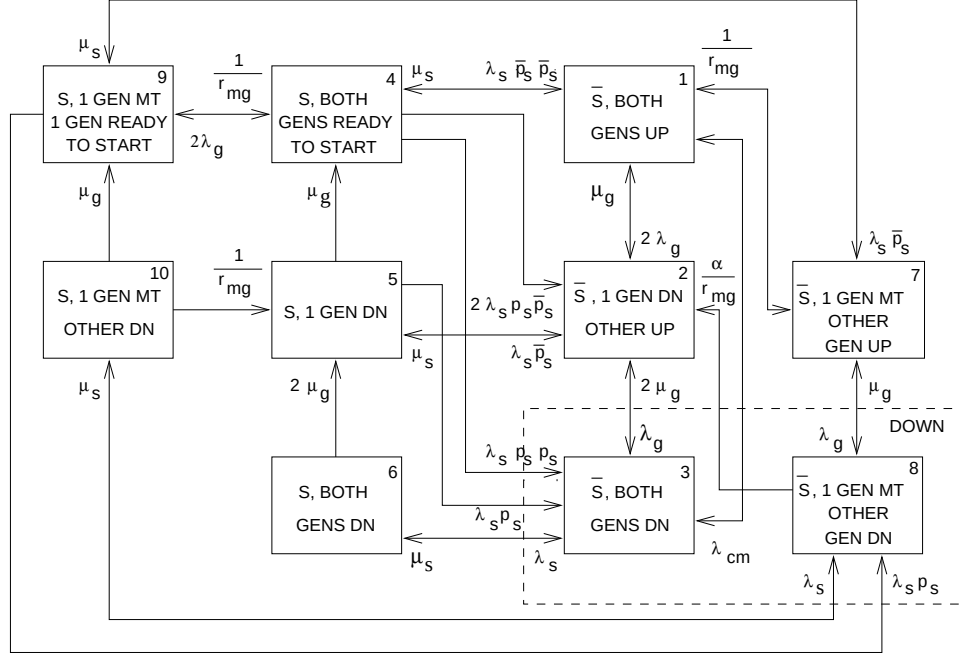


Fig. 5. State transition diagram for utility supply and generators

If all distributions are assumed exponential, then the standard deviations of all the up times and down times would equal the corresponding mean up times and down times. This implies that the standard deviations of the failure rates would also equal the corresponding means computed.

TABLE II: FREQUENCY AND DURATION OF POWER LOSS AT CRITICAL LOAD BUS (CLB)

Cut Set	λ (f/y)	r (h/f)	λr
Power loss at bus A > 4 h	0.003454	5.092	0.017588
Power loss at bus A (0.007576, 5.092)	0.000125	4.845	0.000606
and Failure of [Inverter or battery or STS] (1.3729, 99.812)			
Maintenance on UPS (1.0, 4)	0.000003	2.240	0.000007
and Power loss at bus A (0.007576, 5.092)			
Inverter failure(1.254, 107.0)	0.001643	19.603	0.032208
and STS failure(0.0876, 24.0)			
Σ	0.005225		0.050409

Simulation Results

The simulation method described earlier was used to compute the following statistics for the test system:

1. Frequency λ_p and duration r_p of failure of the utility-generator subsystem (Fig. 4); the standard deviations of λ_p and r_p .

2. Frequency λ_A and duration r_A of power failure at bus A; the standard deviations of λ_A and r_A .
3. Frequency λ_{CLB} and duration r_{CLB} of power failure at the critical load bus; the standard deviations of λ_{CLB} and r_{CLB} .
4. Data for the *Probability Mass Function* of the number of system failures per year, N_f .
5. Data for the *Probability Distribution Function* of the system down time, T_f .

TABLE III compares the indices obtained from simulation with those obtained analytically.

TABLE III: COMPARISON OF SIMULATED AND CALCULATED INDICES

Index	Simulated		Calculated	
	Mean	SD	Mean	SD
λ_p (f/y)	0.001352	0.001497	0.001576	0.001576
r_p (h/f)	5.028	5.503	5.443	5.443
λ_A (f/y)	0.007193	0.006851	0.007576	0.007576
r_A (h/f)	5.216	5.201	5.092	5.092
λ_{CLB} (f/y)	0.004868	0.004888	0.005225	0.005225
r_{CLB} (h/f)	9.7135	13.3312	9.648	9.648

TABLE IV compares the PMF of the number of system failures per year. Now for exponentially distributed up times the failures are Poisson distributed, i.e.,

$$P(N_f = k) = \frac{\lambda_{CLB}^k e^{-\lambda_{CLB}}}{k!} \quad (31)$$

This equation is used to generate the calculated data for the PMF of N_f , in TABLE IV.

Fig. 6 compares the PDF of the system down time. For exponentially distributed down time T_f , the PDF is given by

$$F(t) = P(T_f \leq t) = 1 - e^{-t/r_{CLB}} \quad (32)$$

This equation is used to generate the calculated data for the PDF of T_f , for Fig 5.

TABLE IV: PMF OF NUMBER OF FAILURES PER YEAR

k	$P(N_f = k)$	
	Simulated	Calculated
0	0.9952	0.9947
1	0.4825×10^{-2}	0.5198×10^{-2}
2	0.2167×10^{-2}	0.1358×10^{-2}
3	0.0000	0.2365×10^{-7}
4	0.0000	0.3089×10^{-10}

CONCLUDING REMARKS

This paper has described how a given configuration can be analyzed for producing a quantitative reliability index. The techniques described can help to quantify reliability of alternate design configurations or the same design but with different quality of components, reflecting different failure rates.. Then the reliability can be traded off with cost to produce configurations which have reliability that that consumer is prepared to pay for.

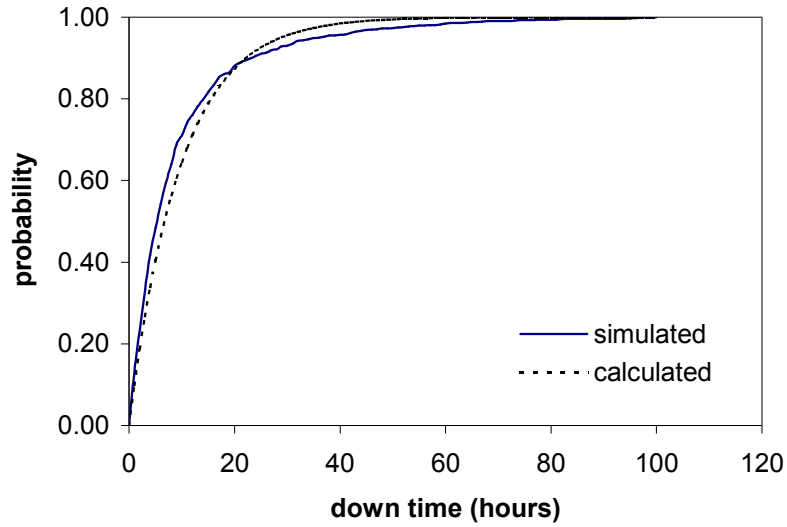


Fig. 6. Probability distribution function of down time

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