

A Survey of Methods of Forming Reduced Order Equivalents for Energy Function Analysis

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Abstract

Energy function analysis, from the perspective of stability assessment of electric power systems, consists of determining the energy injected into a system during a fault, and comparing this with the critical energy to determine whether or not the postfault state of the system will decay into a stable operating point. The critical energy is defined as the lowest of the values of the energy function calculated at all the unstable equilibrium points (*u.e.p.-s*). For a multimachine system of realistic dimensions, enumerating all the *u.e.p.-s* and evaluating the values of the energy function at these points would be a daunting task in terms of the computational labor involved. This necessitates the formation of reduced order equivalents. Several methods have been proposed for achieving this, and this paper attempts to look into the concepts behind some of these methods, specifically those which are independent of the location of the fault. This survey makes no claim to being exhaustive; all it attempts to do is to categorize the divers methods into three classes, outline the basic approach underlying the methods belonging to each category, and cite some representative methods in each class.

1 Introduction

For a m -machine system whose state is described by the vector

$$[\theta_1, \theta_2, \dots, \theta_m, \tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_m]^T$$

with

$$\begin{aligned} \theta_i &= \delta_i - \delta_0, & i &= 1, 2, \dots, m \\ \tilde{\omega}_i &= \omega_i - \omega_0, & \omega_i &= \frac{d\theta_i}{dt} \\ \delta_0 &= \frac{1}{M_T} \sum_{i=1}^m M_i \delta_i, & M_T &= \sum_{i=1}^m M_i \\ \omega_0 &= \frac{1}{M_T} \sum_{i=1}^m M_i \omega_i \end{aligned}$$

the energy function is described by

$$V(\theta, \tilde{\omega}) = \frac{1}{2} \sum_{i=1}^m M_i \tilde{\omega}_i^2 - \sum_{i=1}^m \int_{\theta_i^s}^{\theta_i} M_i \frac{d\tilde{\omega}_i}{dt} d\theta_i \quad (1)$$

where $\theta_i^s = \theta_i$ at postfault stable equilibrium point.

Now for any equilibrium point, stable or otherwise, $\tilde{\omega}_i = 0$, and hence the critical energy can be expressed as

$$V_{cr} = \min \{ V(\theta^u, 0), \forall \text{ u.e.p. } s \} \quad (2)$$

So if V_{inj} is the energy injected during the fault, then the stability criterion would be

$$\begin{aligned} V_{inj} < V_{cr} &\Rightarrow \text{stable} \\ V_{inj} \geq V_{cr} &\Rightarrow \text{unstable} \end{aligned}$$

While this formulation appears reasonable, one must note that the *u.e.p.* for which $V(\theta^u, 0)$ is the lowest might actually lie quite far from the potential energy boundary surface for a given fault, and therefore may not be the *controlling u.e.p.* So the actual critical energy, $V(\theta^{cuep}, 0)$ would be higher than the value of V_{cr} computed from (2). Therefore in addition to (2) being computationally expensive, the result obtained from it also tends to be too conservative.

To circumvent these problems, two approaches have been defined:

1. Determine the *c.u.e.p.* for a given fault, and then calculate $V_{cr} = V(\theta^{cuep}, 0)$
2. Build a reduced order equivalent for the system, and then use equation (2)

Even though the *c.u.e.p.* method seems to be more direct, it should be noted that every fault would have a different *c.u.e.p.*, and determining this is not very straightforward. On the other hand, the reduced order equivalent, once obtained, can usually be applied to all but certain specific faults, and therefore has the advantage of being more generic. In this paper we will classify the various methods for obtaining reduced order equivalents, into three different categories:

1. Identification of coherent groups of generators
2. Partitioning along transmission boundaries
3. Identification of critical modes

Each of these approaches is dealt with in some detail in the following section.

2 Overview of Methods

2.1 Identification of Coherent Groups of Generators

It has been shown [1] that upon occurrence of a fault, the machines in a system separate into two or more groups (a group may comprise a single generator), in such a manner that the trajectories of the angles in a group remain close together. These groups are referred to as coherent groups, or clusters. The number of clusters identified is the order of the reduced equivalent. Several methods have been proposed for this kind of clustering. Some of these are discussed below.

2.1.1 Clustering of Trajectories from Linear Simulation

If the trajectories of all the machines in a system were to be actually determined using conventional transient stability simulation, and the machines clustered based on these trajectories, the procedure would be extremely cumbersome. Podmore [2] showed that the computational burden could be reduced by forming a simplified, linearized model using only $P - \theta$ relationships and performing a one-step trapezoidal integration within every simulation time step; the trajectories thus obtained, though noticeably different from those obtained by exact simulation, retained the same coherency structure. In other words, the linear simulation produced the same clusters as the exact simulation, while being considerably faster.

Once the coherent groups are identified, the network is reduced by replacing the terminal buses for each group by a single equivalent bus, on which the generation units for the group appear in parallel; the load buses are reduced using Gaussian elimination.

It should be noted, however, that because this method entails a time domain simulation, it is still quite slow and inefficient. The remaining methods described in this section overcome this problem by avoiding the time domain simulation altogether.

2.1.2 Clustering Based on Linearized Dynamic Model

Consider the second order differential equation describing the electromechanical dynamic model:

$$M\ddot{\delta} + D\dot{\delta} + F(\delta) = P_m \quad (3)$$

where

- δ = vector of machine angles
- M = $(m \times m)$ diagonal matrix of machine moments of inertia
- D = $(m \times m)$ diagonal matrix of machine damping coefficients
- $F(\cdot)$ = vector function describing dependence of generated real power on machine angles
- P_m = vector of mechanical power inputs into the machines; this is assumed to remain constant through the duration of the fault

Linearizing (3) around the postfault stable equilibrium point δ^s gives us the linearized dynamic model:

$$M(\Delta\ddot{\delta}) + J(\Delta\delta) = 0$$

$$\Rightarrow (\Delta\ddot{\delta}) + M^{-1}J(\Delta\delta) = 0 \quad (4)$$

where $J = \frac{dF}{d\delta}$

Notice that the damping term has vanished, since $\dot{\delta}^s = 0$.

The matrix $[M^{-1}J]$ describes the system Jacobian for the dynamic stability model (4), and can be subjected to eigenvalue analysis for stability determination in the same manner as linear systems. Though this kind of dynamic stability analysis would not necessarily yield the same results as transient stability analysis, clustering methods based on the system Jacobian $[M^{-1}J]$ would be computationally less expensive than the transient simulation method, and can be reasonably expected to yield the same coherency structure.

Noting that the row structure of the system Jacobian $[M^{-1}J]$ is indicative of the coupling between the machine angles, one simple way of clustering the machines would be to do the following:

1. choose an appropriate threshold ϵ
2. beginning with the smallest element in each row, continue to add up the elements, in ascending order, while the sum remains less than ϵ ; at the same time, replace each element added by zero
3. check if the resulting matrix assumes a block diagonal form; if yes, stop; otherwise increase ϵ and repeat the above steps

The machines belonging to each block in the resulting matrix form a cluster. Notice that the number of clusters formed is related to the value of ϵ by a staircase type function, a larger value of ϵ producing a larger number of clusters.

This method is simple, but since it does not examine the oscillatory modes of the system, it may not always produce dynamically coherent clusters. The methods discussed next perform the clustering based on the system modes.

2.1.3 Clustering Based on System Modes

If the right eigenvectors of the system Jacobian $[M^{-1}J]$ were juxtaposed to form the modal matrix, and if the component vectors of the modal matrix could be rearranged to produce a block diagonal structure, then a clustering operation could be performed based on these blocks. However, the modal matrix does not usually decouple so easily.

Alternatively, we can see if the eigenvalues appear in clusters, i.e., if they appear in groups which are separated by relatively large frequency 'gaps', then the clustering of oscillatory modes becomes apparent.

With regard to these two approaches, it is appropriate to point out that a Jordan canonical decomposition of the Jacobian of a multimachine system of realistic size would again be a formidable task.

A clustering method [3] based on an rms coherency measure and a random system disturbance is claimed to be quite robust. This method has the advantage that not only does it obviate the need for performing a Jordan decomposition, it also avoids the calculation of the $[M^{-1}J]$ matrix altogether. While the

algorithmic details are available in [3], a brief description of the philosophy of the technique is given below.

The linear model described by (4) is extended to a more general (i.e., not necessarily around an equilibrium point) linear dynamic system model:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (5)$$

where

$$x = \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} \quad (\text{the state vector})$$

$$u = \begin{bmatrix} \Delta P_m \\ \Delta P_l \end{bmatrix} \quad (\text{the input vector})$$

The input vector consists of deviations in the generator mechanical power inputs and the bus loads, and can be used to model various types of system disturbances. Uncertainty in these disturbances is modeled probabilistically by a mean vector and a covariance matrix:

$$m = E\{u\} = \begin{bmatrix} E\{\Delta P_m\} \\ E\{\Delta P_l\} \end{bmatrix} = \begin{bmatrix} m_m \\ m_l \end{bmatrix} \quad (6)$$

$$R = E\{(u - m)(u - m)^T\} = \begin{bmatrix} R_m & 0 \\ 0 & R_l \end{bmatrix} \quad (7)$$

where $E(\cdot)$ is the expectation operator.

Based on the above, an rms coherency measure between any two machines, say the i -th and the j -th, is defined by

$$C_{ij} = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T^n} E\left\{\int_0^T [\Delta\delta_i(t) - \Delta\delta_j(t)]^2 dt\right\}} \quad (8)$$

where n is a positive integer chosen so that the measure is finite but nonzero over the infinite averaging interval.

Once the matrix $[C]$ has been determined, machines are clustered based on the elements of $[C]$, using some clustering algorithm, such as the one described in section 2.1.2. In [3], the computation of the coherency matrix has been shown to be simpler than that of the system Jacobian.

2.2 Partitioning Along Transmission Boundaries

While the clustering methods reviewed thus far followed a node-based approach, there are other methods which follow a branch-based approach. The logic behind this approach is that angle differences across branches provide a measure of the coupling between the machines. A simplistic way of viewing these transmission boundaries would be to think of them as boundaries separating islands which occur when a faulted system loses synchronism.

A method developed by Lotfalian, et al [5] associates a security measure to every possible pair of generator buses, based on angle differences, and then clusters the generators based on these security measures.

An inertial security measure for a single loss of generation contingency on generator i is defined as:

$$S_M^i(k, l) = [\Delta\delta_M(k, i) - \Delta\delta_M(l, i)]^2 \quad (9)$$

where $\Delta\delta_M(k, i)$ is the change in phase angle of the inertial load flow at bus k from base angle $\delta_0(k)$ at bus k for the loss of generation contingency $\Delta P_m(i)$; $\Delta P_m(i)$ is the lost mechanical power into generator i .

The inertial load flow is the load flow solution which distributes the loss of generation mismatch $\Delta P_m(i)$ among all the generators in proportion to their inertias.

A *Contingency Independent Security Measure (CISM)* is defined for every generator bus pair as the sum of the inertial security measures for a set of single contingencies:

$$S_M(k, l) = \sum_{i=1}^m S_M^i(k, l)$$

$$= \sum_{i=1}^m [\Delta\delta_M(k, i) - \Delta\delta_M(l, i)]^2 \quad (10)$$

The *CISM* has been shown to be a measure of the mean square average of the relative stability limit violations on the equivalent branch connecting the k -th and l -th machines, over all loss of generation contingencies.

Having determined the $[S_M]$ matrix, machines are clustered based on the elements of this matrix. Algorithmic details are available in [5].

2.3 Identification of Critical Modes

A method developed by Feliachi [8] seeks to reduce the order of a system by identifying its critical modes of oscillation. This is achieved by defining a quadratic performance measure and determining the contribution of each eigenvalue to the measure. Feliachi has shown that the reduced order model formed by selecting the critical eigenvalues in this fashion produces an impulse response which is quite close to the impulse response of the full-order model.

While [8] describes the theory in detail, we discuss here a simplified embodiment of the method.

A linear model similar to (5) is assumed.

A quadratic performance measure is defined by the function

$$\tilde{J} = \sum_{i=1}^m \int_0^\infty [x^i(t)]^T [x^i(t)] dt \quad (11)$$

where $[x^i(t)]$ is the trajectory of the state vector

$$x = \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}$$

when an impulse function is applied to the i -th machine when all other inputs (in the u -vector) are zero.

The contribution to $\tilde{J}$ of each eigenvalue of the plant matrix $[A]$ is a measure of the criticality of the eigenvalue. The k eigenvalues with the highest contributions to $\tilde{J}$ are selected to form a reduced order model of order k .

The method proceeds as follows:

1. Assuming a linear model similar to (5), the plant matrix $[A]$ is decomposed into the Jordan form: $\Lambda = V^{-1}AV$; if some of the eigenvalues in Λ happen to lie in the

closed right half plane, these are discarded, and only the remaining part of Λ is used in the following calculations

2. The matrices $R = V^T V$ and $G = V^{-1} B$ are computed
3. The matrix $S = \sum_{i=1}^m G_i G_i^T$ is computed, where G_i is the i -th column of G
4. The Lyapunov equation $\Lambda^* P + P \Lambda = -R$ is solved for P
5. The matrix $M = P S$ is computed, and from this matrix, values are calculated for $\tilde{J}_i = | \operatorname{Re}(M_{ii}) | \quad \forall i = 1, 2, \dots, m$
6. The $\tilde{J}_i$ -s are arranged in decreasing order to correspond to the eigenvalues in decreasing order of criticality; to reduce the model to order k , pick the k most critical eigenvalues

While it is true that for a system of realistic size this method would be extremely cumbersome, it has been included in this survey because of the uniqueness of its approach.

3 Other Methods of Obtaining Reduced Order Models

A report on methods of obtaining reduced order models would be incomplete without at least a passing reference to some other significant methods which were not discussed here only because this study was limited to methods of obtaining models which were not fault-specific. In this section allusions will be made to a few such fault-specific models and methods.

There are, in fact, several methods of forming highly reduced order models, notably two-machine equivalents and single machine infinite bus equivalents, and there is ample literature (such as [4]) available on these methods. These methods are aimed at partitioning a system into two, sometimes three, clusters based on some criterion, such as the directions of the initial accelerations of the machines upon the occurrence of a fault.

Some methods, such as [6, 7], partition the system into an 'internal subsystem' and an 'external subsystem' based on the fault location, and then substitute the 'external subsystem' with a low order equivalent.

There are also some methods, such as [9] and [10], which use such concepts as electrical distances and electromechanical distances. The method given in [9] reduces a system to a two or three machine equivalent, and is fault-specific; the method described in [10] is not fault specific, but its applicability to the stability problem is questionable, since it uses only the network impedances, and does not include the machine inertias, which, we know, are major components in the concept of 'distance' as it applies to the stability problem.

4 Conclusion

In this paper, an attempt was made to present an overview of the various methods available for reducing a multimachine system to a lower order model which can be used for energy

function analysis for stability assessment; attention was focused on methods which are not fault-specific.

While some of the methods reviewed herein were developed specifically for application to the stability problem, others, though developed for more generalized applications, were also included because it was felt that they, too, could be applied to this problem.

Finally, in concluding this paper, it is appropriate to remark that while several methods were discussed in varying degrees of detail, no attempt was made to compare these techniques in terms of performance or validity. Some comparison was made of the computational effort involved in some of these methods.

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